Computer algebra independent integration tests

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/6.3.1-c+d-x-^m-a+b-tanh-^n

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [77]. This is test number [171].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 93.51 (72)	% 6.49 (5)
Mathematica	% 92.21 (71)	% 7.79 (6)
Maple	% 89.61 (69)	% 10.39 (8)
Maxima	% 81.82 (63)	% 18.18 (14)
Fricas	% 83.12 (64)	% 16.88 (13)
Sympy	% 38.96 (30)	% 61.04 (47)
Giac	% 59.74 (46)	% 40.26 (31)
Mupad	% 50.65 (39)	% 49.35 (38)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

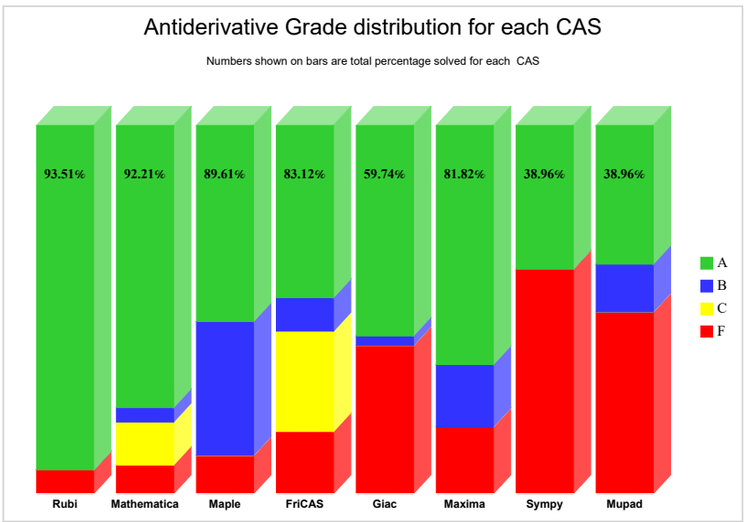
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

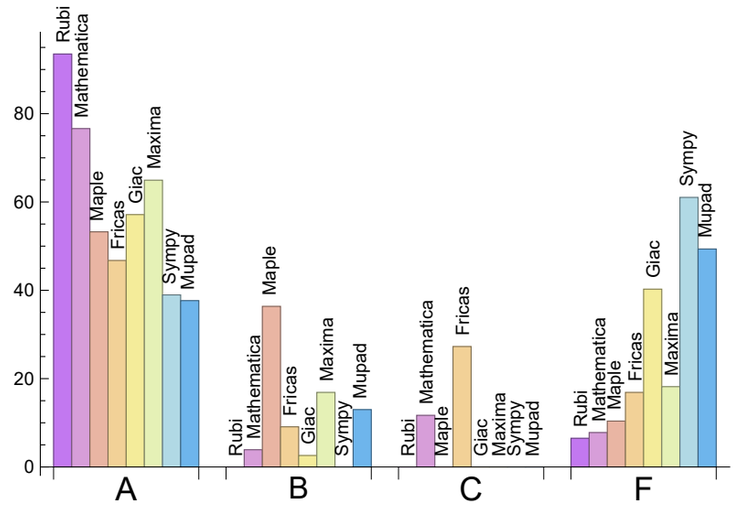
System	% A grade	% B grade	% C grade	% F grade
Rubi	93.51	0.00	0.00	6.49
Mathematica	76.62	3.90	11.69	7.79
Maple	53.25	36.36	0.00	10.39
Maxima	64.94	16.88	0.00	18.18
Fricas	46.75	9.09	27.27	16.88
Sympy	38.96	0.00	0.00	61.04
Giac	57.14	2.60	0.00	40.26
Mupad	37.66	12.99	0.00	49.35

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	5	100.00 %	0.00 %	0.00 %
Mathematica	6	50.00 %	50.00 %	0.00 %
Maple	8	100.00 %	0.00 %	0.00 %
Maxima	14	100.00 %	0.00 %	0.00 %
Fricas	13	0.00 %	0.00 %	100.00 %
Sympy	47	100.00 %	0.00 %	0.00 %
Giac	31	100.00 %	0.00 %	0.00 %
Mupad	38	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

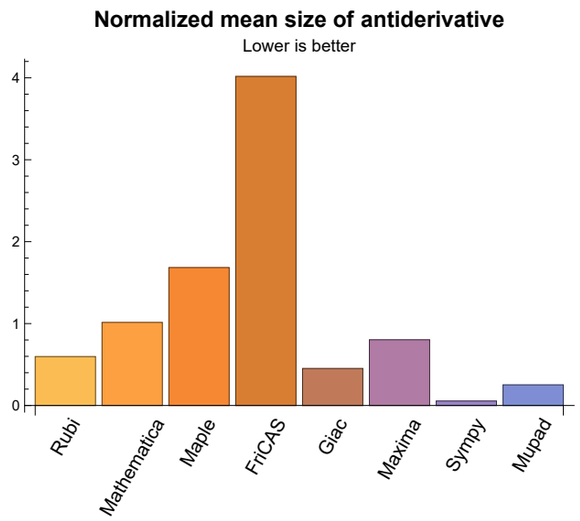
1.3 Performance

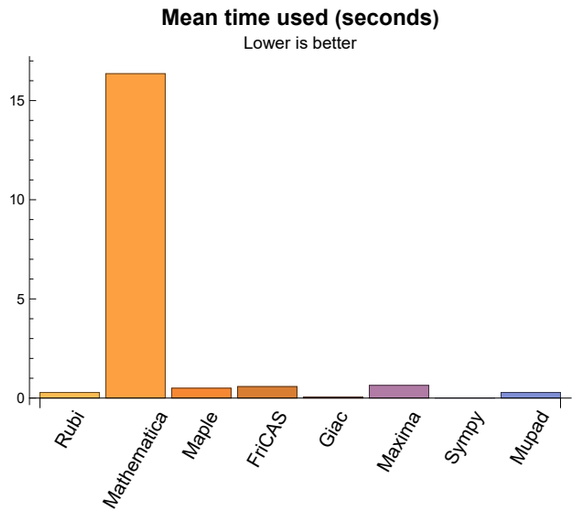
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.28	131.31	0.60	94.50	1.00
Mathematica	16.36	229.35	1.01	161.00	1.05
Maple	0.50	373.03	1.68	129.00	0.57
Maxima	0.64	178.11	0.80	68.00	0.26
Fricas	0.58	1018.34	4.02	240.00	1.62
Sympy	0.01	2.20	0.06	0.00	0.00
Giac	0.06	99.96	0.45	0.00	0.00
Mupad	0.28	46.79	0.25	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{4, 5, 9, 10, 14, 15, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 48, 49, 56, 57, 61, 62, 66, 67, 71, 72, 76, 77}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {3, 7, 11, 12, 13, 18, 19, 47, 55, 75}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

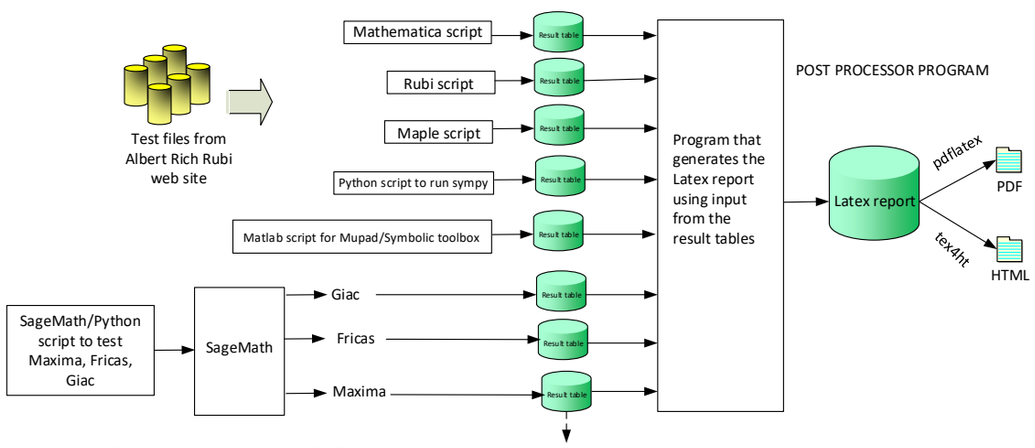
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77 }

B grade: { }

C grade: { }

F grade: { 16, 17, 18, 19, 20 }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 8, 9, 10, 14, 15, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77 }

B grade: { 50, 63, 64 }

C grade: { 3, 7, 11, 12, 13, 18, 19, 55, 75 }

F grade: { 16, 17, 20, 22, 23, 52 }

2.1.3 Maple

A grade: { 4, 5, 8, 9, 10, 13, 14, 15, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 35, 36, 37, 41, 42, 46, 47, 48, 49, 55, 56, 57, 60, 61, 62, 65, 66, 67, 71, 72, 76, 77 }

B grade: { 1, 2, 3, 6, 7, 11, 12, 32, 33, 34, 38, 39, 40, 43, 44, 45, 53, 54, 58, 59, 63, 64, 68, 69, 70, 73, 74, 75 }

C grade: { }

F grade: { 16, 17, 18, 19, 20, 50, 51, 52 }

2.1.4 Maxima

A grade: { 3, 4, 5, 9, 10, 14, 15, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 54, 56, 57, 61, 62, 65, 66, 67, 71, 72, 73, 74, 76, 77 }

B grade: { 1, 2, 6, 8, 11, 12, 53, 58, 59, 63, 64, 68, 69 }

C grade: { }

F grade: { 7, 13, 16, 17, 18, 19, 20, 50, 51, 52, 55, 60, 70, 75 }

2.1.5 FriCAS

A grade: { 4, 5, 9, 10, 14, 15, 29, 30, 31, 32, 33, 34, 35, 36, 37, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 56, 57, 61, 62, 66, 67, 71, 72, 76, 77 }

B grade: { 8, 38, 39, 43, 44, 70, 75 }

C grade: { 1, 2, 3, 6, 7, 11, 12, 13, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 73, 74 }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28 }

2.1.6 Sympy

A grade: { 4, 5, 8, 9, 10, 14, 15, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 48, 49, 56, 57, 61, 62, 66, 67, 71, 72, 76, 77 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

2.1.7 Giac

A grade: { 4, 5, 9, 10, 14, 15, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 57, 61, 62, 66, 67, 71, 72, 76, 77 }

B grade: { 8, 36 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

2.1.8 Mupad

A grade: { 4, 5, 9, 10, 14, 15, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 48, 49, 56, 57, 61, 62, 66, 67, 71, 72, 76, 77 }

B grade: { 8, 32, 33, 34, 38, 39, 40, 43, 44, 45 }

C grade: { }

F grade: { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 35, 36, 37, 41, 42, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	156	394	286	531	0	0	-1
normalized size	1	1.00	1.33	3.37	2.44	4.54	0.00	0.00	-0.01
time (sec)	N/A	0.187	1.733	0.304	0.502	0.598	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	143	234	176	329	0	0	-1
normalized size	1	1.00	1.70	2.79	2.10	3.92	0.00	0.00	-0.01
time (sec)	N/A	0.155	1.464	0.191	0.477	0.496	0.000	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	210	109	78	171	0	0	-1
normalized size	1	1.00	3.68	1.91	1.37	3.00	0.00	0.00	-0.02
time (sec)	N/A	0.090	4.418	0.166	0.483	0.459	0.000	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.022	9.599	0.229	0.000	0.451	0.000	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.022	16.805	0.225	0.000	0.506	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	179	328	343	1505	0	0	-1
normalized size	1	1.00	1.50	2.76	2.88	12.65	0.00	0.00	-0.01
time (sec)	N/A	0.210	2.100	0.197	0.645	0.690	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	301	177	0	840	0	0	-1
normalized size	1	1.00	3.42	2.01	0.00	9.55	0.00	0.00	-0.01
time (sec)	N/A	0.137	6.345	0.164	0.000	0.579	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	77	65	127	232	66	134	56
normalized size	1	1.00	1.92	1.62	3.18	5.80	1.65	3.35	1.40
time (sec)	N/A	0.031	0.247	0.108	0.386	0.505	0.247	0.146	0.095

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.037	19.335	0.237	0.000	0.537	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.035	19.614	0.235	0.000	0.469	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	801	677	595	5569	0	0	-1
normalized size	1	1.00	3.38	2.86	2.51	23.50	0.00	0.00	-0.00
time (sec)	N/A	0.392	12.750	0.311	0.570	0.622	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	458	367	392	3071	0	0	-1
normalized size	1	1.00	2.92	2.34	2.50	19.56	0.00	0.00	-0.01
time (sec)	N/A	0.248	7.940	0.329	0.557	0.586	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	263	166	0	1462	0	0	-1
normalized size	1	1.00	2.63	1.66	0.00	14.62	0.00	0.00	-0.01
time (sec)	N/A	0.137	6.144	0.217	0.000	0.543	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.038	31.072	0.811	0.000	0.546	0.000	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.037	23.147	0.978	0.000	0.513	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1392	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.132	39.854	0.215	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1363	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.106	29.418	0.191	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	F	F	F(-2)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1280	0	556	0	0	0	0	0	-1
normalized size	1	0.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.029	5.320	0.262	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	F	F	F(-2)	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1280	0	556	0	0	0	0	0	-1
normalized size	1	0.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.031	4.386	0.253	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F(-2)	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1365	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.110	27.752	0.206	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1341	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	28.724	0.220	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	180.002	0.290	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	180.003	0.225	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1343	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	35.593	0.243	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	28.754	0.220	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	2.093	0.238	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	2.455	0.256	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.072	26.265	0.284	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	146.540	0.139	0.000	0.488	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.030	9.920	0.105	0.000	0.455	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.017	7.624	0.145	0.000	0.429	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	244	929	186	304	0	193	273
normalized size	1	1.00	1.44	5.50	1.10	1.80	0.00	1.14	1.62
time (sec)	N/A	0.195	0.440	0.197	0.480	0.752	0.000	0.143	1.307

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	169	446	126	192	0	123	187
normalized size	1	1.00	1.39	3.66	1.03	1.57	0.00	1.01	1.53
time (sec)	N/A	0.118	0.293	0.155	0.405	0.480	0.000	0.132	1.182

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	81	165	74	101	0	67	76
normalized size	1	1.00	1.09	2.23	1.00	1.36	0.00	0.91	1.03
time (sec)	N/A	0.054	0.308	0.157	0.344	0.420	0.000	0.161	1.127

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	122	61	48	73	0	49	-1
normalized size	1	1.00	0.78	0.39	0.31	0.46	0.00	0.31	-0.01
time (sec)	N/A	0.295	0.272	0.649	0.528	0.417	0.000	0.147	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	206	90	56	217	0	348	-1
normalized size	1	1.00	1.30	0.57	0.35	1.36	0.00	2.19	-0.01
time (sec)	N/A	0.240	0.805	0.622	0.739	0.456	0.000	0.193	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	264	211	68	342	0	180	-1
normalized size	1	1.00	1.25	1.00	0.32	1.62	0.00	0.85	-0.00
time (sec)	N/A	0.323	1.152	0.648	0.960	0.441	0.000	0.147	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	420	1918	296	573	0	383	267
normalized size	1	1.00	1.83	8.34	1.29	2.49	0.00	1.67	1.16
time (sec)	N/A	0.279	1.138	0.255	0.788	0.524	0.000	0.138	1.251

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	207	913	190	361	0	227	165
normalized size	1	1.00	1.22	5.37	1.12	2.12	0.00	1.34	0.97
time (sec)	N/A	0.189	0.987	0.242	0.635	0.425	0.000	0.154	1.176

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	331	106	192	0	109	89
normalized size	1	1.00	0.86	2.49	0.80	1.44	0.00	0.82	0.67
time (sec)	N/A	0.131	0.544	0.241	1.452	0.483	0.000	0.148	1.136

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	199	106	81	133	0	78	-1
normalized size	1	1.00	0.67	0.36	0.27	0.45	0.00	0.26	-0.00
time (sec)	N/A	0.770	0.472	0.794	1.071	0.454	0.000	0.130	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	442	163	100	609	0	633	-1
normalized size	1	1.00	1.05	0.39	0.24	1.45	0.00	1.51	-0.00
time (sec)	N/A	0.786	1.416	0.907	1.953	0.501	0.000	0.205	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	615	3296	407	844	0	573	376
normalized size	1	1.00	1.83	9.81	1.21	2.51	0.00	1.71	1.12
time (sec)	N/A	0.379	2.854	0.343	2.500	0.533	0.000	0.163	1.352

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	371	1529	255	532	0	331	236
normalized size	1	1.00	1.51	6.22	1.04	2.16	0.00	1.35	0.96
time (sec)	N/A	0.266	1.549	0.297	1.757	0.452	0.000	0.155	1.241

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	185	533	139	286	0	151	129
normalized size	1	1.00	1.01	2.91	0.76	1.56	0.00	0.83	0.70
time (sec)	N/A	0.212	0.798	0.286	1.048	0.420	0.000	0.147	1.128

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	312	151	114	193	0	107	-1
normalized size	1	1.00	0.71	0.35	0.26	0.44	0.00	0.24	-0.00
time (sec)	N/A	1.821	0.720	1.063	4.527	0.469	0.000	0.141	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	794	239	140	1164	0	912	-1
normalized size	1	1.00	1.15	0.35	0.20	1.68	0.00	1.32	-0.00
time (sec)	N/A	1.812	3.131	2.114	7.711	0.464	0.000	0.233	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	34.654	0.255	0.000	0.530	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	13.985	0.335	0.000	0.502	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	186	0	0	148	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	1.66	0.00	0.00	-0.01
time (sec)	N/A	0.115	1.198	0.379	0.000	0.466	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	195	0	0	248	0	0	-1
normalized size	1	1.00	1.27	0.00	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.179	11.173	0.372	0.000	0.476	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	0	0	0	345	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	1.54	0.00	0.00	-0.00
time (sec)	N/A	0.236	180.002	0.418	0.000	0.690	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	161	452	304	583	0	0	-1
normalized size	1	1.00	1.18	3.30	2.22	4.26	0.00	0.00	-0.01
time (sec)	N/A	0.265	2.171	0.296	0.391	0.557	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	147	272	179	364	0	0	-1
normalized size	1	1.00	1.43	2.64	1.74	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.213	1.768	0.279	0.395	0.767	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	226	129	0	191	0	0	-1
normalized size	1	1.00	3.01	1.72	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.128	4.263	0.290	0.000	0.526	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	4.555	0.460	0.000	0.731	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.029	11.399	0.446	0.000	0.507	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	508	873	625	3744	0	0	-1
normalized size	1	1.00	1.83	3.15	2.26	13.52	0.00	0.00	-0.00
time (sec)	N/A	0.548	7.901	0.528	0.434	1.021	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	232	510	411	2123	0	0	-1
normalized size	1	1.00	1.10	2.42	1.95	10.06	0.00	0.00	-0.00
time (sec)	N/A	0.396	5.758	0.466	0.435	0.594	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	192	221	0	944	0	0	-1
normalized size	1	1.00	1.51	1.74	0.00	7.43	0.00	0.00	-0.01
time (sec)	N/A	0.183	2.450	0.411	0.000	0.997	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.056	41.590	0.810	0.000	0.611	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	27.174	0.940	0.000	0.562	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	2010	1788	1297	12909	0	0	-1
normalized size	1	1.00	3.55	3.16	2.29	22.81	0.00	0.00	-0.00
time (sec)	N/A	1.024	14.608	0.668	1.361	1.200	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	1142	1022	869	7298	0	0	-1
normalized size	1	1.00	2.82	2.52	2.15	18.02	0.00	0.00	-0.00
time (sec)	N/A	0.700	11.846	0.587	0.967	0.840	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	265	459	475	3262	0	0	-1
normalized size	1	1.00	1.02	1.76	1.82	12.50	0.00	0.00	-0.00
time (sec)	N/A	0.350	3.684	0.504	0.620	1.030	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.055	63.488	1.092	0.000	0.514	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.053	55.419	1.213	0.000	0.539	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	239	1143	528	740	0	0	-1
normalized size	1	1.00	1.13	5.39	2.49	3.49	0.00	0.00	-0.00
time (sec)	N/A	0.341	3.487	0.679	0.697	0.764	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	191	720	337	500	0	0	-1
normalized size	1	1.00	1.22	4.59	2.15	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.290	3.868	0.660	0.865	0.602	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	144	357	0	306	0	0	-1
normalized size	1	1.00	1.33	3.31	0.00	2.83	0.00	0.00	-0.01
time (sec)	N/A	0.166	3.326	0.660	0.000	0.602	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.063	15.577	0.569	0.000	0.576	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	24.330	0.593	0.000	0.752	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	1019	2278	1060	6160	0	0	-1
normalized size	1	1.00	1.59	3.55	1.65	9.60	0.00	0.00	-0.00
time (sec)	N/A	2.212	13.078	0.942	1.864	0.682	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	506	1352	753	3693	0	0	-1
normalized size	1	1.00	1.06	2.84	1.58	7.76	0.00	0.00	-0.00
time (sec)	N/A	1.651	9.391	0.850	1.925	0.556	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	476	565	0	1790	0	0	-1
normalized size	1	1.00	2.43	2.88	0.00	9.13	0.00	0.00	-0.01
time (sec)	N/A	0.286	7.064	0.783	0.000	0.774	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	156.930	1.620	0.000	0.533	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	128.017	1.598	0.000	0.522	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [.6250]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	6	1.00	14	0.429
2	A	5	5	1.00	14	0.357
3	A	4	4	1.00	12	0.333
4	A	0	0	0.00	0	0.000
5	A	0	0	0.00	0	0.000
6	A	7	7	1.00	16	0.438
7	A	6	6	1.00	16	0.375
8	A	3	2	1.00	14	0.143
9	A	0	0	0.00	0	0.000
10	A	0	0	0.00	0	0.000
11	A	13	10	1.00	16	0.625
12	A	9	7	1.00	16	0.438
13	A	7	7	1.00	14	0.500
14	A	0	0	0.00	0	0.000
15	A	0	0	0.00	0	0.000
16	F	0	0	N/A	0	N/A
17	F	0	0	N/A	0	N/A
18	F	0	0	N/A	0	N/A
19	F	0	0	N/A	0	N/A
20	F	0	0	N/A	0	N/A
21	A	0	0	0.00	0	0.000
22	A	0	0	0.00	0	0.000
23	A	0	0	0.00	0	0.000
24	A	0	0	0.00	0	0.000
25	A	0	0	0.00	0	0.000
26	A	0	0	0.00	0	0.000
27	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	0	0	0.00	0	0.000
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	0	0	0.00	0	0.000
32	A	5	3	1.00	20	0.150
33	A	4	3	1.00	20	0.150
34	A	3	3	1.00	18	0.167
35	A	7	4	1.00	20	0.200
36	A	7	4	1.00	20	0.200
37	A	8	5	1.00	20	0.250
38	A	10	3	1.00	20	0.150
39	A	8	3	1.00	20	0.150
40	A	7	3	1.00	18	0.167
41	A	21	5	1.00	20	0.250
42	A	24	7	1.00	20	0.350
43	A	14	3	1.00	20	0.150
44	A	11	3	1.00	20	0.150
45	A	11	3	1.00	18	0.167
46	A	53	7	1.00	20	0.350
47	A	60	9	1.00	20	0.450
48	A	0	0	0.00	0	0.000
49	A	0	0	0.00	0	0.000
50	A	2	2	1.00	20	0.100
51	A	4	2	1.00	20	0.100
52	A	5	2	1.00	20	0.100
53	A	8	7	1.00	18	0.389
54	A	7	6	1.00	18	0.333
55	A	6	5	1.00	16	0.312
56	A	0	0	0.00	0	0.000
57	A	0	0	0.00	0	0.000
58	A	15	9	1.00	20	0.450
59	A	13	10	1.00	20	0.500
60	A	9	7	1.00	18	0.389
61	A	0	0	0.00	0	0.000
62	A	0	0	0.00	0	0.000
63	A	28	11	1.00	20	0.550

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	22	11	1.00	20	0.550
65	A	16	9	1.00	18	0.500
66	A	0	0	0.00	0	0.000
67	A	0	0	0.00	0	0.000
68	A	6	6	1.00	20	0.300
69	A	5	5	1.00	20	0.250
70	A	4	4	1.00	18	0.222
71	A	0	0	0.00	0	0.000
72	A	0	0	0.00	0	0.000
73	A	28	10	1.00	20	0.500
74	A	24	11	1.00	20	0.550
75	A	5	5	1.00	18	0.278
76	A	0	0	0.00	0	0.000
77	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int (c+dx)^3 \tanh(e+fx)\,dx$

Optimal. Leaf size=117

$$-\frac{3d^2(c+dx)\mathrm{Li}_3\left(-e^{2(e+fx)}\right)}{2f^3}+\frac{3d(c+dx)^2\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}+\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{(c+dx)^4}{4d}+\frac{3d^3\mathrm{Li}_4\left(-e^{2(e}$$

```
[Out] -1/4*(d*x+c)^4/d+(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+3/2*d*(d*x+c)^2*polylog(2
,-exp(2*f*x+2*e))/f^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+3/4*d^
3*polylog(4,-exp(2*f*x+2*e))/f^4
```

Rubi [A] time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{3d(c+dx)^2\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{3d^3\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}+\frac{(c+dx)^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tanh[e + f*x],x]
```

```
[Out] -(c + d*x)^4/(4*d) + ((c + d*x)^3*Log[1 + E^(2*(e + f*x))])/f + (3*d*(c + d
*x)^2*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) - (3*d^2*(c + d*x)*PolyLog[3, -
E^(2*(e + f*x))])/(2*f^3) + (3*d^3*PolyLog[4, -E^(2*(e + f*x))])/(4*f^4)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3 \tanh(e+fx)\,dx &= -\frac{(c+dx)^4}{4d} + 2\int \frac{e^{2(e+fx)}(c+dx)^3}{1+e^{2(e+fx)}}\,dx \\
&= -\frac{(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1+e^{2(e+fx)}\right)}{f} - \frac{(3d)\int (c+dx)^2 \log\left(1+e^{2(e+fx)}\right)\,dx}{f} \\
&= -\frac{(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1+e^{2(e+fx)}\right)}{f} + \frac{3d(c+dx)^2 \mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2} - \frac{\left(3d^2\right)\int (c \\
&= -\frac{(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1+e^{2(e+fx)}\right)}{f} + \frac{3d(c+dx)^2 \mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2} - \frac{3d^2(c+d \\
&= -\frac{(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1+e^{2(e+fx)}\right)}{f} + \frac{3d(c+dx)^2 \mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2} - \frac{3d^2(c+d \\
&= -\frac{(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1+e^{2(e+fx)}\right)}{f} + \frac{3d(c+dx)^2 \mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2} - \frac{3d^2(c+d
\end{aligned}$$

Mathematica [A] time = 1.73, size = 156, normalized size = 1.33

$$\frac{1}{4}x\tanh(e)\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) - \frac{3d\left(2f^2(c+dx)^2\mathrm{Li}_2\left(-e^{-2(e+fx)}\right)+d\left(2f(c+dx)\mathrm{Li}_3\left(-e^{-2(e+fx)}\right)+d\mathrm{L}\right.\right.}{4f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Tanh[e + f*x],x]
```

```
[Out] (c + d*x)^4/(2*d*(1 + E^(2*e))) + ((c + d*x)^3*Log[1 + E^(-2*(e + f*x))])/f
 - (3*d*(2*f^2*(c + d*x)^2*PolyLog[2, -E^(-2*(e + f*x))] + d*(2*f*(c + d*x)
*PolyLog[3, -E^(-2*(e + f*x))] + d*PolyLog[4, -E^(-2*(e + f*x))])))/(4*f^4)
 + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Tanh[e])/4
```

fricas [C] time = 0.60, size = 531, normalized size = 4.54

$$-\frac{d^3 f^4 x^4 + 4\,cd^2 f^4 x^3 + 6\,c^2 d f^4 x^2 + 4\,c^3 f^4 x - 24\,d^3 \mathrm{polylog}\left(4, i\,\cosh\left(fx+e\right) + i\,\sinh\left(fx+e\right)\right) - 24\,d^3 \mathrm{poly}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/4*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 + 4*c^3*f^4*x - 24*d^
3*polylog(4, I*cosh(f*x + e) + I*sinh(f*x + e)) - 24*d^3*polylog(4, -I*cosh
(f*x + e) - I*sinh(f*x + e)) - 12*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)
*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) - 12*(d^3*f^2*x^2 + 2*c*d^2*f^2*x
 + c^2*d*f^2)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) + 4*(d^3*e^3 - 3*c*
d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e) + I)
 + 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(cosh(f*x + e)
+ sinh(f*x + e) - I) - 4*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d
^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*log(I*cosh(f*x + e) + I*sinh(f*x +
e) + 1) - 4*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*
d^2*e^2*f + 3*c^2*d*e*f^2)*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1) + 24
*(d^3*f*x + c*d^2*f)*polylog(3, I*cosh(f*x + e) + I*sinh(f*x + e)) + 24*(d^
3*f*x + c*d^2*f)*polylog(3, -I*cosh(f*x + e) - I*sinh(f*x + e)))/f^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \tanh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tanh(f*x + e), x)
```

maple [B] time = 0.30, size = 394, normalized size = 3.37

$$-c\,d^2 x^3 + \frac{3d^3\,\mathrm{polylog}\left(4, -\mathrm{e}^{2fx+2e}\right)}{4f^4} - \frac{6d\,c^2 ex}{f} + \frac{6d^2 c\,e^2 x}{f^2} + \frac{3c\,d^2 \ln\left(\mathrm{e}^{2fx+2e}+1\right)x^2}{f} + \frac{3c\,d^2\,\mathrm{polylog}\left(2, -\mathrm{e}^{2fx+2e}\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tanh(f*x+e),x)
```

```
[Out] -c*d^2*x^3+3/4*d^3*polylog(4,-exp(2*f*x+2*e))/f^4-6/f*d*c^2*e*x+6/f^2*d^2*c
*e^2*x+3/f*c*d^2*ln(exp(2*f*x+2*e)+1)*x^2+3/f^2*c*d^2*polylog(2,-exp(2*f*x+
2*e))*x+3/f*c^2*d*ln(exp(2*f*x+2*e)+1)*x-6/f^3*c*d^2*e^2*ln(exp(f*x+e))+6/f
^2*c^2*d*e*ln(exp(f*x+e))-3/f^2*d*c^2*e^2-2/f^3*d^3*e^3*x+2/f^4*d^3*e^3*ln(
exp(f*x+e))-3/2/f^3*c*d^2*polylog(3,-exp(2*f*x+2*e))+3/2/f^2*c^2*d*polylog(
2,-exp(2*f*x+2*e))+1/f*d^3*ln(exp(2*f*x+2*e)+1)*x^3+3/2/f^2*d^3*polylog(2,-
exp(2*f*x+2*e))*x^2-3/2/f^3*d^3*polylog(3,-exp(2*f*x+2*e))*x+4/f^3*d^2*c*e^
3-3/2*c^2*d*x^2-3/2/f^4*d^3*e^4+1/f*c^3*ln(exp(2*f*x+2*e)+1)-2/f*c^3*ln(exp
(f*x+e))-1/4*d^3*x^4+c^3*x
```

maxima [B] time = 0.50, size = 286, normalized size = 2.44

$$\frac{1}{4}\,d^3 x^4 + cd^2 x^3 + \frac{3}{2}\,c^2 d x^2 + \frac{c^3 \log\left(e^{\left(2fx+2e\right)}+1\right)}{2f} + \frac{c^3 \log\left(e^{\left(-2fx-2e\right)}+1\right)}{2f} + \frac{3\left(2fx \log\left(e^{\left(2fx+2e\right)}+1\right) + \mathrm{Li}_2\left(-e^{\left(\right.\right.}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tanh(f*x+e),x, algorithm="maxima")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + 1/2*c^3*log(e^(2*f*x + 2*e) + 1)/
f + 1/2*c^3*log(e^(-2*f*x - 2*e) + 1)/f + 3/2*(2*f*x*log(e^(2*f*x + 2*e) +
1) + dilog(-e^(2*f*x + 2*e)))*c^2*d/f^2 + 3/2*(2*f^2*x^2*log(e^(2*f*x + 2*e
) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*c*d^
2/f^3 + 1/3*(4*f^3*x^3*log(e^(2*f*x + 2*e) + 1) + 6*f^2*x^2*dilog(-e^(2*f*x
 + 2*e)) - 6*f*x*polylog(3, -e^(2*f*x + 2*e)) + 3*polylog(4, -e^(2*f*x + 2*
e)))*d^3/f^4 - 1/2*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2)/f^4
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e + f\,x\right)\,(c + d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)*(c + d*x)^3,x)
```

```
[Out] int(tanh(e + f*x)*(c + d*x)^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \tanh\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tanh(f*x+e),x)
```

```
[Out] Integral((c + d*x)**3*tanh(e + f*x), x)
```

3.2 $\int (c+dx)^2 \tanh(e+fx)\,dx$

Optimal. Leaf size=84

$$\frac{d(c+dx)\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2}+\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{(c+dx)^3}{3d}-\frac{d^2\mathrm{Li}_3\left(-e^{2(e+fx)}\right)}{2f^3}$$

```
[Out] -1/3*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+d*(d*x+c)*polylog(2,-exp(
2*f*x+2*e))/f^2-1/2*d^2*polylog(3,-exp(2*f*x+2*e))/f^3
```

Rubi [A] time = 0.16, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3718, 2190, 2531, 2282, 6589}

$$\frac{d(c+dx)\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{d^2\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tanh[e + f*x],x]
```

```
[Out] -(c + d*x)^3/(3*d) + ((c + d*x)^2*Log[1 + E^(2*(e + f*x))])/f + (d*(c + d*x
)*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (d^2*PolyLog[3, -E^(2*(e + f*x))])/(2
*f^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2 \tanh(e+fx)\,dx &= -\frac{(c+dx)^3}{3d} + 2\int \frac{e^{2(e+fx)}(c+dx)^2}{1+e^{2(e+fx)}}\,dx \\ &= -\frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} - \frac{(2d)\int (c+dx)\log\left(1+e^{2(e+fx)}\right)\,dx}{f} \\ &= -\frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d(c+dx)\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2} - \frac{d^2\int \mathrm{Li}_2\left(-e\right.}{f^2} \\ &= -\frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d(c+dx)\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2} - \frac{d^2\,\mathrm{Subst}\left(\int\right.}{} \\ &= -\frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d(c+dx)\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2} - \frac{d^2\mathrm{Li}_3\left(-e^{2(e}\right.}{2f^3}\end{aligned}$$

Mathematica [A] time = 1.46, size = 143, normalized size = 1.70

$$\frac{1}{3}x\tanh(e)\left(3c^2+3cdx+d^2x^2\right)+\frac{e^{2e}\left(-\frac{3d\left(e^{-2e}+1\right)\left(2f(c+dx)\mathrm{Li}_2\left(-e^{-2(e+fx)}\right)+d\mathrm{Li}_3\left(-e^{-2(e+fx)}\right)\right)}{f^3}+\frac{6\left(e^{-2e}+1\right)(c+dx)^2\log\left(e^{-2(e+fx)}+1\right)}{f}\right.}{6\left(e^{2e}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Tanh[e + f*x],x]
```

```
[Out] (E^(2*e)*((4*(c + d*x)^3)/(d*E^(2*e)) + (6*(1 + E^(-2*e))*(c + d*x)^2*Log[1
 + E^(-2*(e + f*x))])/f - (3*d*(1 + E^(-2*e))*(2*f*(c + d*x)*PolyLog[2, -E^
(-2*(e + f*x))] + d*PolyLog[3, -E^(-2*(e + f*x))]))/f^3))/(6*(1 + E^(2*e)))
 + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tanh[e])/3
```

fricas [C] time = 0.50, size = 329, normalized size = 3.92

$$-\frac{d^2f^3x^3+3\,cdf^3x^2+3\,c^2f^3x+6\,d^2\mathrm{polylog}\left(3,i\,\cosh\left(fx+e\right)+i\,\sinh\left(fx+e\right)\right)+6\,d^2\mathrm{polylog}\left(3,-i\,\cosh\left(f\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/3*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x + 6*d^2*polylog(3, I*cosh(f
*x + e) + I*sinh(f*x + e)) + 6*d^2*polylog(3, -I*cosh(f*x + e) - I*sinh(f*x
 + e)) - 6*(d^2*f*x + c*d*f)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) - 6*(
d^2*f*x + c*d*f)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 3*(d^2*e^2 - 2
*c*d*e*f + c^2*f^2)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 3*(d^2*e^2 - 2
*c*d*e*f + c^2*f^2)*log(cosh(f*x + e) + sinh(f*x + e) - I) - 3*(d^2*f^2*x^2
 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*log(I*cosh(f*x + e) + I*sinh(f*x + e)
 + 1) - 3*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*log(-I*cosh(f*x
 + e) - I*sinh(f*x + e) + 1))/f^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\tanh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tanh(f*x + e), x)
```

maple [B] time = 0.19, size = 234, normalized size = 2.79

$$-\frac{d^2x^3}{3}-cd\,x^2+c^2x+\frac{c^2\ln\left(\mathrm{e}^{2fx+2e}+1\right)}{f}-\frac{2c^2\ln\left(\mathrm{e}^{fx+e}\right)}{f}-\frac{2d^2e^2\ln\left(\mathrm{e}^{fx+e}\right)}{f^3}+\frac{2d^2e^2x}{f^2}+\frac{4d^2e^3}{3f^3}+\frac{d^2\ln\left(\mathrm{e}^{2fx+2e}+1\right)x^2}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tanh(f*x+e),x)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2+c^2*x+1/f*c^2*ln(exp(2*f*x+2*e)+1)-2/f*c^2*ln(exp(f*x+
e))-2/f^3*d^2*e^2*ln(exp(f*x+e))+2/f^2*d^2*e^2*x+4/3/f^3*d^2*e^3+1/f*d^2*ln
(exp(2*f*x+2*e)+1)*x^2+1/f^2*d^2*polylog(2,-exp(2*f*x+2*e))*x-1/2*d^2*polyl
og(3,-exp(2*f*x+2*e))/f^3+4/f^2*c*d*e*ln(exp(f*x+e))-4/f*d*c*e*x-2/f^2*d*c*
e^2+2/f*c*d*ln(exp(2*f*x+2*e)+1)*x+1/f^2*c*d*polylog(2,-exp(2*f*x+2*e))
```

maxima [B] time = 0.48, size = 176, normalized size = 2.10

$$\frac{1}{3}\,d^2x^3+cdx^2+\frac{c^2\log\left(e^{\left(2fx+2e\right)}+1\right)}{2f}+\frac{c^2\log\left(e^{\left(-2fx-2e\right)}+1\right)}{2f}+\frac{\left(2fx\log\left(e^{\left(2fx+2e\right)}+1\right)+\mathrm{Li}_2\left(-e^{\left(2fx+2e\right)}\right)\right)cd}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tanh(f*x+e),x, algorithm="maxima")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + 1/2*c^2*log(e^(2*f*x + 2*e) + 1)/f + 1/2*c^2*log(e^
(-2*f*x - 2*e) + 1)/f + (2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x +
 2*e)))*c*d/f^2 + 1/2*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^
(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*d^2/f^3 - 2/3*(d^2*f^3*x^3 +
 3*c*d*f^3*x^2)/f^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e+f\,x\right)\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)*(c + d*x)^2,x)
```

```
[Out] int(tanh(e + f*x)*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\tanh\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tanh(f*x+e),x)
```

```
[Out] Integral((c + d*x)**2*tanh(e + f*x), x)
```

3.3 $\int (c+dx)\tanh(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{(c+dx)^2}{2d}+\frac{d\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}$$

```
[Out] -1/2*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*f*x+2*e))/f+1/2*d*polylog(2,-exp(2*f*x+
2*e))/f^2
```

Rubi [A] time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3718, 2190, 2279, 2391}

$$\frac{d\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tanh[e + f*x],x]
```

```
[Out] -(c + d*x)^2/(2*d) + ((c + d*x)*Log[1 + E^(2*(e + f*x))])/f + (d*PolyLog[2,
 -E^(2*(e + f*x))])/(2*f^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\tanh(e+fx)\,dx = -\frac{(c+dx)^2}{2d} + 2\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}\,dx$$
$$= -\frac{(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} - \frac{d\int \log\left(1+e^{2(e+fx)}\right)\,dx}{f}$$
$$= -\frac{(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} - \frac{d\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2(e+fx)}\right)}{2f^2}$$
$$= -\frac{(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}$$

Mathematica [C] time = 4.42, size = 210, normalized size = 3.68

$$\frac{c\log(\cosh(e+fx))}{f} + \frac{d\mathrm{csch}(e)\mathrm{sech}(e)\left(f^2x^2e^{-\tanh^{-1}(\coth(e))} - \frac{i\coth(e)\left(i\mathrm{Li}_2\left(e^{2i\left(ifx+i\tanh^{-1}(\coth(e))\right)}\right) - fx\left(-\pi+2i\tanh^{-1}(\coth(e\right.\right.\right.\right.}{2f^2\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Tanh[e + f*x],x]
```

```
[Out] (c*Log[Cosh[e + f*x]])/f + (d*Csch[e]*((f^2*x^2)/E^ArcTanh[Coth[e]] - (I*Co
th[e]*(-(f*x*(-Pi + (2*I)*ArcTanh[Coth[e]])) - Pi*Log[1 + E^(2*f*x)] - 2*(I
*f*x + I*ArcTanh[Coth[e]])*Log[1 - E^((2*I)*(I*f*x + I*ArcTanh[Coth[e]]))]
+ Pi*Log[Cosh[f*x]] + (2*I)*ArcTanh[Coth[e]]*Log[I*Sinh[f*x + ArcTanh[Coth[
e]]]] + I*PolyLog[2, E^((2*I)*(I*f*x + I*ArcTanh[Coth[e]]))]))/Sqrt[1 - Cot
h[e]^2])*Sech[e])/(2*f^2*Sqrt[Csch[e]^2*(-Cosh[e]^2 + Sinh[e]^2)]) + (d*x^2
*Tanh[e])/2
```

fricas [C] time = 0.46, size = 171, normalized size = 3.00

$$-\frac{df^2x^2 + 2cf^2x - 2d\mathrm{Li}_2\left(i\cosh\left(fx+e\right) + i\sinh\left(fx+e\right)\right) - 2d\mathrm{Li}_2\left(-i\cosh\left(fx+e\right) - i\sinh\left(fx+e\right)\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] -1/2*(d*f^2*x^2 + 2*c*f^2*x - 2*d*dilog(I*cosh(f*x + e) + I*sinh(f*x + e))
- 2*d*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) + 2*(d*e - c*f)*log(cosh(f*
x + e) + sinh(f*x + e) + I) + 2*(d*e - c*f)*log(cosh(f*x + e) + sinh(f*x +
e) - I) - 2*(d*f*x + d*e)*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) - 2*(d
*f*x + d*e)*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1))/f^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\tanh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*tanh(f*x + e), x)
```

maple [B] time = 0.17, size = 109, normalized size = 1.91

$$-\frac{d\,x^2}{2}+cx+\frac{c\ln\left(\mathrm{e}^{2fx+2e}+1\right)}{f}-\frac{2c\ln\left(\mathrm{e}^{fx+e}\right)}{f}-\frac{2dex}{f}-\frac{d\,e^2}{f^2}+\frac{d\ln\left(\mathrm{e}^{2fx+2e}+1\right)x}{f}+\frac{d\,\mathrm{polylog}\left(2,-\mathrm{e}^{2fx+2e}\right)}{2f^2}+\frac{2de\ln\left(\mathrm{e}\right.}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tanh(f*x+e),x)
```

```
[Out] -1/2*d*x^2+c*x+1/f*c*ln(exp(2*f*x+2*e)+1)-2/f*c*ln(exp(f*x+e))-2/f*d*e*x-1/
f^2*d*e^2+1/f*d*ln(exp(2*f*x+2*e)+1)*x+1/2*d*polylog(2,-exp(2*f*x+2*e))/f^2
+2/f^2*d*e*ln(exp(f*x+e))
```

maxima [A] time = 0.48, size = 78, normalized size = 1.37

$$-\frac{1}{2}dx^2+\frac{c\log\left(e^{\left(2fx+2e\right)}+1\right)}{2f}+\frac{c\log\left(e^{\left(-2fx-2e\right)}+1\right)}{2f}+\frac{\left(2fx\log\left(e^{\left(2fx+2e\right)}+1\right)+\mathrm{Li}_2\left(-e^{\left(2fx+2e\right)}\right)\right)d}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/2*d*x^2 + 1/2*c*log(e^(2*f*x + 2*e) + 1)/f + 1/2*c*log(e^(-2*f*x - 2*e)
+ 1)/f + 1/2*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))*d/f
^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tanh\left(e+f\,x\right)\,\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)*(c + d*x),x)
```

```
[Out] int(tanh(e + f*x)*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tanh\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e),x)
```

```
[Out] Integral((c + d*x)*tanh(e + f*x), x)
```

3.4 $\int \frac{\tanh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\tanh(e+fx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tanh(f*x+e)/(d*x+c),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[e + f*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Tanh[e + f*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh(e+fx)}{c+dx}\,dx$$

Mathematica [A] time = 9.60, size = 0, normalized size = 0.00

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[e + f*x]/(c + d*x),x]
```

```
[Out] Integrate[Tanh[e + f*x]/(c + d*x), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tanh\left(fx+e\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)/(d*x + c), x)
```

maple [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(d*x+c),x)
```

```
[Out] int(tanh(f*x+e)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log\left(dx+c\right)}{d}-2\int \frac{1}{dx+\left(dxe^{(2e)}+ce^{(2e)}\right)e^{\left(2fx\right)}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(d*x+c),x, algorithm="maxima")
```

```
[Out] log(d*x + c)/d - 2*integrate(1/(d*x + (d*x*e^(2*e) + c*e^(2*e))*e^(2*f*x) +
 c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\tanh\left(e+f\,x\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(c + d*x),x)
```

```
[Out] int(tanh(e + f*x)/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(e+fx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(d*x+c),x)
```

```
[Out] Integral(tanh(e + f*x)/(c + d*x), x)
```

3.5 $\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\tanh(e+fx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(tanh(f*x+e)/(d*x+c)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[e + f*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Tanh[e + f*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 16.81, size = 0, normalized size = 0.00

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[e + f*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Tanh[e + f*x]/(c + d*x)^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tanh\left(fx+e\right)}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)/(d*x + c)^2, x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(d*x+c)^2,x)
```

```
[Out] int(tanh(f*x+e)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{d^2x+cd}-2\int \frac{1}{d^2x^2+2\,cdx+c^2+\left(d^2x^2e^{(2\,e)}+2\,cdxe^{(2\,e)}+c^2e^{(2\,e)}\right)e^{\left(2\,fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/(d^2*x + c*d) - 2*integrate(1/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2*e^(2*e
) + 2*c*d*x*e^(2*e) + c^2*e^(2*e))*e^(2*f*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\tanh\left(e+f\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)/(c + d*x)^2,x)
```

```
[Out] int(tanh(e + f*x)/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(e+fx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(d*x+c)**2,x)
```

```
[Out] Integral(tanh(e + f*x)/(c + d*x)**2, x)
```

3.6 $\int (c+dx)^3 \tanh^2(e+fx)\,dx$

Optimal. Leaf size=119

$$\frac{3d^2(c+dx)\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^3}+\frac{3d(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{(c+dx)^3\tanh(e+fx)}{f}-\frac{(c+dx)^3}{f}+\frac{(c+dx)^4}{4d}-\frac{3d^3\mathrm{L}}{}$$

```
[Out] -(d*x+c)^3/f+1/4*(d*x+c)^4/d+3*d*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f^2+3*d^2*(
d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^3-3/2*d^3*polylog(3,-exp(2*f*x+2*e))/f^
4-(d*x+c)^3*tanh(f*x+e)/f
```

Rubi [A] time = 0.21, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3720, 3718, 2190, 2531, 2282, 6589, 32}

$$\frac{3d^2(c+dx)\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}-\frac{3d^3\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4}+\frac{3d(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{(c+dx)^3\tanh(}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tanh[e + f*x]^2,x]
```

```
[Out] -((c + d*x)^3/f) + (c + d*x)^4/(4*d) + (3*d*(c + d*x)^2*Log[1 + E^(2*(e + f
*x))])/f^2 + (3*d^2*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^3 - (3*d^3*Po
lyLog[3, -E^(2*(e + f*x))])/(2*f^4) - ((c + d*x)^3*Tanh[e + f*x])/f
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
```

```
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = -\frac{(c+dx)^3\tanh(e+fx)}{f} + \frac{(3d)\int (c+dx)^2\tanh(e+fx)\,dx}{f} + \int (c+dx)^3\,dx$$

$$= -\frac{(c+dx)^3}{f} + \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\tanh(e+fx)}{f} + \frac{(6d)\int \frac{e^{2(e+fx)}(c+dx)^2}{1+e^{2(e+fx)}}\,dx}{f}$$

$$= -\frac{(c+dx)^3}{f} + \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f^2} - \frac{(c+dx)^3\tanh(e+fx)}{f}$$

$$= -\frac{(c+dx)^3}{f} + \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f^2} + \frac{3d^2(c+dx)\mathrm{Li}_2\left(-e^{2(e+f}\right.}{f^3}$$

$$= -\frac{(c+dx)^3}{f} + \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f^2} + \frac{3d^2(c+dx)\mathrm{Li}_2\left(-e^{2(e+f}\right.}{f^3}$$

$$= -\frac{(c+dx)^3}{f} + \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f^2} + \frac{3d^2(c+dx)\mathrm{Li}_2\left(-e^{2(e+f}\right.}{f^3}$$

Mathematica [A] time = 2.10, size = 179, normalized size = 1.50

$$\frac{1}{4}\left(x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) + \frac{2de^{2e}\left(-\frac{3d\left(e^{-2e}+1\right)\left(2f(c+dx)\mathrm{Li}_2\left(-e^{-2(e+fx)}\right)+d\mathrm{Li}_3\left(-e^{-2(e+fx)}\right)\right)}{f^3} + \frac{6\left(e^{-2e}+1\right)(c+dx)^2\log\left(e^{-2(e}\right.\right.}{f}}{\left(e^{2e}+1\right)f}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Tanh[e + f*x]^2,x]
```

```
[Out] (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (2*d*E^(2*e)*((4*(c + d*x)
^3)/(d*E^(2*e)) + (6*(1 + E^(-2*e))*(c + d*x)^2*Log[1 + E^(-2*(e + f*x))])/
f - (3*d*(1 + E^(-2*e))*(2*f*(c + d*x)*PolyLog[2, -E^(-2*(e + f*x))] + d*Po
lyLog[3, -E^(-2*(e + f*x))]))/f^3))/((1 + E^(2*e))*f) - (4*(c + d*x)^3*Sech
[e]*Sech[e + f*x]*Sinh[f*x])/f)/4
```

fricas [C] time = 0.69, size = 1505, normalized size = 12.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 + 4*c^3*f^4*x - 8*d^3*
e^3 + 24*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 8*c^3*f^3 + (d^3*f^4*x^4 - 8*d^3*e^
3 + 24*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 4*(c*d^2*f^4 - 2*d^3*f^3)*x^3 + 6*(c^
2*d*f^4 - 4*c*d^2*f^3)*x^2 + 4*(c^3*f^4 - 6*c^2*d*f^3)*x)*cosh(f*x + e)^2 +
 2*(d^3*f^4*x^4 - 8*d^3*e^3 + 24*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 4*(c*d^2*f^
4 - 2*d^3*f^3)*x^3 + 6*(c^2*d*f^4 - 4*c*d^2*f^3)*x^2 + 4*(c^3*f^4 - 6*c^2*d
*f^3)*x)*cosh(f*x + e)*sinh(f*x + e) + (d^3*f^4*x^4 - 8*d^3*e^3 + 24*c*d^2*
e^2*f - 24*c^2*d*e*f^2 + 4*(c*d^2*f^4 - 2*d^3*f^3)*x^3 + 6*(c^2*d*f^4 - 4*c
*d^2*f^3)*x^2 + 4*(c^3*f^4 - 6*c^2*d*f^3)*x)*sinh(f*x + e)^2 + 24*(d^3*f*x
+ c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e)^2 + 2*(d^3*f*x + c*d^2*f)*cos
h(f*x + e)*sinh(f*x + e) + (d^3*f*x + c*d^2*f)*sinh(f*x + e)^2)*dilog(I*cos
h(f*x + e) + I*sinh(f*x + e)) + 24*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)
*cosh(f*x + e)^2 + 2*(d^3*f*x + c*d^2*f)*cosh(f*x + e)*sinh(f*x + e) + (d^3
*f*x + c*d^2*f)*sinh(f*x + e)^2)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e))
+ 12*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^
2)*cosh(f*x + e)^2 + 2*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cosh(f*x + e)*si
nh(f*x + e) + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sinh(f*x + e)^2)*log(cosh
(f*x + e) + sinh(f*x + e) + I) + 12*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 + (d
^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cosh(f*x + e)^2 + 2*(d^3*e^2 - 2*c*d^2*e*
f + c^2*d*f^2)*cosh(f*x + e)*sinh(f*x + e) + (d^3*e^2 - 2*c*d^2*e*f + c^2*d
*f^2)*sinh(f*x + e)^2)*log(cosh(f*x + e) + sinh(f*x + e) - I) + 12*(d^3*f^2
*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x
 - d^3*e^2 + 2*c*d^2*e*f)*cosh(f*x + e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x
- d^3*e^2 + 2*c*d^2*e*f)*cosh(f*x + e)*sinh(f*x + e) + (d^3*f^2*x^2 + 2*c*d
^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sinh(f*x + e)^2)*log(I*cosh(f*x + e) + I*
sinh(f*x + e) + 1) + 12*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*
f + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cosh(f*x + e)^2 +
 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*cosh(f*x + e)*sinh
(f*x + e) + (d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*sinh(f*x
+ e)^2)*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1) - 24*(d^3*cosh(f*x + e)
^2 + 2*d^3*cosh(f*x + e)*sinh(f*x + e) + d^3*sinh(f*x + e)^2 + d^3)*polylog
(3, I*cosh(f*x + e) + I*sinh(f*x + e)) - 24*(d^3*cosh(f*x + e)^2 + 2*d^3*co
sh(f*x + e)*sinh(f*x + e) + d^3*sinh(f*x + e)^2 + d^3)*polylog(3, -I*cosh(f
*x + e) - I*sinh(f*x + e)))/(f^4*cosh(f*x + e)^2 + 2*f^4*cosh(f*x + e)*sinh
(f*x + e) + f^4*sinh(f*x + e)^2 + f^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \tanh\left(fx + e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tanh(f*x + e)^2, x)
```

maple [B] time = 0.20, size = 328, normalized size = 2.76

$$\frac{d^3x^4}{4}+c\,d^2x^3+\frac{3c^2d\,x^2}{2}+c^3x+\frac{2d^3x^3+6c\,d^2x^2+6c^2dx+2c^3}{f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{3d\,c^2\ln\left(\mathrm{e}^{2fx+2e}+1\right)}{f^2}-\frac{6d\,c^2\ln\left(\mathrm{e}^{fx+e}\right)}{f^2}-\frac{6d^3e^2\ln}{f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tanh(f*x+e)^2,x)
```

```
[Out] 1/4*d^3*x^4+c*d^2*x^3+3/2*c^2*d*x^2+c^3*x+2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+
c^3)/f/(exp(2*f*x+2*e)+1)+3/f^2*d*c^2*ln(exp(2*f*x+2*e)+1)-6/f^2*d*c^2*ln(e
xp(f*x+e))-6/f^4*d^3*e^2*ln(exp(f*x+e))-2/f*d^3*x^3+6/f^3*d^3*e^2*x+4/f^4*d
```

```
^3*e^3+3/f^2*d^3*ln(exp(2*f*x+2*e)+1)*x^2+3/f^3*d^3*polylog(2,-exp(2*f*x+2*
e))*x-3/2*d^3*polylog(3,-exp(2*f*x+2*e))/f^4+12/f^3*d^2*c*e*ln(exp(f*x+e))-
6/f*d^2*c*x^2-12/f^2*d^2*c*e*x-6/f^3*d^2*c*e^2+6/f^2*d^2*c*ln(exp(2*f*x+2*e
)+1)*x+3/f^3*d^2*c*polylog(2,-exp(2*f*x+2*e))
```

maxima [B] time = 0.64, size = 343, normalized size = 2.88

$$c^3\left(x+\frac{e}{f}-\frac{2}{f\left(e^{\left(-2fx-2e\right)}+1\right)}\right)-\frac{3}{2}c^2d\left(\frac{2xe^{\left(2fx+2e\right)}}{fe^{\left(2fx+2e\right)}+f}-\frac{fx^2+\left(fx^2e^{\left(2e\right)}-2xe^{\left(2e\right)}\right)e^{\left(2fx\right)}}{fe^{\left(2fx+2e\right)}+f}-\frac{2\log\left(\left(e^{\left(2fx+2e\right)}+1\right)\right.}{f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] c^3*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) - 3/2*c^2*d*(2*x*e^(2*f*x + 2*
e)/(f*e^(2*f*x + 2*e) + f) - (f*x^2 + (f*x^2*e^(2*e) - 2*x*e^(2*e))*e^(2*f*
x))/(f*e^(2*f*x + 2*e) + f) - 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2) +
3*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))*c*d^2/f^3 + 1/
4*(d^3*f*x^4 + 24*c*d^2*x^2 + 4*(c*d^2*f + 2*d^3)*x^3 + (d^3*f*x^4*e^(2*e)
+ 4*c*d^2*f*x^3*e^(2*e))*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 3/2*(2*f^2*x^
2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^
(2*f*x + 2*e)))*d^3/f^4 - 2*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/f^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e+f\,x\right)^2\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2*(c + d*x)^3,x)
```

```
[Out] int(tanh(e + f*x)^2*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\tanh^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tanh(f*x+e)**2,x)
```

```
[Out] Integral((c + d*x)**3*tanh(e + f*x)**2, x)
```

3.7 $\int (c+dx)^2 \tanh^2(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{2d(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{(c+dx)^2\tanh(e+fx)}{f}-\frac{(c+dx)^2}{f}+\frac{(c+dx)^3}{3d}+\frac{d^2\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^3}$$

```
[Out] -(d*x+c)^2/f+1/3*(d*x+c)^3/d+2*d*(d*x+c)*ln(1+exp(2*f*x+2*e))/f^2+d^2*polyl
og(2,-exp(2*f*x+2*e))/f^3-(d*x+c)^2*tanh(f*x+e)/f
```

Rubi [A] time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3720, 3718, 2190, 2279, 2391, 32}

$$\frac{d^2\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}+\frac{2d(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{(c+dx)^2\tanh(e+fx)}{f}-\frac{(c+dx)^2}{f}+\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tanh[e + f*x]^2,x]
```

```
[Out] -((c + d*x)^2/f) + (c + d*x)^3/(3*d) + (2*d*(c + d*x)*Log[1 + E^(2*(e + f*x
))])/f^2 + (d^2*PolyLog[2, -E^(2*(e + f*x))])/f^3 - ((c + d*x)^2*Tanh[e + f
*x])/f
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
```

```
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2 \tanh^2(e+fx)\,dx &= -\frac{(c+dx)^2\tanh(e+fx)}{f} + \frac{(2d)\int (c+dx)\tanh(e+fx)\,dx}{f} + \int (c+dx)^2\,dx \\ &= -\frac{(c+dx)^2}{f} + \frac{(c+dx)^3}{3d} - \frac{(c+dx)^2\tanh(e+fx)}{f} + \frac{(4d)\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}\,dx}{f} \\ &= -\frac{(c+dx)^2}{f} + \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^2} - \frac{(c+dx)^2\tanh(e+fx)}{f} \\ &= -\frac{(c+dx)^2}{f} + \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^2} - \frac{(c+dx)^2\tanh(e+fx)}{f} \\ &= -\frac{(c+dx)^2}{f} + \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^2} + \frac{d^2\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^3} - \frac{(c+\end{aligned}$$

Mathematica [C] time = 6.34, size = 301, normalized size = 3.42

$$\frac{\operatorname{sech}(e)\operatorname{sech}(e+fx)\left(c^2(-\sinh(fx)) - 2cdx\sinh(fx) - d^2x^2\sinh(fx)\right)}{f} + \frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right) + \frac{2cd\operatorname{sech}(e)(\cos}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tanh[e + f*x]^2,x]
```

```
[Out] (x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 + (2*c*d*Sech[e]*(Cosh[e]*Log[Cosh[e]*Cos
h[f*x] + Sinh[e]*Sinh[f*x]] - f*x*Sinh[e]))/(f^2*(Cosh[e]^2 - Sinh[e]^2)) +
 (d^2*Csch[e]*((f^2*x^2)/E^ArcTanh[Coth[e]] - (I*Coth[e]*(-(f*x*(-Pi + (2*I
)*ArcTanh[Coth[e]])) - Pi*Log[1 + E^(2*f*x)] - 2*(I*f*x + I*ArcTanh[Coth[e]
])*Log[1 - E^((2*I)*(I*f*x + I*ArcTanh[Coth[e]]))] + Pi*Log[Cosh[f*x]] + (2
*I)*ArcTanh[Coth[e]]*Log[I*Sinh[f*x + ArcTanh[Coth[e]]]] + I*PolyLog[2, E^(
(2*I)*(I*f*x + I*ArcTanh[Coth[e]]))]))/Sqrt[1 - Coth[e]^2])*Sech[e])/(f^3*S
qrt[Csch[e]^2*(-Cosh[e]^2 + Sinh[e]^2)]) + (Sech[e]*Sech[e + f*x]*(-(c^2*Si
nh[f*x]) - 2*c*d*x*Sinh[f*x] - d^2*x^2*Sinh[f*x]))/f
```

fricas [C] time = 0.58, size = 840, normalized size = 9.55

$$\frac{d^2f^3x^3 + 3\,cdf^3x^2 + 3\,c^2f^3x + 6\,d^2e^2 - 12\,cdef + 6\,c^2f^2 + \left(d^2f^3x^3 + 6\,d^2e^2 - 12\,cdef + 3\left(cdf^3 - 2\,d^2f^2\right)x^2 + 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x + 6*d^2*e^2 - 12*c*d*e*f + 6
*c^2*f^2 + (d^2*f^3*x^3 + 6*d^2*e^2 - 12*c*d*e*f + 3*(c*d*f^3 - 2*d^2*f^2)*
x^2 + 3*(c^2*f^3 - 4*c*d*f^2)*x)*cosh(f*x + e)^2 + 2*(d^2*f^3*x^3 + 6*d^2*e
^2 - 12*c*d*e*f + 3*(c*d*f^3 - 2*d^2*f^2)*x^2 + 3*(c^2*f^3 - 4*c*d*f^2)*x)*
```

```
cosh(f*x + e)*sinh(f*x + e) + (d^2*f^3*x^3 + 6*d^2*e^2 - 12*c*d*e*f + 3*(c*
d*f^3 - 2*d^2*f^2)*x^2 + 3*(c^2*f^3 - 4*c*d*f^2)*x)*sinh(f*x + e)^2 + 6*(d^
2*cosh(f*x + e)^2 + 2*d^2*cosh(f*x + e)*sinh(f*x + e) + d^2*sinh(f*x + e)^2
 + d^2)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) + 6*(d^2*cosh(f*x + e)^2 +
 2*d^2*cosh(f*x + e)*sinh(f*x + e) + d^2*sinh(f*x + e)^2 + d^2)*dilog(-I*co
sh(f*x + e) - I*sinh(f*x + e)) - 6*(d^2*e - c*d*f + (d^2*e - c*d*f)*cosh(f*
x + e)^2 + 2*(d^2*e - c*d*f)*cosh(f*x + e)*sinh(f*x + e) + (d^2*e - c*d*f)*
sinh(f*x + e)^2)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 6*(d^2*e - c*d*f
+ (d^2*e - c*d*f)*cosh(f*x + e)^2 + 2*(d^2*e - c*d*f)*cosh(f*x + e)*sinh(f*
x + e) + (d^2*e - c*d*f)*sinh(f*x + e)^2)*log(cosh(f*x + e) + sinh(f*x + e)
 - I) + 6*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cosh(f*x + e)^2 + 2*(d^2*f*x
 + d^2*e)*cosh(f*x + e)*sinh(f*x + e) + (d^2*f*x + d^2*e)*sinh(f*x + e)^2)*
log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) + 6*(d^2*f*x + d^2*e + (d^2*f*x
+ d^2*e)*cosh(f*x + e)^2 + 2*(d^2*f*x + d^2*e)*cosh(f*x + e)*sinh(f*x + e)
+ (d^2*f*x + d^2*e)*sinh(f*x + e)^2)*log(-I*cosh(f*x + e) - I*sinh(f*x + e)
 + 1))/(f^3*cosh(f*x + e)^2 + 2*f^3*cosh(f*x + e)*sinh(f*x + e) + f^3*sinh(
f*x + e)^2 + f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \tanh\left(fx + e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tanh(f*x + e)^2, x)
```

maple [B] time = 0.16, size = 177, normalized size = 2.01

$$\frac{d^2x^3}{3}+cd\,x^2+c^2x+\frac{2d^2x^2+4cdx+2c^2}{f\left(\mathrm{e}^{2fx+2e}+1\right)}+\frac{2dc\ln\left(\mathrm{e}^{2fx+2e}+1\right)}{f^2}-\frac{4dc\ln\left(\mathrm{e}^{fx+e}\right)}{f^2}-\frac{2d^2x^2}{f}-\frac{4d^2ex}{f^2}-\frac{2d^2e^2}{f^3}+\frac{2d^2\ln\left(\mathrm{e}^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tanh(f*x+e)^2,x)
```

```
[Out] 1/3*d^2*x^3+c*d*x^2+c^2*x+2*(d^2*x^2+2*c*d*x+c^2)/f/(exp(2*f*x+2*e)+1)+2/f^
2*d*c*ln(exp(2*f*x+2*e)+1)-4/f^2*d*c*ln(exp(f*x+e))-2*d^2*x^2/f-4/f^2*d^2*e
*x-2/f^3*d^2*e^2+2/f^2*d^2*ln(exp(2*f*x+2*e)+1)*x+d^2*polylog(2,-exp(2*f*x+
2*e))/f^3+4/f^3*d^2*e*ln(exp(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(x+\frac{e}{f}-\frac{2}{f\left(e^{\left(-2fx-2e\right)}+1\right)}\right)-cd\left(\frac{2\,xe^{\left(2fx+2e\right)}}{fe^{\left(2fx+2e\right)}+f}-\frac{fx^2+\left(fx^2e^{\left(2e\right)}-2\,xe^{\left(2e\right)}\right)e^{\left(2fx\right)}}{fe^{\left(2fx+2e\right)}+f}-\frac{2\log\left(\left(e^{\left(2fx+2e\right)}+1\right)\right.}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] c^2*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) - c*d*(2*x*e^(2*f*x + 2*e)/(f*
e^(2*f*x + 2*e) + f) - (f*x^2 + (f*x^2*e^(2*e) - 2*x*e^(2*e))*e^(2*f*x))/(f
*e^(2*f*x + 2*e) + f) - 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2) + 1/3*d^
2*((f*x^3*e^(2*f*x + 2*e) + f*x^3 + 6*x^2)/(f*e^(2*f*x + 2*e) + f) - 12*int
egrate(x/(f*e^(2*f*x + 2*e) + f), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e + f\,x\right)^2 (c + d\,x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2*(c + d*x)^2,x)
[Out] int(tanh(e + f*x)^2*(c + d*x)^2, x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \tanh^2\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tanh(f*x+e)**2,x)
[Out] Integral((c + d*x)**2*tanh(e + f*x)**2, x)
```

3.8 $\int (c+dx)\tanh^2(e+fx)\,dx$

Optimal. Leaf size=40

$$-\frac{(c+dx)\tanh(e+fx)}{f}+cx+\frac{d\log(\cosh(e+fx))}{f^2}+\frac{dx^2}{2}$$

```
[Out] c*x+1/2*d*x^2+d*ln(cosh(f*x+e))/f^2-(d*x+c)*tanh(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3720, 3475}

$$-\frac{(c+dx)\tanh(e+fx)}{f}+cx+\frac{d\log(\cosh(e+fx))}{f^2}+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tanh[e + f*x]^2,x]
```

```
[Out] c*x + (d*x^2)/2 + (d*Log[Cosh[e + f*x]])/f^2 - ((c + d*x)*Tanh[e + f*x])/f
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\tanh^2(e+fx)\,dx &= -\frac{(c+dx)\tanh(e+fx)}{f}+\frac{d\int \tanh(e+fx)\,dx}{f}+\int (c+dx)\,dx\\ &= cx+\frac{dx^2}{2}+\frac{d\log(\cosh(e+fx))}{f^2}-\frac{(c+dx)\tanh(e+fx)}{f}\end{aligned}$$

Mathematica [A] time = 0.25, size = 77, normalized size = 1.92

$$\frac{c\tanh^{-1}(\tanh(e+fx))}{f}-\frac{c\tanh(e+fx)}{f}+\frac{d\log(\cosh(e+fx))}{f^2}-\frac{dx\text{sech}(e)\sinh(fx)\text{sech}(e+fx)}{f}+\frac{dx\text{sech}(e)(f$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Tanh[e + f*x]^2,x]
```

```
[Out] (c*ArcTanh[Tanh[e + f*x]])/f + (d*Log[Cosh[e + f*x]])/f^2 + (d*x*Sech[e]*(f
*x*Cosh[e] - 2*Sinh[e]))/(2*f) - (d*x*Sech[e]*Sech[e + f*x]*Sinh[f*x])/f -
(c*Tanh[e + f*x])/f
```

fricas [B] time = 0.50, size = 232, normalized size = 5.80

$$\frac{df^2x^2 + 2cf^2x + \left(df^2x^2 + 2\left(cf^2 - 2df\right)x\right)\cosh\left(fx+e\right)^2 + 2\left(df^2x^2 + 2\left(cf^2 - 2df\right)x\right)\cosh\left(fx+e\right)\sinh\left(fx}{2\left(f^2\cosh\left(fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(d*f^2*x^2 + 2*c*f^2*x + (d*f^2*x^2 + 2*(c*f^2 - 2*d*f)*x)*cosh(f*x + e
)^2 + 2*(d*f^2*x^2 + 2*(c*f^2 - 2*d*f)*x)*cosh(f*x + e)*sinh(f*x + e) + (d*
f^2*x^2 + 2*(c*f^2 - 2*d*f)*x)*sinh(f*x + e)^2 + 4*c*f + 2*(d*cosh(f*x + e)
^2 + 2*d*cosh(f*x + e)*sinh(f*x + e) + d*sinh(f*x + e)^2 + d)*log(2*cosh(f*
x + e)/(cosh(f*x + e) - sinh(f*x + e))))/(f^2*cosh(f*x + e)^2 + 2*f^2*cosh(
f*x + e)*sinh(f*x + e) + f^2*sinh(f*x + e)^2 + f^2)
```

giac [B] time = 0.15, size = 134, normalized size = 3.35

$$\frac{df^2x^2e^{\left(2fx+2e\right)} + df^2x^2 + 2cf^2xe^{\left(2fx+2e\right)} + 2cf^2x - 4dfxe^{\left(2fx+2e\right)} + 2de^{\left(2fx+2e\right)}\log\left(e^{\left(2fx+2e\right)}+1\right) + 4cf + 2}{2\left(f^2e^{\left(2fx+2e\right)} + f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d*f^2*x^2*e^(2*f*x + 2*e) + d*f^2*x^2 + 2*c*f^2*x*e^(2*f*x + 2*e) + 2*
c*f^2*x - 4*d*f*x*e^(2*f*x + 2*e) + 2*d*e^(2*f*x + 2*e)*log(e^(2*f*x + 2*e)
 + 1) + 4*c*f + 2*d*log(e^(2*f*x + 2*e) + 1))/(f^2*e^(2*f*x + 2*e) + f^2)
```

maple [A] time = 0.11, size = 65, normalized size = 1.62

$$\frac{d\,x^2}{2} + cx - \frac{2dx}{f} - \frac{2de}{f^2} + \frac{2dx + 2c}{f\left(\mathrm{e}^{2fx+2e}+1\right)} + \frac{d\ln\left(\mathrm{e}^{2fx+2e}+1\right)}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tanh(f*x+e)^2,x)
```

```
[Out] 1/2*d*x^2+c*x-2*d*x/f-2*d/f^2*e+2*(d*x+c)/f/(exp(2*f*x+2*e)+1)+d/f^2*ln(exp
(2*f*x+2*e)+1)
```

maxima [B] time = 0.39, size = 127, normalized size = 3.18

$$c\left(x + \frac{e}{f} - \frac{2}{f\left(e^{\left(-2fx-2e\right)}+1\right)}\right) - \frac{1}{2}d\left(\frac{2xe^{\left(2fx+2e\right)}}{fe^{\left(2fx+2e\right)}+f} - \frac{fx^2 + \left(fx^2e^{\left(2e\right)} - 2xe^{\left(2e\right)}\right)e^{\left(2fx\right)}}{fe^{\left(2fx+2e\right)}+f} - \frac{2\log\left(\left(e^{\left(2fx+2e\right)}+1\right)e^{\left(-2\right.}\right.}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] c*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) - 1/2*d*(2*x*e^(2*f*x + 2*e)/(f*
e^(2*f*x + 2*e) + f) - (f*x^2 + (f*x^2*e^(2*e) - 2*x*e^(2*e))*e^(2*f*x))/(f
*e^(2*f*x + 2*e) + f) - 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2)
```

mupad [B] time = 0.09, size = 56, normalized size = 1.40

$$x\left(c + \frac{d}{f}\right) + \frac{d\,x^2}{2} - \frac{d\,\ln\left(\tanh\left(e+f\,x\right)+1\right)}{f^2} - \frac{c\tanh\left(e+f\,x\right)}{f} - \frac{d\,x\tanh\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2*(c + d*x),x)
```

```
[Out] x*(c + d/f) + (d*x^2)/2 - (d*log(tanh(e + f*x) + 1))/f^2 - (c*tanh(e + f*x)
)/f - (d*x*tanh(e + f*x))/f
```

sympy [**A**] time = 0.25, size = 66, normalized size = 1.65

$$\begin{cases} cx - \frac{c\tanh\left(e+fx\right)}{f} + \frac{dx^2}{2} - \frac{dx\tanh\left(e+fx\right)}{f} + \frac{dx}{f} - \frac{d\log\left(\tanh\left(e+fx\right)+1\right)}{f^2} & \text{for } f \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\tanh^2\left(e\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e)**2,x)
```

```
[Out] Piecewise((c*x - c*tanh(e + f*x)/f + d*x**2/2 - d*x*tanh(e + f*x)/f + d*x/f
 - d*log(tanh(e + f*x) + 1)/f**2, Ne(f, 0)), ((c*x + d*x**2/2)*tanh(e)**2,
True))
```

3.9 $\int \frac{\tanh^2(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^2(e+fx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tanh(f*x+e)^2/(d*x+c),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[e + f*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Tanh[e + f*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \int \frac{\tanh^2(e+fx)}{c+dx}\,dx$$

Mathematica [A] time = 19.34, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[e + f*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Tanh[e + f*x]^2/(c + d*x), x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tanh\left(fx+e\right)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^2/(d*x + c), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2\left(fx+e\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(d*x+c),x)
[Out] int(tanh(f*x+e)^2/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$2\,d\int \frac{1}{d^2fx^2+2\,cdfx+c^2f+\left(d^2fx^2e^{(2\,e)}+2\,cdfxe^{(2\,e)}+c^2fe^{(2\,e)}\right)e^{\left(2\,fx\right)}}\,dx+\frac{\log\left(dx+c\right)}{d}+\frac{}{dfx+cf+\left(dfxe^{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(d*x+c),x, algorithm="maxima")
[Out] 2*d*integrate(1/(d^2*f*x^2 + 2*c*d*f*x + c^2*f + (d^2*f*x^2*e^(2*e) + 2*c*d
*f*x*e^(2*e) + c^2*f*e^(2*e))*e^(2*f*x)), x) + log(d*x + c)/d + 2/(d*f*x +
c*f + (d*f*x*e^(2*e) + c*f*e^(2*e))*e^(2*f*x))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tanh\left(e+f\,x\right)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(c + d*x),x)
[Out] int(tanh(e + f*x)^2/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2\left(e+fx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(d*x+c),x)
[Out] Integral(tanh(e + f*x)**2/(c + d*x), x)
```

3.10 $\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^2(e+fx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(tanh(f*x+e)^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[e + f*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Tanh[e + f*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 19.61, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[e + f*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Tanh[e + f*x]^2/(c + d*x)^2, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tanh\left(fx+e\right)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^2}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^2/(d*x + c)^2, x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2\left(fx+e\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(d*x+c)^2,x)
```

```
[Out] int(tanh(f*x+e)^2/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$4\,d\int \frac{1}{d^3fx^3+3\,cd^2fx^2+3\,c^2dfx+c^3f+\left(d^3fx^3e^{(2\,e)}+3\,cd^2fx^2e^{(2\,e)}+3\,c^2dfxe^{(2\,e)}+c^3fe^{(2\,e)}\right)e^{\left(2\,fx\right)}}\,dx-\frac{}{d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 4*d*integrate(1/(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f + (d^3*f*x
^3*e^(2*e) + 3*c*d^2*f*x^2*e^(2*e) + 3*c^2*d*f*x*e^(2*e) + c^3*f*e^(2*e))*e
^(2*f*x)), x) - (d*f*x + c*f + (d*f*x*e^(2*e) + c*f*e^(2*e))*e^(2*f*x) - 2*
d)/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2*e^(2*e) + 2*c*d^2*f*x*e^
(2*e) + c^2*d*f*e^(2*e))*e^(2*f*x))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tanh\left(e+f\,x\right)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^2/(c + d*x)^2,x)
```

```
[Out] int(tanh(e + f*x)^2/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2\left(e+fx\right)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(d*x+c)**2,x)
```

```
[Out] Integral(tanh(e + f*x)**2/(c + d*x)**2, x)
```

3.11 $\int (c+dx)^3 \tanh^3(e+fx)\,dx$

Optimal. Leaf size=237

$$-\frac{3d^2(c+dx)\mathrm{Li}_3\left(-e^{2(e+fx)}\right)}{2f^3}+\frac{3d^2(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f^3}+\frac{3d(c+dx)^2\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}-\frac{3d(c+dx)^2\tanh(e+fx)}{2f^2}$$

```
[Out] -3/2*d*(d*x+c)^2/f^2+1/2*(d*x+c)^3/f-1/4*(d*x+c)^4/d+3*d^2*(d*x+c)*ln(1+exp
(2*f*x+2*e))/f^3+(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+3/2*d^3*polylog(2,-exp(2*
f*x+2*e))/f^4+3/2*d*(d*x+c)^2*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*d^2*(d*x+c
)*polylog(3,-exp(2*f*x+2*e))/f^3+3/4*d^3*polylog(4,-exp(2*f*x+2*e))/f^4-3/2
*d*(d*x+c)^2*tanh(f*x+e)/f^2-1/2*(d*x+c)^3*tanh(f*x+e)^2/f
```

Rubi [A] time = 0.39, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3720, 3718, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{3d(c+dx)^2\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{3d^3\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^4}+\frac{3d^3\mathrm{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tanh[e + f*x]^3,x]
```

```
[Out] (-3*d*(c + d*x)^2)/(2*f^2) + (c + d*x)^3/(2*f) - (c + d*x)^4/(4*d) + (3*d^2
*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f^3 + ((c + d*x)^3*Log[1 + E^(2*(e + f
*x))])/f + (3*d^3*PolyLog[2, -E^(2*(e + f*x))])/(2*f^4) + (3*d*(c + d*x)^2*
PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) - (3*d^2*(c + d*x)*PolyLog[3, -E^(2*(
e + f*x))])/(2*f^3) + (3*d^3*PolyLog[4, -E^(2*(e + f*x))])/(4*f^4) - (3*d*(
c + d*x)^2*Tanh[e + f*x])/(2*f^2) - ((c + d*x)^3*Tanh[e + f*x]^2)/(2*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = -\frac{(c+dx)^3 \tanh^2(e+fx)}{2f} + \frac{(3d)\int (c+dx)^2 \tanh^2(e+fx)\,dx}{2f} + \int (c+dx)^3 \tanh$$

$$= -\frac{(c+dx)^4}{4d} - \frac{3d(c+dx)^2 \tanh(e+fx)}{2f^2} - \frac{(c+dx)^3 \tanh^2(e+fx)}{2f} + 2\int \frac{e^{2(e+fx)}(}{1+e^2}$$

$$= -\frac{3d(c+dx)^2}{2f^2} + \frac{(c+dx)^3}{2f} - \frac{(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1+e^{2(e+fx)}\right)}{f} - \frac{3d(c+dx)^2}{}$$

$$= -\frac{3d(c+dx)^2}{2f^2} + \frac{(c+dx)^3}{2f} - \frac{(c+dx)^4}{4d} + \frac{3d^2(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^3} + \frac{(c+dx)^2}{}$$

$$= -\frac{3d(c+dx)^2}{2f^2} + \frac{(c+dx)^3}{2f} - \frac{(c+dx)^4}{4d} + \frac{3d^2(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^3} + \frac{(c+dx)^2}{}$$

$$= -\frac{3d(c+dx)^2}{2f^2} + \frac{(c+dx)^3}{2f} - \frac{(c+dx)^4}{4d} + \frac{3d^2(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^3} + \frac{(c+dx)^2}{}$$

$$= -\frac{3d(c+dx)^2}{2f^2} + \frac{(c+dx)^3}{2f} - \frac{(c+dx)^4}{4d} + \frac{3d^2(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^3} + \frac{(c+dx)^2}{}$$

Mathematica [C] time = 12.75, size = 801, normalized size = 3.38

$$\frac{\operatorname{sech}(e)(\cosh(e)\log(\cosh(e)\cosh(fx)+\sinh(e)\sinh(fx)) - fx\sinh(e))c^3}{f\left(\cosh^2(e)-\sinh^2(e)\right)} + \frac{3d\operatorname{csch}(e)\left(e^{-\tanh^{-1}(\coth(e))}f^2x^2 - \frac{i\cot}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Tanh[e + f*x]^3,x]
```

```
[Out] (c*d^2*(2*f^2*x^2*(2*f*x + 3*(1 + E^(2*e))*Log[1 + E^(-2*(e + f*x))]) - 6*(
1 + E^(2*e))*f*x*PolyLog[2, -E^(-2*(e + f*x))] - 3*(1 + E^(2*e))*PolyLog[3,
 -E^(-2*(e + f*x))])*Sech[e])/(4*E^e*f^3) + (d^3*E^e*((2*x^4)/E^(2*e) + (4*
(1 + E^(-2*e))*x^3*Log[1 + E^(-2*(e + f*x))])/f - (3*(1 + E^(2*e))*(2*f^2*x
^2*PolyLog[2, -E^(-2*(e + f*x))] + 2*f*x*PolyLog[3, -E^(-2*(e + f*x))] + Po
lyLog[4, -E^(-2*(e + f*x))]))/(E^(2*e)*f^4))*Sech[e])/8 + ((c + d*x)^3*Sech
[e + f*x]^2)/(2*f) + (3*c*d^2*Sech[e]*(Cosh[e]*Log[Cosh[e]*Cosh[f*x] + Sinh
[e]*Sinh[f*x]] - f*x*Sinh[e]))/(f^3*(Cosh[e]^2 - Sinh[e]^2)) + (c^3*Sech[e]
*(Cosh[e]*Log[Cosh[e]*Cosh[f*x] + Sinh[e]*Sinh[f*x]] - f*x*Sinh[e]))/(f*(Co
sh[e]^2 - Sinh[e]^2)) + (3*d^3*Csch[e]*((f^2*x^2)/E^ArcTanh[Coth[e]] - (I*C
oth[e]*(-(f*x*(-Pi + (2*I)*ArcTanh[Coth[e]])) - Pi*Log[1 + E^(2*f*x)] - 2*(
I*f*x + I*ArcTanh[Coth[e]])*Log[1 - E^((2*I)*(I*f*x + I*ArcTanh[Coth[e]]))]
 + Pi*Log[Cosh[f*x]] + (2*I)*ArcTanh[Coth[e]]*Log[I*Sinh[f*x + ArcTanh[Coth
[e]]]] + I*PolyLog[2, E^((2*I)*(I*f*x + I*ArcTanh[Coth[e]]))]))/Sqrt[1 - Co
th[e]^2])*Sech[e])/(2*f^4*Sqrt[Csch[e]^2*(-Cosh[e]^2 + Sinh[e]^2)]) + (3*c^
2*d*Csch[e]*((f^2*x^2)/E^ArcTanh[Coth[e]] - (I*Coth[e]*(-(f*x*(-Pi + (2*I)*
ArcTanh[Coth[e]])) - Pi*Log[1 + E^(2*f*x)] - 2*(I*f*x + I*ArcTanh[Coth[e]])
*Log[1 - E^((2*I)*(I*f*x + I*ArcTanh[Coth[e]]))] + Pi*Log[Cosh[f*x]] + (2*I
)*ArcTanh[Coth[e]]*Log[I*Sinh[f*x + ArcTanh[Coth[e]]]] + I*PolyLog[2, E^((2
*I)*(I*f*x + I*ArcTanh[Coth[e]]))]))/Sqrt[1 - Coth[e]^2])*Sech[e])/(2*f^2*S
qrt[Csch[e]^2*(-Cosh[e]^2 + Sinh[e]^2)]) - (3*Sech[e]*Sech[e + f*x]*(c^2*d*
Sinh[f*x] + 2*c*d^2*x*Sinh[f*x] + d^3*x^2*Sinh[f*x]))/(2*f^2) + (x*(4*c^3 +
 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Tanh[e])/4
```

fricas [C] time = 0.62, size = 5569, normalized size = 23.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 + 4*c^3*f^4*x - 2*d^3
*e^4 + 8*c^3*e*f^3 - 12*d^3*e^2 + (d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 - 2*d^3*e^
4 - 12*c^2*d*e^2*f^2 + 8*c^3*e*f^3 - 12*d^3*e^2 + 6*(c^2*d*f^4 + 2*d^3*f^2)
*x^2 + 8*(c*d^2*e^3 + 3*c*d^2*e)*f + 4*(c^3*f^4 + 6*c*d^2*f^2)*x)*cosh(f*x
+ e)^4 + 4*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 - 2*d^3*e^4 - 12*c^2*d*e^2*f^2 +
8*c^3*e*f^3 - 12*d^3*e^2 + 6*(c^2*d*f^4 + 2*d^3*f^2)*x^2 + 8*(c*d^2*e^3 + 3
*c*d^2*e)*f + 4*(c^3*f^4 + 6*c*d^2*f^2)*x)*cosh(f*x + e)*sinh(f*x + e)^3 +
(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 - 2*d^3*e^4 - 12*c^2*d*e^2*f^2 + 8*c^3*e*f^3
 - 12*d^3*e^2 + 6*(c^2*d*f^4 + 2*d^3*f^2)*x^2 + 8*(c*d^2*e^3 + 3*c*d^2*e)*f
 + 4*(c^3*f^4 + 6*c*d^2*f^2)*x)*sinh(f*x + e)^4 - 12*(c^2*d*e^2 + c^2*d)*f^
2 + 2*(d^3*f^4*x^4 - 2*d^3*e^4 - 12*d^3*e^2 + 4*(2*c^3*e - c^3)*f^3 + 4*(c*
d^2*f^4 - d^3*f^3)*x^3 - 6*(2*c^2*d*e^2 + c^2*d)*f^2 + 6*(c^2*d*f^4 - 2*c*d
^2*f^3 + d^3*f^2)*x^2 + 8*(c*d^2*e^3 + 3*c*d^2*e)*f + 4*(c^3*f^4 - 3*c^2*d*
f^3 + 3*c*d^2*f^2)*x)*cosh(f*x + e)^2 + 2*(d^3*f^4*x^4 - 2*d^3*e^4 - 12*d^3
*e^2 + 4*(2*c^3*e - c^3)*f^3 + 4*(c*d^2*f^4 - d^3*f^3)*x^3 - 6*(2*c^2*d*e^2
 + c^2*d)*f^2 + 6*(c^2*d*f^4 - 2*c*d^2*f^3 + d^3*f^2)*x^2 + 3*(d^3*f^4*x^4
+ 4*c*d^2*f^4*x^3 - 2*d^3*e^4 - 12*c^2*d*e^2*f^2 + 8*c^3*e*f^3 - 12*d^3*e^2
 + 6*(c^2*d*f^4 + 2*d^3*f^2)*x^2 + 8*(c*d^2*e^3 + 3*c*d^2*e)*f + 4*(c^3*f^4
 + 6*c*d^2*f^2)*x)*cosh(f*x + e)^2 + 8*(c*d^2*e^3 + 3*c*d^2*e)*f + 4*(c^3*f
^4 - 3*c^2*d*f^3 + 3*c*d^2*f^2)*x)*sinh(f*x + e)^2 + 8*(c*d^2*e^3 + 3*c*d^2
*e)*f - 12*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^
2*f^2*x + c^2*d*f^2 + d^3)*cosh(f*x + e)^4 + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x
 + c^2*d*f^2 + d^3)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^3*f^2*x^2 + 2*c*d^2*
f^2*x + c^2*d*f^2 + d^3)*sinh(f*x + e)^4 + d^3 + 2*(d^3*f^2*x^2 + 2*c*d^2*f
^2*x + c^2*d*f^2 + d^3)*cosh(f*x + e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x +
c^2*d*f^2 + d^3 + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + d^3)*cosh(f*
x + e)^2)*sinh(f*x + e)^2 + 4*((d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + d
^3)*cosh(f*x + e)^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + d^3)*cosh(
f*x + e))*sinh(f*x + e))*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) - 12*(d^3
*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d
*f^2 + d^3)*cosh(f*x + e)^4 + 4*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 +
d^3)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f
^2 + d^3)*sinh(f*x + e)^4 + d^3 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^
2 + d^3)*cosh(f*x + e)^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + d^3
 + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + d^3)*cosh(f*x + e)^2)*sinh(
f*x + e)^2 + 4*((d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + d^3)*cosh(f*x +
e)^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + d^3)*cosh(f*x + e))*sinh(
f*x + e))*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) + 4*(d^3*e^3 + 3*c^2*d*
e*f^2 - c^3*f^3 + (d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e
^2 + c*d^2)*f)*cosh(f*x + e)^4 + 4*(d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d
^3*e - 3*(c*d^2*e^2 + c*d^2)*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^3*e^3 +
3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e^2 + c*d^2)*f)*sinh(f*x + e)^
4 + 3*d^3*e + 2*(d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e^2
 + c*d^2)*f)*cosh(f*x + e)^2 + 2*(d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3
*e + 3*(d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e^2 + c*d^2)
*f)*cosh(f*x + e)^2 - 3*(c*d^2*e^2 + c*d^2)*f)*sinh(f*x + e)^2 - 3*(c*d^2*e
^2 + c*d^2)*f + 4*((d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*
e^2 + c*d^2)*f)*cosh(f*x + e)^3 + (d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^
3*e - 3*(c*d^2*e^2 + c*d^2)*f)*cosh(f*x + e))*sinh(f*x + e))*log(cosh(f*x +
 e) + sinh(f*x + e) + I) + 4*(d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + (d^3*e^3
+ 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e^2 + c*d^2)*f)*cosh(f*x + e
)^4 + 4*(d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e^2 + c*d^2
```

```
)*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3
*d^3*e - 3*(c*d^2*e^2 + c*d^2)*f)*sinh(f*x + e)^4 + 3*d^3*e + 2*(d^3*e^3 +
3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e^2 + c*d^2)*f)*cosh(f*x + e)^
2 + 2*(d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e + 3*(d^3*e^3 + 3*c^2*d*e
*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e^2 + c*d^2)*f)*cosh(f*x + e)^2 - 3*(c*
d^2*e^2 + c*d^2)*f)*sinh(f*x + e)^2 - 3*(c*d^2*e^2 + c*d^2)*f + 4*((d^3*e^3
 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e^2 + c*d^2)*f)*cosh(f*x +
e)^3 + (d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 + 3*d^3*e - 3*(c*d^2*e^2 + c*d^2)
*f)*cosh(f*x + e))*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) - I) -
4*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2
+ (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2
+ 3*d^3*e + 3*(c^2*d*f^3 + d^3*f)*x)*cosh(f*x + e)^4 + 4*(d^3*f^3*x^3 + 3*c
*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(c^2*d
*f^3 + d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^3*f^3*x^3 + 3*c*d^2*f^3
*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(c^2*d*f^3 + d
^3*f)*x)*sinh(f*x + e)^4 + 3*d^3*e + 2*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3
*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(c^2*d*f^3 + d^3*f)*x)*c
osh(f*x + e)^2 + 2*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f
 + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3
*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(c^2*d*f^3 + d^3*f)*x)*cosh(f*x
+ e)^2 + 3*(c^2*d*f^3 + d^3*f)*x)*sinh(f*x + e)^2 + 3*(c^2*d*f^3 + d^3*f)*x
 + 4*((d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*
f^2 + 3*d^3*e + 3*(c^2*d*f^3 + d^3*f)*x)*cosh(f*x + e)^3 + (d^3*f^3*x^3 + 3
*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(c^2
*d*f^3 + d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e))*log(I*cosh(f*x + e) + I*si
nh(f*x + e) + 1) - 4*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2
*f + 3*c^2*d*e*f^2 + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2
*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(c^2*d*f^3 + d^3*f)*x)*cosh(f*x + e)^4 + 4
*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 +
 3*d^3*e + 3*(c^2*d*f^3 + d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^3*f^
3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e
 + 3*(c^2*d*f^3 + d^3*f)*x)*sinh(f*x + e)^4 + 3*d^3*e + 2*(d^3*f^3*x^3 + 3*
c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(c^2*
d*f^3 + d^3*f)*x)*cosh(f*x + e)^2 + 2*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*
e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(d^3*f^3*x^3 + 3*c*d^2*f^
3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(c^2*d*f^3 +
d^3*f)*x)*cosh(f*x + e)^2 + 3*(c^2*d*f^3 + d^3*f)*x)*sinh(f*x + e)^2 + 3*(c
^2*d*f^3 + d^3*f)*x + 4*((d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2
*e^2*f + 3*c^2*d*e*f^2 + 3*d^3*e + 3*(c^2*d*f^3 + d^3*f)*x)*cosh(f*x + e)^3
 + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2
 + 3*d^3*e + 3*(c^2*d*f^3 + d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e))*log(-I*
cosh(f*x + e) - I*sinh(f*x + e) + 1) - 24*(d^3*cosh(f*x + e)^4 + 4*d^3*cosh
(f*x + e)*sinh(f*x + e)^3 + d^3*sinh(f*x + e)^4 + 2*d^3*cosh(f*x + e)^2 + d
^3 + 2*(3*d^3*cosh(f*x + e)^2 + d^3)*sinh(f*x + e)^2 + 4*(d^3*cosh(f*x + e)
^3 + d^3*cosh(f*x + e))*sinh(f*x + e))*polylog(4, I*cosh(f*x + e) + I*sinh(
f*x + e)) - 24*(d^3*cosh(f*x + e)^4 + 4*d^3*cosh(f*x + e)*sinh(f*x + e)^3 +
 d^3*sinh(f*x + e)^4 + 2*d^3*cosh(f*x + e)^2 + d^3 + 2*(3*d^3*cosh(f*x + e)
^2 + d^3)*sinh(f*x + e)^2 + 4*(d^3*cosh(f*x + e)^3 + d^3*cosh(f*x + e))*sin
h(f*x + e))*polylog(4, -I*cosh(f*x + e) - I*sinh(f*x + e)) + 24*(d^3*f*x +
(d^3*f*x + c*d^2*f)*cosh(f*x + e)^4 + 4*(d^3*f*x + c*d^2*f)*cosh(f*x + e)*s
inh(f*x + e)^3 + (d^3*f*x + c*d^2*f)*sinh(f*x + e)^4 + c*d^2*f + 2*(d^3*f*x
 + c*d^2*f)*cosh(f*x + e)^2 + 2*(d^3*f*x + c*d^2*f + 3*(d^3*f*x + c*d^2*f)*
cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((d^3*f*x + c*d^2*f)*cosh(f*x + e)^3 +
 (d^3*f*x + c*d^2*f)*cosh(f*x + e))*sinh(f*x + e))*polylog(3, I*cosh(f*x +
e) + I*sinh(f*x + e)) + 24*(d^3*f*x + (d^3*f*x + c*d^2*f)*cosh(f*x + e)^4 +
 4*(d^3*f*x + c*d^2*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^3*f*x + c*d^2*f)*
sinh(f*x + e)^4 + c*d^2*f + 2*(d^3*f*x + c*d^2*f)*cosh(f*x + e)^2 + 2*(d^3*
f*x + c*d^2*f + 3*(d^3*f*x + c*d^2*f)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*
((d^3*f*x + c*d^2*f)*cosh(f*x + e)^3 + (d^3*f*x + c*d^2*f)*cosh(f*x + e))*s
```

```
inh(f*x + e))*polylog(3, -I*cosh(f*x + e) - I*sinh(f*x + e)) + 4*((d^3*f^4*
x^4 + 4*c*d^2*f^4*x^3 - 2*d^3*e^4 - 12*c^2*d*e^2*f^2 + 8*c^3*e*f^3 - 12*d^3
*e^2 + 6*(c^2*d*f^4 + 2*d^3*f^2)*x^2 + 8*(c*d^2*e^3 + 3*c*d^2*e)*f + 4*(c^3
*f^4 + 6*c*d^2*f^2)*x)*cosh(f*x + e)^3 + (d^3*f^4*x^4 - 2*d^3*e^4 - 12*d^3*
e^2 + 4*(2*c^3*e - c^3)*f^3 + 4*(c*d^2*f^4 - d^3*f^3)*x^3 - 6*(2*c^2*d*e^2
+ c^2*d)*f^2 + 6*(c^2*d*f^4 - 2*c*d^2*f^3 + d^3*f^2)*x^2 + 8*(c*d^2*e^3 + 3
*c*d^2*e)*f + 4*(c^3*f^4 - 3*c^2*d*f^3 + 3*c*d^2*f^2)*x)*cosh(f*x + e))*sin
h(f*x + e))/(f^4*cosh(f*x + e)^4 + 4*f^4*cosh(f*x + e)*sinh(f*x + e)^3 + f^
4*sinh(f*x + e)^4 + 2*f^4*cosh(f*x + e)^2 + f^4 + 2*(3*f^4*cosh(f*x + e)^2
+ f^4)*sinh(f*x + e)^2 + 4*(f^4*cosh(f*x + e)^3 + f^4*cosh(f*x + e))*sinh(f
*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \tanh\left(fx + e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tanh(f*x + e)^3, x)
```

maple [B] time = 0.31, size = 677, normalized size = 2.86

$$-c\,d^2x^3-\frac{6d\,c^2ex}{f}+\frac{6d^2c\,e^2x}{f^2}+\frac{3c\,d^2\ln\left(\mathrm{e}^{2fx+2e}+1\right)x^2}{f}+\frac{3c^2d\ln\left(\mathrm{e}^{2fx+2e}+1\right)x}{f}-\frac{6c\,d^2e^2\ln\left(\mathrm{e}^{fx+e}\right)}{f^3}+\frac{6c^2de\ln\left(\mathrm{e}^{f}\right.}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tanh(f*x+e)^3,x)
```

```
[Out] -c*d^2*x^3+3/2*d^3*polylog(2,-exp(2*f*x+2*e))/f^4+3/4*d^3*polylog(4,-exp(2*
f*x+2*e))/f^4-6/f*d*c^2*e*x+6/f^2*d^2*c*e^2*x+3/f*c*d^2*ln(exp(2*f*x+2*e)+1
)*x^2+3/f^2*c*d^2*polylog(2,-exp(2*f*x+2*e))*x+3/f*c^2*d*ln(exp(2*f*x+2*e)+
1)*x-6/f^3*c*d^2*e^2*ln(exp(f*x+e))+6/f^2*c^2*d*e*ln(exp(f*x+e))-3/f^2*d*c^
2*e^2-2/f^3*d^3*e^3*x+2/f^4*d^3*e^3*ln(exp(f*x+e))-3/2/f^3*c*d^2*polylog(3,
-exp(2*f*x+2*e))+3/2/f^2*c^2*d*polylog(2,-exp(2*f*x+2*e))+1/f*d^3*ln(exp(2*
f*x+2*e)+1)*x^3+3/2/f^2*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3/2/f^3*d^3*poly
log(3,-exp(2*f*x+2*e))*x-6/f^3*d^3*e*x-6/f^3*c*d^2*ln(exp(f*x+e))+6/f^4*d^3
*e*ln(exp(f*x+e))+3/f^3*d^3*ln(exp(2*f*x+2*e)+1)*x+3/f^3*c*d^2*ln(exp(2*f*x
+2*e)+1)+4/f^3*d^2*c*e^3-3/f^4*e^2*d^3-3/f^2*d^3*x^2+(2*d^3*f*x^3*exp(2*f*x
+2*e)+6*c*d^2*f*x^2*exp(2*f*x+2*e)+6*c^2*d*f*x*exp(2*f*x+2*e)+3*d^3*x^2*exp
(2*f*x+2*e)+2*c^3*f*exp(2*f*x+2*e)+6*c*d^2*x*exp(2*f*x+2*e)+3*c^2*d*exp(2*f
*x+2*e)+3*d^3*x^2+6*c*d^2*x+3*c^2*d)/f^2/(exp(2*f*x+2*e)+1)^2-3/2*c^2*d*x^2
-3/2/f^4*d^3*e^4+1/f*c^3*ln(exp(2*f*x+2*e)+1)-2/f*c^3*ln(exp(f*x+e))-1/4*d^
3*x^4+c^3*x
```

maxima [B] time = 0.57, size = 595, normalized size = 2.51

$$c^3\left(x+\frac{e}{f}+\frac{\log\left(e^{\left(-2fx-2e\right)}+1\right)}{f}+\frac{2\,e^{\left(-2fx-2e\right)}}{f\left(2\,e^{\left(-2fx-2e\right)}+e^{\left(-4fx-4e\right)}+1\right)}\right)-\frac{6\,cd^2x}{f^2}+\frac{3\left(2\,f^2x^2\log\left(e^{\left(2fx+2e\right)}+1\right)+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] c^3*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2
*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) - 6*c*d^2*x/f^2 + 3/2*(2*f^2*x^2*log(
e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x
```

```
 + 2*e)))*c*d^2/f^3 + 3*c*d^2*log(e^(2*f*x + 2*e) + 1)/f^3 + 1/4*(d^3*f^2*x
^4 + 4*c*d^2*f^2*x^3 + 24*c*d^2*x + 12*c^2*d + 6*(c^2*d*f^2 + 2*d^3)*x^2 +
(d^3*f^2*x^4*e^(4*e) + 4*c*d^2*f^2*x^3*e^(4*e) + 6*c^2*d*f^2*x^2*e^(4*e))*e
^(4*f*x) + 2*(d^3*f^2*x^4*e^(2*e) + 4*(c*d^2*f^2*e^(2*e) + d^3*f*e^(2*e))*x
^3 + 6*c^2*d*e^(2*e) + 6*(c^2*d*f^2*e^(2*e) + 2*c*d^2*f*e^(2*e) + d^3*e^(2*
e))*x^2 + 12*(c^2*d*f*e^(2*e) + c*d^2*e^(2*e))*x)*e^(2*f*x))/(f^2*e^(4*f*x
+ 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) + 1/3*(4*f^3*x^3*log(e^(2*f*x + 2*e)
+ 1) + 6*f^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*polylog(3, -e^(2*f*x + 2*e
)) + 3*polylog(4, -e^(2*f*x + 2*e)))*d^3/f^4 + 3/2*(c^2*d*f^2 + d^3)*(2*f*x
*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f^4 - 1/2*(d^3*f^4*x^4
 + 4*c*d^2*f^4*x^3 + 6*(c^2*d*f^2 + d^3)*f^2*x^2)/f^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tanh\left(e + f\,x\right)^3 (c + d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3*(c + d*x)^3,x)
```

```
[Out] int(tanh(e + f*x)^3*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \tanh^3\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tanh(f*x+e)**3,x)
```

```
[Out] Integral((c + d*x)**3*tanh(e + f*x)**3, x)
```

3.12 $\int (c+dx)^2 \tanh^3(e+fx)\,dx$

Optimal. Leaf size=157

$$\frac{d(c+dx)\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2}-\frac{d(c+dx)\tanh(e+fx)}{f^2}+\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{(c+dx)^2\tanh^2(e+fx)}{2f}+\frac{cdx}{f}-$$

```
[Out] c*d*x/f+1/2*d^2*x^2/f-1/3*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+d^2*
ln(cosh(f*x+e))/f^3+d*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2-1/2*d^2*polylo
g(3,-exp(2*f*x+2*e))/f^3-d*(d*x+c)*tanh(f*x+e)/f^2-1/2*(d*x+c)^2*tanh(f*x+e
)^2/f
```

Rubi [A] time = 0.25, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3720, 3475, 3718, 2190, 2531, 2282, 6589}

$$\frac{d(c+dx)\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{d^2\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}-\frac{d(c+dx)\tanh(e+fx)}{f^2}+\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tanh[e + f*x]^3,x]
```

```
[Out] (c*d*x)/f + (d^2*x^2)/(2*f) - (c + d*x)^3/(3*d) + ((c + d*x)^2*Log[1 + E^(2
*(e + f*x))])/f + (d^2*Log[Cosh[e + f*x]])/f^3 + (d*(c + d*x)*PolyLog[2, -E
^(2*(e + f*x))])/f^2 - (d^2*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3) - (d*(c +
 d*x)*Tanh[e + f*x])/f^2 - ((c + d*x)^2*Tanh[e + f*x]^2)/(2*f)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = -\frac{(c+dx)^2\tanh^2(e+fx)}{2f} + \frac{d\int (c+dx)\tanh^2(e+fx)\,dx}{f} + \int (c+dx)^2\tanh(e+$$

$$= -\frac{(c+dx)^3}{3d} - \frac{d(c+dx)\tanh(e+fx)}{f^2} - \frac{(c+dx)^2\tanh^2(e+fx)}{2f} + 2\int \frac{e^{2(e+fx)}(c+$$

$$= \frac{cdx}{f} + \frac{d^2x^2}{2f} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d^2\log(\cosh(e+fx))}{f^3} - \frac{d}{}$$

$$= \frac{cdx}{f} + \frac{d^2x^2}{2f} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d^2\log(\cosh(e+fx))}{f^3} + \frac{d}{}$$

$$= \frac{cdx}{f} + \frac{d^2x^2}{2f} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d^2\log(\cosh(e+fx))}{f^3} + \frac{d}{}$$

$$= \frac{cdx}{f} + \frac{d^2x^2}{2f} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d^2\log(\cosh(e+fx))}{f^3} + \frac{d}{}$$

Mathematica [C] time = 7.94, size = 458, normalized size = 2.92

$$\frac{1}{3}x\tanh(e)\left(3c^2+3cdx+d^2x^2\right) + \frac{c^2\operatorname{sech}(e)(\cosh(e)\log(\sinh(e)\sinh(fx)+\cosh(e)\cosh(fx)) - fx\sinh(e))}{f\left(\cosh^2(e)-\sinh^2(e)\right)} + \frac{\sec}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tanh[e + f*x]^3,x]
```

```
[Out] (d^2*(2*f^2*x^2*(2*f*x + 3*(1 + E^(2*e))*Log[1 + E^(-2*(e + f*x))]) - 6*(1
+ E^(2*e))*f*x*PolyLog[2, -E^(-2*(e + f*x))] - 3*(1 + E^(2*e))*PolyLog[3, -
E^(-2*(e + f*x))])*Sech[e]/(12*E^e*f^3) + ((c + d*x)^2*Sech[e + f*x]^2)/(2
*f) + (d^2*Sech[e]*(Cosh[e]*Log[Cosh[e]*Cosh[f*x] + Sinh[e]*Sinh[f*x]] - f*
x*Sinh[e]))/(f^3*(Cosh[e]^2 - Sinh[e]^2)) + (c^2*Sech[e]*(Cosh[e]*Log[Cosh[
e]*Cosh[f*x] + Sinh[e]*Sinh[f*x]] - f*x*Sinh[e]))/(f*(Cosh[e]^2 - Sinh[e]^2
)) + (c*d*Csch[e]*((f^2*x^2)/E^ArcTanh[Coth[e]] - (I*Coth[e]*(-(f*x*(-Pi +
(2*I)*ArcTanh[Coth[e]])) - Pi*Log[1 + E^(2*f*x)] - 2*(I*f*x + I*ArcTanh[Cot
```

```
h[e]])*Log[1 - E^((2*I)*(I*f*x + I*ArcTanh[Coth[e]]))] + Pi*Log[Cosh[f*x]]
+ (2*I)*ArcTanh[Coth[e]]*Log[I*Sinh[f*x + ArcTanh[Coth[e]]]] + I*PolyLog[2,
 E^((2*I)*(I*f*x + I*ArcTanh[Coth[e]]))])/Sqrt[1 - Coth[e]^2])*Sech[e])/(f
^2*Sqrt[Csch[e]^2*(-Cosh[e]^2 + Sinh[e]^2)]) + (Sech[e]*Sech[e + f*x]*(-(c*
d*Sinh[f*x]) - d^2*x*Sinh[f*x]))/f^2 + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tanh[
e])/3
```

fricas [C] time = 0.59, size = 3071, normalized size = 19.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/3*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x + 2*d^2*e^3 + 6*c^2*e*f^2 +
 (d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 2*d^2*e^3 - 6*c*d*e^2*f + 6*c^2*e*f^2 + 6*d
^2*e + 3*(c^2*f^3 + 2*d^2*f)*x)*cosh(f*x + e)^4 + 4*(d^2*f^3*x^3 + 3*c*d*f^
3*x^2 + 2*d^2*e^3 - 6*c*d*e^2*f + 6*c^2*e*f^2 + 6*d^2*e + 3*(c^2*f^3 + 2*d^
2*f)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 2*d^
2*e^3 - 6*c*d*e^2*f + 6*c^2*e*f^2 + 6*d^2*e + 3*(c^2*f^3 + 2*d^2*f)*x)*sinh
(f*x + e)^4 + 6*d^2*e + 2*(d^2*f^3*x^3 + 2*d^2*e^3 + 6*d^2*e + 3*(2*c^2*e -
 c^2)*f^2 + 3*(c*d*f^3 - d^2*f^2)*x^2 - 3*(2*c*d*e^2 + c*d)*f + 3*(c^2*f^3
- 2*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)^2 + 2*(d^2*f^3*x^3 + 2*d^2*e^3 + 6*d^
2*e + 3*(2*c^2*e - c^2)*f^2 + 3*(c*d*f^3 - d^2*f^2)*x^2 + 3*(d^2*f^3*x^3 +
3*c*d*f^3*x^2 + 2*d^2*e^3 - 6*c*d*e^2*f + 6*c^2*e*f^2 + 6*d^2*e + 3*(c^2*f^
3 + 2*d^2*f)*x)*cosh(f*x + e)^2 - 3*(2*c*d*e^2 + c*d)*f + 3*(c^2*f^3 - 2*c*
d*f^2 + d^2*f)*x)*sinh(f*x + e)^2 - 6*(c*d*e^2 + c*d)*f - 6*((d^2*f*x + c*d
*f)*cosh(f*x + e)^4 + 4*(d^2*f*x + c*d*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (
d^2*f*x + c*d*f)*sinh(f*x + e)^4 + d^2*f*x + c*d*f + 2*(d^2*f*x + c*d*f)*co
sh(f*x + e)^2 + 2*(d^2*f*x + c*d*f + 3*(d^2*f*x + c*d*f)*cosh(f*x + e)^2)*s
inh(f*x + e)^2 + 4*((d^2*f*x + c*d*f)*cosh(f*x + e)^3 + (d^2*f*x + c*d*f)*c
osh(f*x + e))*sinh(f*x + e))*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) - 6*(
(d^2*f*x + c*d*f)*cosh(f*x + e)^4 + 4*(d^2*f*x + c*d*f)*cosh(f*x + e)*sinh(
f*x + e)^3 + (d^2*f*x + c*d*f)*sinh(f*x + e)^4 + d^2*f*x + c*d*f + 2*(d^2*f
*x + c*d*f)*cosh(f*x + e)^2 + 2*(d^2*f*x + c*d*f + 3*(d^2*f*x + c*d*f)*cosh
(f*x + e)^2)*sinh(f*x + e)^2 + 4*((d^2*f*x + c*d*f)*cosh(f*x + e)^3 + (d^2*
f*x + c*d*f)*cosh(f*x + e))*sinh(f*x + e))*dilog(-I*cosh(f*x + e) - I*sinh(
f*x + e)) - 3*((d^2*e^2 - 2*c*d*e*f + c^2*f^2 + d^2)*cosh(f*x + e)^4 + 4*(d
^2*e^2 - 2*c*d*e*f + c^2*f^2 + d^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^2*e^
2 - 2*c*d*e*f + c^2*f^2 + d^2)*sinh(f*x + e)^4 + d^2*e^2 - 2*c*d*e*f + c^2*
f^2 + 2*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 + d^2)*cosh(f*x + e)^2 + 2*(d^2*e^2
- 2*c*d*e*f + c^2*f^2 + 3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 + d^2)*cosh(f*x +
e)^2 + d^2)*sinh(f*x + e)^2 + d^2 + 4*((d^2*e^2 - 2*c*d*e*f + c^2*f^2 + d^2
)*cosh(f*x + e)^3 + (d^2*e^2 - 2*c*d*e*f + c^2*f^2 + d^2)*cosh(f*x + e))*si
nh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) + I) - 3*((d^2*e^2 - 2*c*d*e
*f + c^2*f^2 + d^2)*cosh(f*x + e)^4 + 4*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 + d^
2)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^2*e^2 - 2*c*d*e*f + c^2*f^2 + d^2)*si
nh(f*x + e)^4 + d^2*e^2 - 2*c*d*e*f + c^2*f^2 + 2*(d^2*e^2 - 2*c*d*e*f + c^
2*f^2 + d^2)*cosh(f*x + e)^2 + 2*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 + 3*(d^2*e^
2 - 2*c*d*e*f + c^2*f^2 + d^2)*cosh(f*x + e)^2 + d^2)*sinh(f*x + e)^2 + d^2
 + 4*((d^2*e^2 - 2*c*d*e*f + c^2*f^2 + d^2)*cosh(f*x + e)^3 + (d^2*e^2 - 2*
c*d*e*f + c^2*f^2 + d^2)*cosh(f*x + e))*sinh(f*x + e))*log(cosh(f*x + e) +
sinh(f*x + e) - I) - 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + (d^2*f^2*x^2 + 2*c*d*f^
2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e)^4 + 4*(d^2*f^2*x^2 + 2*c*d*f^2*x -
 d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^2*f^2*x^2 + 2*c*d*
f^2*x - d^2*e^2 + 2*c*d*e*f)*sinh(f*x + e)^4 - d^2*e^2 + 2*c*d*e*f + 2*(d^2
*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e)^2 + 2*(d^2*f^2*
x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f + 3*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^
2*e^2 + 2*c*d*e*f)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((d^2*f^2*x^2 + 2*c
*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e)^3 + (d^2*f^2*x^2 + 2*c*d*f^2*
```

```
x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e))*sinh(f*x + e))*log(I*cosh(f*x + e)
+ I*sinh(f*x + e) + 1) - 3*(d^2*f^2*x^2 + 2*c*d*f^2*x + (d^2*f^2*x^2 + 2*c*
d*f^2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e)^4 + 4*(d^2*f^2*x^2 + 2*c*d*f^2
*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (d^2*f^2*x^2 + 2*
c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*sinh(f*x + e)^4 - d^2*e^2 + 2*c*d*e*f + 2*
(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e)^2 + 2*(d^2*
f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f + 3*(d^2*f^2*x^2 + 2*c*d*f^2*x
- d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((d^2*f^2*x^2 +
 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e)^3 + (d^2*f^2*x^2 + 2*c*d*
f^2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e))*sinh(f*x + e))*log(-I*cosh(f*x
+ e) - I*sinh(f*x + e) + 1) + 6*(d^2*cosh(f*x + e)^4 + 4*d^2*cosh(f*x + e)*
sinh(f*x + e)^3 + d^2*sinh(f*x + e)^4 + 2*d^2*cosh(f*x + e)^2 + 2*(3*d^2*co
sh(f*x + e)^2 + d^2)*sinh(f*x + e)^2 + d^2 + 4*(d^2*cosh(f*x + e)^3 + d^2*c
osh(f*x + e))*sinh(f*x + e))*polylog(3, I*cosh(f*x + e) + I*sinh(f*x + e))
+ 6*(d^2*cosh(f*x + e)^4 + 4*d^2*cosh(f*x + e)*sinh(f*x + e)^3 + d^2*sinh(f
*x + e)^4 + 2*d^2*cosh(f*x + e)^2 + 2*(3*d^2*cosh(f*x + e)^2 + d^2)*sinh(f*
x + e)^2 + d^2 + 4*(d^2*cosh(f*x + e)^3 + d^2*cosh(f*x + e))*sinh(f*x + e))
*polylog(3, -I*cosh(f*x + e) - I*sinh(f*x + e)) + 4*((d^2*f^3*x^3 + 3*c*d*f
^3*x^2 + 2*d^2*e^3 - 6*c*d*e^2*f + 6*c^2*e*f^2 + 6*d^2*e + 3*(c^2*f^3 + 2*d
^2*f)*x)*cosh(f*x + e)^3 + (d^2*f^3*x^3 + 2*d^2*e^3 + 6*d^2*e + 3*(2*c^2*e
- c^2)*f^2 + 3*(c*d*f^3 - d^2*f^2)*x^2 - 3*(2*c*d*e^2 + c*d)*f + 3*(c^2*f^3
 - 2*c*d*f^2 + d^2*f)*x)*cosh(f*x + e))*sinh(f*x + e))/(f^3*cosh(f*x + e)^4
 + 4*f^3*cosh(f*x + e)*sinh(f*x + e)^3 + f^3*sinh(f*x + e)^4 + 2*f^3*cosh(f
*x + e)^2 + f^3 + 2*(3*f^3*cosh(f*x + e)^2 + f^3)*sinh(f*x + e)^2 + 4*(f^3*
cosh(f*x + e)^3 + f^3*cosh(f*x + e))*sinh(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \tanh\left(fx+e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tanh(f*x + e)^3, x)
```

maple [B] time = 0.33, size = 367, normalized size = 2.34

$$-\frac{d^2x^3}{3}-cd\,x^2+c^2x+\frac{2d^2f\,x^2\mathrm{e}^{2fx+2e}+4cdfx\,\mathrm{e}^{2fx+2e}+2c^2f\,\mathrm{e}^{2fx+2e}+2d^2x\,\mathrm{e}^{2fx+2e}+2cd\,\mathrm{e}^{2fx+2e}+2d^2x+2cd}{f^2\left(\mathrm{e}^{2fx+2e}+1\right)^2}+\frac{cd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tanh(f*x+e)^3,x)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2+c^2*x+2*(d^2*f*x^2*exp(2*f*x+2*e)+2*c*d*f*x*exp(2*f*x+
2*e)+c^2*f*exp(2*f*x+2*e)+d^2*x*exp(2*f*x+2*e)+c*d*exp(2*f*x+2*e)+d^2*x+c*d
)/f^2/(exp(2*f*x+2*e)+1)^2+1/f^2*c*d*polylog(2,-exp(2*f*x+2*e))-2/f^2*d*c*e
^2+1/f*d^2*ln(exp(2*f*x+2*e)+1)*x^2+1/f^2*d^2*polylog(2,-exp(2*f*x+2*e))*x+
4/3/f^3*d^2*e^3+2/f*c*d*ln(exp(2*f*x+2*e)+1)*x+2/f^2*d^2*e^2*x-4/f*d*c*e*x-
2/f*c^2*ln(exp(f*x+e))+1/f*c^2*ln(exp(2*f*x+2*e)+1)-2/f^3*d^2*e^2*ln(exp(f*
x+e))-2/f^3*d^2*ln(exp(f*x+e))+1/f^3*d^2*ln(exp(2*f*x+2*e)+1)-1/2*d^2*polyl
og(3,-exp(2*f*x+2*e))/f^3+4/f^2*c*d*e*ln(exp(f*x+e))
```

maxima [B] time = 0.56, size = 392, normalized size = 2.50

$$c^2\left(x+\frac{e}{f}+\frac{\log\left(e^{\left(-2fx-2e\right)}+1\right)}{f}+\frac{2\,e^{\left(-2fx-2e\right)}}{f\left(2\,e^{\left(-2fx-2e\right)}+e^{\left(-4fx-4e\right)}+1\right)}\right)+\frac{\left(2fx\log\left(e^{\left(2fx+2e\right)}+1\right)+\mathrm{Li}_2\left(-e^{\left(2fx+2e\right)}\right)\right.}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] c^2*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2
*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) + (2*f*x*log(e^(2*f*x + 2*e) + 1) + d
ilog(-e^(2*f*x + 2*e)))*c*d/f^2 - 2*d^2*x/f^2 + 1/3*(d^2*f^2*x^3 + 3*c*d*f^
2*x^2 + 6*d^2*x + 6*c*d + (d^2*f^2*x^3*e^(4*e) + 3*c*d*f^2*x^2*e^(4*e))*e^(
4*f*x) + 2*(d^2*f^2*x^3*e^(2*e) + 3*(c*d*f^2*e^(2*e) + d^2*f*e^(2*e))*x^2 +
 3*c*d*e^(2*e) + 3*(2*c*d*f*e^(2*e) + d^2*e^(2*e))*x)*e^(2*f*x))/(f^2*e^(4*
f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) + 1/2*(2*f^2*x^2*log(e^(2*f*x + 2
*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*d^
2/f^3 + d^2*log(e^(2*f*x + 2*e) + 1)/f^3 - 2/3*(d^2*f^3*x^3 + 3*c*d*f^3*x^2
)/f^3
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e + f\,x\right)^3 (c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3*(c + d*x)^2,x)
```

```
[Out] int(tanh(e + f*x)^3*(c + d*x)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \tanh^3\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tanh(f*x+e)**3,x)
```

```
[Out] Integral((c + d*x)**2*tanh(e + f*x)**3, x)
```

3.13 $\int (c+dx)\tanh^3(e+fx)\,dx$

Optimal. Leaf size=100

$$\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{(c+dx)\tanh^2(e+fx)}{2f}-\frac{(c+dx)^2}{2d}+\frac{d\,\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}-\frac{d\tanh(e+fx)}{2f^2}+\frac{dx}{2f}$$

```
[Out] 1/2*d*x/f-1/2*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*f*x+2*e))/f+1/2*d*polylog(2,-e
xp(2*f*x+2*e))/f^2-1/2*d*tanh(f*x+e)/f^2-1/2*(d*x+c)*tanh(f*x+e)^2/f
```

Rubi [A] time = 0.14, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3720, 3473, 8, 3718, 2190, 2279, 2391}

$$\frac{d\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{(c+dx)\tanh^2(e+fx)}{2f}-\frac{(c+dx)^2}{2d}-\frac{d\tanh(e+fx)}{2f^2}+\frac{dx}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tanh[e + f*x]^3,x]
```

```
[Out] (d*x)/(2*f) - (c + d*x)^2/(2*d) + ((c + d*x)*Log[1 + E^(2*(e + f*x))])/f +
(d*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) - (d*Tanh[e + f*x])/(2*f^2) - ((c
+ d*x)*Tanh[e + f*x]^2)/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
```

```
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\tanh^3(e+fx)\,dx = -\frac{(c+dx)\tanh^2(e+fx)}{2f} + \frac{d\int \tanh^2(e+fx)\,dx}{2f} + \int (c+dx)\tanh(e+fx)\,dx$$

$$= -\frac{(c+dx)^2}{2d} - \frac{d\tanh(e+fx)}{2f^2} - \frac{(c+dx)\tanh^2(e+fx)}{2f} + 2\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}\,d$$

$$= \frac{dx}{2f} - \frac{(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} - \frac{d\tanh(e+fx)}{2f^2} - \frac{(c+dx)\tanh^2($$

$$= \frac{dx}{2f} - \frac{(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} - \frac{d\tanh(e+fx)}{2f^2} - \frac{(c+dx)\tanh^2($$

$$= \frac{dx}{2f} - \frac{(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2} - \frac{d\tanh(e+fx}{2f^2}$$

Mathematica [C] time = 6.14, size = 263, normalized size = 2.63

$$-\frac{c\tanh^2(e+fx)}{2f} + \frac{c\log(\cosh(e+fx))}{f} + d\,\mathrm{csch}(e)\mathrm{sech}(e)\left(f^2x^2e^{-\tanh^{-1}(\coth(e))} - \frac{i\coth(e)\left(i\mathrm{Li}_2\left(e^{2i\left(ifx+i\tanh^{-1}(\coth(e))\right)}\right)\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Tanh[e + f*x]^3,x]
```

```
[Out] (c*Log[Cosh[e + f*x]])/f + (d*x*Sech[e + f*x]^2)/(2*f) + (d*Csch[e]*((f^2*x
^2)/E^ArcTanh[Coth[e]] - (I*Coth[e]*(-(f*x*(-Pi + (2*I)*ArcTanh[Coth[e]]))
- Pi*Log[1 + E^(2*f*x)] - 2*(I*f*x + I*ArcTanh[Coth[e]])*Log[1 - E^((2*I)*(
I*f*x + I*ArcTanh[Coth[e]]))] + Pi*Log[Cosh[f*x]] + (2*I)*ArcTanh[Coth[e]]*
Log[I*Sinh[f*x + ArcTanh[Coth[e]]]] + I*PolyLog[2, E^((2*I)*(I*f*x + I*ArcT
anh[Coth[e]]))]))/Sqrt[1 - Coth[e]^2])*Sech[e])/(2*f^2*Sqrt[Csch[e]^2*(-Cos
h[e]^2 + Sinh[e]^2)]) - (d*Sech[e]*Sech[e + f*x]*Sinh[f*x])/(2*f^2) + (d*x^
2*Tanh[e])/2 - (c*Tanh[e + f*x]^2)/(2*f)
```

fricas [C] time = 0.54, size = 1462, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(d*f^2*x^2 + (d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*cosh(f*x + e)
^4 + 4*(d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*cosh(f*x + e)*sinh(f*x +
```

```
 e)^3 + (d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*sinh(f*x + e)^4 + 2*c*f
^2*x - 2*d*e^2 + 4*c*e*f + 2*(d*f^2*x^2 - 2*d*e^2 + 2*(2*c*e - c)*f + 2*(c*
f^2 - d*f)*x - d)*cosh(f*x + e)^2 + 2*(d*f^2*x^2 - 2*d*e^2 + 3*(d*f^2*x^2 +
 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*cosh(f*x + e)^2 + 2*(2*c*e - c)*f + 2*(c*f^
2 - d*f)*x - d)*sinh(f*x + e)^2 - 2*(d*cosh(f*x + e)^4 + 4*d*cosh(f*x + e)*
sinh(f*x + e)^3 + d*sinh(f*x + e)^4 + 2*d*cosh(f*x + e)^2 + 2*(3*d*cosh(f*x
 + e)^2 + d)*sinh(f*x + e)^2 + 4*(d*cosh(f*x + e)^3 + d*cosh(f*x + e))*sinh
(f*x + e) + d)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) - 2*(d*cosh(f*x + e
)^4 + 4*d*cosh(f*x + e)*sinh(f*x + e)^3 + d*sinh(f*x + e)^4 + 2*d*cosh(f*x
+ e)^2 + 2*(3*d*cosh(f*x + e)^2 + d)*sinh(f*x + e)^2 + 4*(d*cosh(f*x + e)^3
 + d*cosh(f*x + e))*sinh(f*x + e) + d)*dilog(-I*cosh(f*x + e) - I*sinh(f*x
+ e)) + 2*((d*e - c*f)*cosh(f*x + e)^4 + 4*(d*e - c*f)*cosh(f*x + e)*sinh(f
*x + e)^3 + (d*e - c*f)*sinh(f*x + e)^4 + 2*(d*e - c*f)*cosh(f*x + e)^2 + 2
*(3*(d*e - c*f)*cosh(f*x + e)^2 + d*e - c*f)*sinh(f*x + e)^2 + d*e - c*f +
4*((d*e - c*f)*cosh(f*x + e)^3 + (d*e - c*f)*cosh(f*x + e))*sinh(f*x + e))*
log(cosh(f*x + e) + sinh(f*x + e) + I) + 2*((d*e - c*f)*cosh(f*x + e)^4 + 4
*(d*e - c*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (d*e - c*f)*sinh(f*x + e)^4 +
2*(d*e - c*f)*cosh(f*x + e)^2 + 2*(3*(d*e - c*f)*cosh(f*x + e)^2 + d*e - c*
f)*sinh(f*x + e)^2 + d*e - c*f + 4*((d*e - c*f)*cosh(f*x + e)^3 + (d*e - c*
f)*cosh(f*x + e))*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) - I) - 2
*((d*f*x + d*e)*cosh(f*x + e)^4 + 4*(d*f*x + d*e)*cosh(f*x + e)*sinh(f*x +
e)^3 + (d*f*x + d*e)*sinh(f*x + e)^4 + d*f*x + 2*(d*f*x + d*e)*cosh(f*x + e
)^2 + 2*(d*f*x + 3*(d*f*x + d*e)*cosh(f*x + e)^2 + d*e)*sinh(f*x + e)^2 + d
*e + 4*((d*f*x + d*e)*cosh(f*x + e)^3 + (d*f*x + d*e)*cosh(f*x + e))*sinh(f
*x + e))*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) - 2*((d*f*x + d*e)*cosh
(f*x + e)^4 + 4*(d*f*x + d*e)*cosh(f*x + e)*sinh(f*x + e)^3 + (d*f*x + d*e)
*sinh(f*x + e)^4 + d*f*x + 2*(d*f*x + d*e)*cosh(f*x + e)^2 + 2*(d*f*x + 3*(
d*f*x + d*e)*cosh(f*x + e)^2 + d*e)*sinh(f*x + e)^2 + d*e + 4*((d*f*x + d*e
)*cosh(f*x + e)^3 + (d*f*x + d*e)*cosh(f*x + e))*sinh(f*x + e))*log(-I*cosh
(f*x + e) - I*sinh(f*x + e) + 1) + 4*((d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*
c*e*f)*cosh(f*x + e)^3 + (d*f^2*x^2 - 2*d*e^2 + 2*(2*c*e - c)*f + 2*(c*f^2
- d*f)*x - d)*cosh(f*x + e))*sinh(f*x + e) - 2*d)/(f^2*cosh(f*x + e)^4 + 4*
f^2*cosh(f*x + e)*sinh(f*x + e)^3 + f^2*sinh(f*x + e)^4 + 2*f^2*cosh(f*x +
e)^2 + 2*(3*f^2*cosh(f*x + e)^2 + f^2)*sinh(f*x + e)^2 + f^2 + 4*(f^2*cosh(
f*x + e)^3 + f^2*cosh(f*x + e))*sinh(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\tanh\left(fx + e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*tanh(f*x + e)^3, x)
```

maple [A] time = 0.22, size = 166, normalized size = 1.66

$$-\frac{d\,x^2}{2}+cx+\frac{2dfx\,\mathrm{e}^{2fx+2e}+2cf\,\mathrm{e}^{2fx+2e}+\mathrm{e}^{2fx+2e}d+d}{f^2\left(\mathrm{e}^{2fx+2e}+1\right)^2}+\frac{c\ln\left(\mathrm{e}^{2fx+2e}+1\right)}{f}-\frac{2c\ln\left(\mathrm{e}^{fx+e}\right)}{f}-\frac{2dex}{f}-\frac{d\,e^2}{f^2}+\frac{d\ln\left(\mathrm{e}^{2fx+2}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tanh(f*x+e)^3,x)
```

```
[Out] -1/2*d*x^2+c*x+(2*d*f*x*exp(2*f*x+2*e)+2*c*f*exp(2*f*x+2*e)+exp(2*f*x+2*e)*
d+d)/f^2/(exp(2*f*x+2*e)+1)^2+1/f*c*ln(exp(2*f*x+2*e)+1)-2/f*c*ln(exp(f*x+e
))-2/f*d*e*x-1/f^2*d*e^2+1/f*d*ln(exp(2*f*x+2*e)+1)*x+1/2*d*polylog(2,-exp(
2*f*x+2*e))/f^2+2/f^2*d*e*ln(exp(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c\left(x+\frac{e}{f}+\frac{\log\left(e^{\left(-2fx-2e\right)}+1\right)}{f}+\frac{2e^{\left(-2fx-2e\right)}}{f\left(2e^{\left(-2fx-2e\right)}+e^{\left(-4fx-4e\right)}+1\right)}\right)+\frac{1}{2}d\left(\frac{f^2x^2e^{\left(4fx+4e\right)}+f^2x^2+2\left(f^2x^2e^{(2e)}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] c*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2*f
*x - 2*e) + e^(-4*f*x - 4*e) + 1))) + 1/2*d*((f^2*x^2*e^(4*f*x + 4*e) + f^2
*x^2 + 2*(f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + 2)/(f^2*e^
(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) - 4*integrate(x/(e^(2*f*x + 2*
e) + 1), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh\left(e+f\,x\right)^3\,\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3*(c + d*x),x)
```

```
[Out] int(tanh(e + f*x)^3*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tanh^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tanh(f*x+e)**3,x)
```

```
[Out] Integral((c + d*x)*tanh(e + f*x)**3, x)
```

3.14 $\int \frac{\tanh^3(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^3(e+fx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tanh(f*x+e)^3/(d*x+c),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[e + f*x]^3/(c + d*x),x]
```

```
[Out] Defer[Int][Tanh[e + f*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh^3(e+fx)}{c+dx}\,dx$$

Mathematica [A] time = 31.07, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[e + f*x]^3/(c + d*x),x]
```

```
[Out] Integrate[Tanh[e + f*x]^3/(c + d*x), x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tanh\left(fx+e\right)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)^3/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^3/(d*x + c), x)
```

maple [A] time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3\left(fx+e\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(d*x+c),x)
```

```
[Out] int(tanh(f*x+e)^3/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2\,dfxe^{(2e)}+2\,cfe^{(2e)}-de^{(2e)}\right)e^{\left(2fx\right)}-d}{d^2f^2x^2+2\,cdf^2x+c^2f^2+\left(d^2f^2x^2e^{(4e)}+2\,cdf^2xe^{(4e)}+c^2f^2e^{(4e)}\right)e^{\left(4fx\right)}+2\left(d^2f^2x^2e^{(2e)}+2\,cdf^2xe^{(2e)}+c^2f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((2*d*f*x*e^(2*e) + 2*c*f*e^(2*e) - d*e^(2*e))*e^(2*f*x) - d)/(d^2*f^2*x^2
+ 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2*e^(4*e) + 2*c*d*f^2*x*e^(4*e) + c^2*
f^2*e^(4*e))*e^(4*f*x) + 2*(d^2*f^2*x^2*e^(2*e) + 2*c*d*f^2*x*e^(2*e) + c^2
*f^2*e^(2*e))*e^(2*f*x)) + log(d*x + c)/d - integrate(2*(d^2*f^2*x^2 + 2*c*
d*f^2*x + c^2*f^2 + d^2)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*x + c
^3*f^2 + (d^3*f^2*x^3*e^(2*e) + 3*c*d^2*f^2*x^2*e^(2*e) + 3*c^2*d*f^2*x*e^(
2*e) + c^3*f^2*e^(2*e))*e^(2*f*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tanh\left(e+f\,x\right)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(c + d*x),x)
```

```
[Out] int(tanh(e + f*x)^3/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3\left(e+fx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(d*x+c),x)
```

```
[Out] Integral(tanh(e + f*x)**3/(c + d*x), x)
```

3.15 $\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tanh^3(e+fx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(tanh(f*x+e)^3/(d*x+c)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tanh[e + f*x]^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Tanh[e + f*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 23.15, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tanh[e + f*x]^3/(c + d*x)^2,x]
```

```
[Out] Integrate[Tanh[e + f*x]^3/(c + d*x)^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tanh\left(fx+e\right)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh\left(fx+e\right)^3}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^3/(d*x + c)^2, x)
```

maple [A] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3\left(fx+e\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(d*x+c)^2,x)
```

```
[Out] int(tanh(f*x+e)^3/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{d^2f^2x^2+2\,cdf^2x+c^2f^2+2\,d^2+\left(d^2f^2x^2e^{(4\,e)}+2\,cdf^2xe^{(4\,e)}+c^2f^2e^{(4\,e)}\right)e^{\left(4\,fx\right)}+2\left(d^2f^2x^2e^{(2\,e)}+c^2\right.}{d^4f^2x^3+3\,cd^3f^2x^2+3\,c^2d^2f^2x+c^3df^2+\left(d^4f^2x^3e^{(4\,e)}+3\,cd^3f^2x^2e^{(4\,e)}+3\,c^2d^2f^2xe^{(4\,e)}+c^3df^2e^{(4\,e)}\right)e^{\left(4\,fx\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2 + (d^2*f^2*x^2*e^(4*e) + 2*c*
d*f^2*x*e^(4*e) + c^2*f^2*e^(4*e))*e^(4*f*x) + 2*(d^2*f^2*x^2*e^(2*e) + c^2
*f^2*e^(2*e) - c*d*f*e^(2*e) + d^2*e^(2*e) + (2*c*d*f^2*e^(2*e) - d^2*f*e^(
2*e))*x)*e^(2*f*x))/(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*
d*f^2 + (d^4*f^2*x^3*e^(4*e) + 3*c*d^3*f^2*x^2*e^(4*e) + 3*c^2*d^2*f^2*x*e^
(4*e) + c^3*d*f^2*e^(4*e))*e^(4*f*x) + 2*(d^4*f^2*x^3*e^(2*e) + 3*c*d^3*f^2
*x^2*e^(2*e) + 3*c^2*d^2*f^2*x*e^(2*e) + c^3*d*f^2*e^(2*e))*e^(2*f*x)) - in
tegrate(2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 3*d^2)/(d^4*f^2*x^4 + 4*c*
d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4*e^
(2*e) + 4*c*d^3*f^2*x^3*e^(2*e) + 6*c^2*d^2*f^2*x^2*e^(2*e) + 4*c^3*d*f^2*x
*e^(2*e) + c^4*f^2*e^(2*e))*e^(2*f*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tanh\left(e+f\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(e + f*x)^3/(c + d*x)^2,x)
```

```
[Out] int(tanh(e + f*x)^3/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3\left(e+fx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(d*x+c)**2,x)
```

```
[Out] Integral(tanh(e + f*x)**3/(c + d*x)**2, x)
```

3.16 $\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$

Optimal. Leaf size=1392

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2(-b)^{5/2}}{2f^2}-\frac{(c+dx)\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)(-b)^{5/2}}{f}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\log\left(\frac{2}{1-\frac{\sqrt{b\tanh(e}}{\sqrt{-b}}}\right.}{f^2}$$

```
[Out] 2/3*b^(5/2)*d*arctan((b*tanh(f*x+e))^(1/2)/b^(1/2))/f^2-(-b)^(5/2)*(d*x+c)*
arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f-1/2*(-b)^(5/2)*d*arctanh((b*tan
h(f*x+e))^(1/2)/(-b)^(1/2))^2/f^2+2/3*b^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/
2)/b^(1/2))/f^2+b^(5/2)*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f+1/
2*b^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/f^2-b^(5/2)*d*arctanh(
(b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2))
)/f^2+b^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2
)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b
^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2)
)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(5/2)*d*arctanh((b*tanh(f*x+e)
)^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2
)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+(-b)^(5/2)*d*arctanh((b*tan
h(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-
1/2*(-b)^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b
*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/
2)))/f^2-1/2*(-b)^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*(
b^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2
)/(-b)^(1/2)))/f^2-(-b)^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*l
n(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/2*b^(5/2)*d*polylog(2,1-2*b
^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(5/2)*d*polylog(2,1-2*b^(
1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+1/4*b^(5/2)*d*polylog(2,1-2*b^(1/
2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh
(f*x+e))^(1/2)))/f^2+1/4*b^(5/2)*d*polylog(2,1-2*b^(1/2)*((-b)^(1/2)+(b*tan
h(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+
1/2*(-b)^(5/2)*d*polylog(2,1-2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/
4*(-b)^(5/2)*d*polylog(2,1-2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^
(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/4*(-b)^(5/2)*d*polylog(2
,1+2*(b^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e)
)^(1/2)/(-b)^(1/2)))/f^2+1/2*(-b)^(5/2)*d*polylog(2,1-2/(1+(b*tanh(f*x+e))^
(1/2)/(-b)^(1/2)))/f^2-4/3*b^2*d*(b*tanh(f*x+e))^(1/2)/f^2-2/3*b*(d*x+c)*(b
*tanh(f*x+e))^(3/2)/f
```

Rubi [F] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)*(b*Tanh[e + f*x])^(5/2),x]
```

```
[Out] (2*b^(5/2)*d*ArcTan[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/(3*f^2) + (2*b^(5/2)*d*
ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/(3*f^2) - (4*b^2*d*Sqrt[b*Tanh[e +
f*x]])/(3*f^2) - (2*b*(c + d*x)*(b*Tanh[e + f*x])^(3/2))/(3*f) + b^2*Defer[
Int][(c + d*x)*Sqrt[b*Tanh[e + f*x]], x]
```

Rubi steps

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx = -\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f} + b^2\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx + \frac{(2bd)}{}$$

$$= -\frac{4b^2d\sqrt{b\tanh(e+fx)}}{3f^2} - \frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f} + b^2\int (c+dx)\sqrt{b}$$

$$= -\frac{4b^2d\sqrt{b\tanh(e+fx)}}{3f^2} - \frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f} + b^2\int (c+dx)\sqrt{b}$$

$$= -\frac{4b^2d\sqrt{b\tanh(e+fx)}}{3f^2} - \frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f} + b^2\int (c+dx)\sqrt{b}$$

$$= -\frac{4b^2d\sqrt{b\tanh(e+fx)}}{3f^2} - \frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f} + b^2\int (c+dx)\sqrt{b}$$

$$= \frac{2b^{5/2}d\tan^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{3f^2} + \frac{2b^{5/2}d\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{3f^2} - \frac{4b^2d\sqrt{b\tanh}}{3f^2}$$

Mathematica **[F]** time = 39.85, size = 0, normalized size = 0.00

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)*(b*Tanh[e + f*x])^(5/2),x]
```

```
[Out] Integrate[(c + d*x)*(b*Tanh[e + f*x])^(5/2), x]
```

fricas **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*tanh(f*x + e))^(5/2), x)
```

maple **[F]** time = 0.22, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(b*tanh(f*x+e))^(5/2),x)
```

```
[Out] int((d*x+c)*(b*tanh(f*x+e))^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \left(b \tanh\left(fx + e\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((d*x + c)*(b*tanh(f*x + e))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b \tanh\left(e + f\,x\right)\right)^{5/2} (c + d\,x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(e + f*x))^(5/2)*(c + d*x),x)
[Out] int((b*tanh(e + f*x))^(5/2)*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tanh\left(e + fx\right)\right)^{\frac{5}{2}} (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))**(5/2),x)
[Out] Integral((b*tanh(e + f*x))**(5/2)*(c + d*x), x)
```

3.17 $\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$

Optimal. Leaf size=1363

$$-\frac{(-b)^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2}{2f^2}-\frac{(-b)^{3/2}(c+dx)\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{(-b)^{3/2}d\log\left(\frac{2}{1-\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}}\right)\tanh^{-1}\left(\right.}{f^2}$$

```
[Out] -2*b^(3/2)*d*arctan((b*tanh(f*x+e))^(1/2)/b^(1/2))/f^2-(-b)^(3/2)*(d*x+c)*a
rctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f-1/2*(-b)^(3/2)*d*arctanh((b*tanh
(f*x+e))^(1/2)/(-b)^(1/2))^2/f^2+2*b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/
b^(1/2))/f^2+b^(3/2)*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f+1/2*b
^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/f^2-b^(3/2)*d*arctanh((b*
tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f
^2+b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(
b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1
/2))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(
b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(3/2)*d*arctanh((b*tanh(f*x+e))^(
1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b
^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+(-b)^(3/2)*d*arctanh((b*tanh(f
*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/2
*(-b)^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*ta
nh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2))
)/f^2-1/2*(-b)^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*(b^(
1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(
-b)^(1/2)))/f^2-(-b)^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2
/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/2*b^(3/2)*d*polylog(2,1-2*b^(1
/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(3/2)*d*polylog(2,1-2*b^(1/2
)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+1/4*b^(3/2)*d*polylog(2,1-2*b^(1/2)*
((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*
x+e))^(1/2)))/f^2+1/4*b^(3/2)*d*polylog(2,1-2*b^(1/2)*((-b)^(1/2)+(b*tanh(f
*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+1/2
*(-b)^(3/2)*d*polylog(2,1-2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/4*(
-b)^(3/2)*d*polylog(2,1-2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/
2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/4*(-b)^(3/2)*d*polylog(2,1+
2*(b^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(
1/2)/(-b)^(1/2)))/f^2+1/2*(-b)^(3/2)*d*polylog(2,1-2/(1+(b*tanh(f*x+e))^(1/
2)/(-b)^(1/2)))/f^2-2*b*(d*x+c)*(b*tanh(f*x+e))^(1/2)/f
```

Rubi [F] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)*(b*Tanh[e + f*x])^(3/2),x]
```

```
[Out] (-2*b^(3/2)*d*ArcTan[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/f^2 + (2*b^(3/2)*d*Arc
Tanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/f^2 - (2*b*(c + d*x)*Sqrt[b*Tanh[e + f
*x]])/f + b^2*Defer[Int][(c + d*x)/Sqrt[b*Tanh[e + f*x]], x]
```

Rubi steps

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = -\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f} + b^2\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx + \frac{(2bd)\int \sqrt{b\tanh($$

$$= -\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f} + b^2\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx - \frac{\left(2b^2d\right)\text{Subst}\left(\int$$

$$= -\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f} + b^2\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx - \frac{\left(4b^2d\right)\text{Subst}\left(\int$$

$$= -\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f} + b^2\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx + \frac{\left(2b^2d\right)\text{Subst}\left(\int$$

$$= -\frac{2b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f^2} + \frac{2b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f^2} - \frac{2b(c+dx)\sqrt{b\,t}}{f}$$

Mathematica [F] time = 29.42, size = 0, normalized size = 0.00

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)*(b*Tanh[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(c + d*x)*(b*Tanh[e + f*x])^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*tanh(f*x + e))^(3/2), x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(b*tanh(f*x+e))^(3/2),x)
```

```
[Out] int((d*x+c)*(b*tanh(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \left(b \tanh\left(fx + e\right)\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*x + c)*(b*tanh(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b \tanh\left(e + f\,x\right)\right)^{3/2} \, (c + d\,x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(e + f*x))^(3/2)*(c + d*x),x)
[Out] int((b*tanh(e + f*x))^(3/2)*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tanh\left(e + fx\right)\right)^{\frac{3}{2}} (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))**(3/2),x)
[Out] Integral((b*tanh(e + f*x))**(3/2)*(c + d*x), x)
```

3.18 $\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$

Optimal. Leaf size=1280

$$-\frac{\sqrt{-b}\,d\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2}{2f^2}-\frac{\sqrt{-b}\,(c+dx)\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{-b}\,d\log\left(\frac{2}{1-\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}}\right)\tanh^{-1}\left(\frac{\sqrt{b\tanh(e}}{\sqrt{-b}}\right.}{f^2}$$

```
[Out] -(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*(-b)^(1/2)/f-1/2*d*arcta
nh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2*(-b)^(1/2)/f^2+d*arctanh((b*tanh(f*x
+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*(-b)^(1/2
)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*tanh
(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*
(-b)^(1/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*(b^(1/
2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b
)^(1/2)))*(-b)^(1/2)/f^2-d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(
1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*(-b)^(1/2)/f^2+1/2*d*polylog(2,1-2/(1-
(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*(-b)^(1/2)/f^2-1/4*d*polylog(2,1-2*(b^(1
/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-
b)^(1/2)))*(-b)^(1/2)/f^2-1/4*d*polylog(2,1+2*(b^(1/2)+(b*tanh(f*x+e))^(1/2
))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*(-b)^(1/2)/f^
2+1/2*d*polylog(2,1-2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))*(-b)^(1/2)/f^2+
(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*b^(1/2)/f+1/2*d*arctanh((b*t
anh(f*x+e))^(1/2)/b^(1/2))^2*b^(1/2)/f^2-d*arctanh((b*tanh(f*x+e))^(1/2)/b^
(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))*b^(1/2)/f^2+d*arctanh(
(b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2))
)*b^(1/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-
b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e
))^(1/2)))*b^(1/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^
(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*t
anh(f*x+e))^(1/2)))*b^(1/2)/f^2-1/2*d*polylog(2,1-2*b^(1/2)/(b^(1/2)-(b*tan
h(f*x+e))^(1/2)))*b^(1/2)/f^2-1/2*d*polylog(2,1-2*b^(1/2)/(b^(1/2)+(b*tanh(
f*x+e))^(1/2)))*b^(1/2)/f^2+1/4*d*polylog(2,1-2*b^(1/2)*((-b)^(1/2)-(b*tanh
(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))*b^(1/
2)/f^2+1/4*d*polylog(2,1-2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)
^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))*b^(1/2)/f^2
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)*Sqrt[b*Tanh[e + f*x]],x]
```

```
[Out] Defer[Int][(c + d*x)*Sqrt[b*Tanh[e + f*x]], x]
```

Rubi steps

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$$

Mathematica [C] time = 5.32, size = 556, normalized size = 0.43

$$\frac{\sqrt{b\tanh(e+fx)}\left(-4f(c+dx)\left(\log\left(1-\sqrt{\tanh(e+fx)}\right)-\log\left(\sqrt{\tanh(e+fx)}+1\right)+2\tan^{-1}\left(\sqrt{\tanh(e+fx)}\right)\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sqrt[b*Tanh[e + f*x]],x]
```

```
[Out] ((-4*f*(c + d*x)*(2*ArcTan[Sqrt[Tanh[e + f*x]]] + Log[1 - Sqrt[Tanh[e + f*x
]]] - Log[1 + Sqrt[Tanh[e + f*x]]]) + d*((4*I)*ArcTan[Sqrt[Tanh[e + f*x]]]^
2 - 4*ArcTan[Sqrt[Tanh[e + f*x]]]*Log[1 + E^((4*I)*ArcTan[Sqrt[Tanh[e + f*x
]]])] - Log[1 - Sqrt[Tanh[e + f*x]]]^2 + 2*Log[1 - Sqrt[Tanh[e + f*x]]]*Log
[(1/2 + I/2)*(-I + Sqrt[Tanh[e + f*x]])] + 2*Log[1 - Sqrt[Tanh[e + f*x]]]*L
og[(1/2 - I/2)*(I + Sqrt[Tanh[e + f*x]])] - 2*Log[1 - Sqrt[Tanh[e + f*x]]]*
Log[(1 + Sqrt[Tanh[e + f*x]])/2] - 2*Log[1 - (1/2 - I/2)*(1 + Sqrt[Tanh[e +
 f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + 2*Log[(1 - Sqrt[Tanh[e + f*x]])/2]*
Log[1 + Sqrt[Tanh[e + f*x]]] - 2*Log[(-1/2 - I/2)*(I + Sqrt[Tanh[e + f*x]])
]*Log[1 + Sqrt[Tanh[e + f*x]]] + Log[1 + Sqrt[Tanh[e + f*x]]]^2 + I*PolyLog
[2, -E^((4*I)*ArcTan[Sqrt[Tanh[e + f*x]]])] - 2*PolyLog[2, (1 - Sqrt[Tanh[e
 + f*x]])/2] + 2*PolyLog[2, (-1/2 - I/2)*(-1 + Sqrt[Tanh[e + f*x]])] + 2*Po
lyLog[2, (-1/2 + I/2)*(-1 + Sqrt[Tanh[e + f*x]])] + 2*PolyLog[2, (1 + Sqrt[
Tanh[e + f*x]])/2] - 2*PolyLog[2, (1/2 - I/2)*(1 + Sqrt[Tanh[e + f*x]])] -
2*PolyLog[2, (1/2 + I/2)*(1 + Sqrt[Tanh[e + f*x]])]))*Sqrt[b*Tanh[e + f*x]]
)/(8*f^2*Sqrt[Tanh[e + f*x]])
```

fricas **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\sqrt{b \tanh\left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sqrt(b*tanh(f*x + e)), x)
```

maple **[F]** time = 0.26, size = 0, normalized size = 0.00

$$\int (dx + c)\sqrt{b \tanh\left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(b*tanh(f*x+e))^(1/2),x)
```

```
[Out] int((d*x+c)*(b*tanh(f*x+e))^(1/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\sqrt{b \tanh\left(fx + e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*sqrt(b*tanh(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{b \tanh\left(e + f\,x\right)}\ (c + d\,x)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(e + f*x))^(1/2)*(c + d*x),x)
```

```
[Out] int((b*tanh(e + f*x))^(1/2)*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh\left(e + fx\right)}\ (c + dx)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*tanh(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(b*tanh(e + f*x))*(c + d*x), x)
```

3.19 $\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$

Optimal. Leaf size=1280

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2}{2\sqrt{-b}\,f^2}-\frac{(c+dx)\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}\,f}+\frac{d\log\left(\frac{2}{1-\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}}\right)\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}\,f^2}-\frac{d\log}{}$$

```
[Out] -(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f/(-b)^(1/2)-1/2*d*arcta
nh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^2/(-b)^(1/2)+d*arctanh((b*tanh(f*x
+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2/(-b)^
(1/2)-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*tanh
(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/
f^2/(-b)^(1/2)-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*(b^(1/
2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b
)^(1/2)))/f^2/(-b)^(1/2)-d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(
1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2/(-b)^(1/2)+1/2*d*polylog(2,1-2/(1-
(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2/(-b)^(1/2)-1/4*d*polylog(2,1-2*(b^(1
/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-
b)^(1/2)))/f^2/(-b)^(1/2)-1/4*d*polylog(2,1+2*(b^(1/2)+(b*tanh(f*x+e))^(1/2
))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2/(-b)^(1/2
)+1/2*d*polylog(2,1-2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2/(-b)^(1/2)+
(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f/b^(1/2)+1/2*d*arctanh((b*t
anh(f*x+e))^(1/2)/b^(1/2))^2/f^2/b^(1/2)-d*arctanh((b*tanh(f*x+e))^(1/2)/b^
(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^2/b^(1/2)+d*arctanh(
(b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2))
)/f^2/b^(1/2)-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-
b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e
))^(1/2)))/f^2/b^(1/2)-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^
(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*t
anh(f*x+e))^(1/2)))/f^2/b^(1/2)-1/2*d*polylog(2,1-2*b^(1/2)/(b^(1/2)-(b*tan
h(f*x+e))^(1/2)))/f^2/b^(1/2)-1/2*d*polylog(2,1-2*b^(1/2)/(b^(1/2)+(b*tanh(
f*x+e))^(1/2)))/f^2/b^(1/2)+1/4*d*polylog(2,1-2*b^(1/2)*((-b)^(1/2)-(b*tanh
(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2/b
^(1/2)+1/4*d*polylog(2,1-2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)
^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2/b^(1/2)
```

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)/Sqrt[b*Tanh[e + f*x]],x]
```

```
[Out] Defer[Int][(c + d*x)/Sqrt[b*Tanh[e + f*x]], x]
```

Rubi steps

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$$

Mathematica [C] time = 4.39, size = 556, normalized size = 0.43

$$\frac{\sqrt{\tanh(e+fx)}\left(4f(c+dx)\left(-\log\left(1-\sqrt{\tanh(e+fx)}\right)+\log\left(\sqrt{\tanh(e+fx)}+1\right)+2\tan^{-1}\left(\sqrt{\tanh(e+fx)}\right)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)/Sqrt[b*Tanh[e + f*x]],x]
```

```
[Out] ((4*f*(c + d*x)*(2*ArcTan[Sqrt[Tanh[e + f*x]]] - Log[1 - Sqrt[Tanh[e + f*x]
]] + Log[1 + Sqrt[Tanh[e + f*x]]]) + d*((-4*I)*ArcTan[Sqrt[Tanh[e + f*x]]]^
2 + 4*ArcTan[Sqrt[Tanh[e + f*x]]]*Log[1 + E^((4*I)*ArcTan[Sqrt[Tanh[e + f*x
]]])] - Log[1 - Sqrt[Tanh[e + f*x]]]^2 + 2*Log[1 - Sqrt[Tanh[e + f*x]]]*Log
[(1/2 + I/2)*(-I + Sqrt[Tanh[e + f*x]])] + 2*Log[1 - Sqrt[Tanh[e + f*x]]]*L
og[(1/2 - I/2)*(I + Sqrt[Tanh[e + f*x]])] - 2*Log[1 - Sqrt[Tanh[e + f*x]]]*
Log[(1 + Sqrt[Tanh[e + f*x]])/2] - 2*Log[1 - (1/2 - I/2)*(1 + Sqrt[Tanh[e +
 f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + 2*Log[(1 - Sqrt[Tanh[e + f*x]])/2]*
Log[1 + Sqrt[Tanh[e + f*x]]] - 2*Log[(-1/2 - I/2)*(I + Sqrt[Tanh[e + f*x]])
]*Log[1 + Sqrt[Tanh[e + f*x]]] + Log[1 + Sqrt[Tanh[e + f*x]]]^2 - I*PolyLog
[2, -E^((4*I)*ArcTan[Sqrt[Tanh[e + f*x]]])] - 2*PolyLog[2, (1 - Sqrt[Tanh[e
 + f*x]])/2] + 2*PolyLog[2, (-1/2 - I/2)*(-1 + Sqrt[Tanh[e + f*x]])] + 2*Po
lyLog[2, (-1/2 + I/2)*(-1 + Sqrt[Tanh[e + f*x]])] + 2*PolyLog[2, (1 + Sqrt[
Tanh[e + f*x]])/2] - 2*PolyLog[2, (1/2 - I/2)*(1 + Sqrt[Tanh[e + f*x]])] -
2*PolyLog[2, (1/2 + I/2)*(1 + Sqrt[Tanh[e + f*x]])]))*Sqrt[Tanh[e + f*x]])/
(8*f^2*Sqrt[b*Tanh[e + f*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/sqrt(b*tanh(f*x + e)), x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

```
[Out] int((d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\sqrt{b \tanh\left(fx + e\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/sqrt(b*tanh(f*x + e)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c + d\,x}{\sqrt{b \tanh\left(e + f\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(b*tanh(e + f*x))^(1/2),x)
[Out] int((c + d*x)/(b*tanh(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx}{\sqrt{b \tanh\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*tanh(f*x+e))**(1/2),x)
[Out] Integral((c + d*x)/sqrt(b*tanh(e + f*x)), x)
```

3.20 $\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=1365

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2}{2(-b)^{3/2}f^2}-\frac{(c+dx)\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{(-b)^{3/2}f}+\frac{d\log\left(\frac{2}{1-\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}}\right)\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{(-b)^{3/2}f^2}-\frac{d\log\left($$

```
[Out] 2*d*arctan((b*tanh(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f^2-(d*x+c)*arctanh((b*ta
nh(f*x+e))^(1/2)/(-b)^(1/2))/(-b)^(3/2)/f-1/2*d*arctanh((b*tanh(f*x+e))^(1/
2)/(-b)^(1/2))^2/(-b)^(3/2)/f^2+2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/
b^(3/2)/f^2+(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f+1/2*d*
arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/b^(3/2)/f^2-d*arctanh((b*tanh(f*x+
e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^
2+d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*
x+e))^(1/2)))/b^(3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2
*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(
b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1
/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(
b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2+d*arctanh((b*tanh(f*x+e))^(1/2)
/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^2-1/2*
d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*tanh(f*x+e))^(
1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)
/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*(b^(1/2)+(b*tanh
(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/
(-b)^(3/2)/f^2-d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1+(b*tanh(
f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^2-1/2*d*polylog(2,1-2*b^(1/2)/(b^(1
/2)-(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2-1/2*d*polylog(2,1-2*b^(1/2)/(b^(1/2
)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2+1/4*d*polylog(2,1-2*b^(1/2)*((-b)^(1/
2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/
2)))/b^(3/2)/f^2+1/4*d*polylog(2,1-2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1
/2))/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2+1/2*
d*polylog(2,1-2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^2-1/4*d*
polylog(2,1-2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*ta
nh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^2-1/4*d*polylog(2,1+2*(b^(1/2)+(
b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1
/2)))/(-b)^(3/2)/f^2+1/2*d*polylog(2,1-2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2
)))/(-b)^(3/2)/f^2-2*(d*x+c)/b/f/(b*tanh(f*x+e))^(1/2)
```

Rubi [F] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)/(b*Tanh[e + f*x])^(3/2),x]
```

```
[Out] (2*d*ArcTan[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/(b^(3/2)*f^2) + (2*d*ArcTanh[Sq
rt[b*Tanh[e + f*x]]/Sqrt[b]])/(b^(3/2)*f^2) - (2*(c + d*x))/(b*f*Sqrt[b*Tan
h[e + f*x]]) + Defer[Int][(c + d*x)*Sqrt[b*Tanh[e + f*x]], x]/b^2
```

Rubi steps

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = -\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}} + \frac{\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx}{b^2} + \frac{(2d)\int \frac{1}{\sqrt{b\tanh(e+fx)}}\,dx}{bf}$$

$$= -\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}} + \frac{\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx}{b^2} - \frac{(2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-b^2+x^2\right)}\right.}{f^2}$$

$$= -\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}} + \frac{\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx}{b^2} - \frac{(4d)\,\text{Subst}\left(\int \frac{1}{-b^2+x^4}\,dx,\right.}{f^2}$$

$$= -\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}} + \frac{\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx}{b^2} + \frac{(2d)\,\text{Subst}\left(\int \frac{1}{b-x^2}\,dx, x,\right.}{bf^2}$$

$$= \frac{2d\tan^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f^2} + \frac{2d\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{b^{3/2}f^2} - \frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}} + \frac{\int (c$$

Mathematica [F] time = 27.75, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)/(b*Tanh[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(c + d*x)/(b*Tanh[e + f*x])^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*tanh(f*x + e))^(3/2), x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*tanh(f*x+e))^(3/2),x)
[Out] int((d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\left(b \tanh\left(fx + e\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*x + c)/(b*tanh(f*x + e))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c + d\,x}{\left(b \tanh\left(e + f\,x\right)\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(b*tanh(e + f*x))^(3/2),x)
[Out] int((c + d*x)/(b*tanh(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c + dx}{\left(b \tanh\left(e + fx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*tanh(f*x+e))**(3/2),x)
[Out] Integral((c + d*x)/(b*tanh(e + f*x))**(3/2), x)
```

3.21 $\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx$

Optimal. Leaf size=1341

$$\text{Int}\left(\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}},x\right)b^2+\frac{2d^2\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2 b^{3/2}}{f^3}+\frac{4d(c+dx)\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)b^{3/2}}{f^2}-\frac{4d^2\tanh$$

```
[Out] 4*(-b)^(3/2)*d*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f^2+2*(-b)
^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^3+4*b^(3/2)*d*(d*x
+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f^2+2*b^(3/2)*d^2*arctanh((b*tan
h(f*x+e))^(1/2)/b^(1/2))^2/f^3-4*b^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/
b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^3+4*b^(3/2)*d^2*ar
ctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^
(1/2)))/f^3-2*b^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/
2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh
(f*x+e))^(1/2)))/f^3-2*b^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*l
n(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1/2
)+(b*tanh(f*x+e))^(1/2)))/f^3-4*(-b)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2
)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^3+2*(-b)^(3/2)*d
^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*tanh(f*x+e))^
(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^3+2*(-b
)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*(b^(1/2)+(b*tan
h(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))
/f^3+4*(-b)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1+(b*
tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^3-2*b^(3/2)*d^2*polylog(2,1-2*b^(1/2)/(b^
(1/2)-(b*tanh(f*x+e))^(1/2)))/f^3-2*b^(3/2)*d^2*polylog(2,1-2*b^(1/2)/(b^(1
/2)+(b*tanh(f*x+e))^(1/2)))/f^3+b^(3/2)*d^2*polylog(2,1-2*b^(1/2)*((-b)^(1/
2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/
2)))/f^3+b^(3/2)*d^2*polylog(2,1-2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2
))/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^3-2*(-b)^(3/2)*d
^2*polylog(2,1-2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^3+(-b)^(3/2)*d^2*p
olylog(2,1-2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tan
h(f*x+e))^(1/2)/(-b)^(1/2)))/f^3+(-b)^(3/2)*d^2*polylog(2,1+2*(b^(1/2)+(b*t
anh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)
))/f^3-2*(-b)^(3/2)*d^2*polylog(2,1-2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))
/f^3-2*b*(d*x+c)^2*(b*tanh(f*x+e))^(1/2)/f+b^2*Unintegrable((d*x+c)^2/(b*ta
nh(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^2*(b*Tanh[e + f*x])^(3/2),x]
```

```
[Out] (-2*b*(c + d*x)^2*Sqrt[b*Tanh[e + f*x]])/f + b^2*Defer[Int][(c + d*x)^2/Sqr
t[b*Tanh[e + f*x]], x] + (4*b*d*Defer[Int][(c + d*x)*Sqrt[b*Tanh[e + f*x]],
 x])/f
```

Rubi steps

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx = -\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}+b^2\int\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx+\frac{(4bd)\int(c+$$

Mathematica [A] time = 28.72, size = 0, normalized size = 0.00

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^2*(b*Tanh[e + f*x])^(3/2),x]
[Out] Integrate[(c + d*x)^2*(b*Tanh[e + f*x])^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
[Out] integrate((d*x + c)^2*(b*tanh(f*x + e))^(3/2), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x)
[Out] int((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^2*(b*tanh(f*x + e))^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(b\tanh\left(e+fx\right)\right)^{3/2} (c+dx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(e + f*x))^(3/2)*(c + d*x)^2,x)
[Out] int((b*tanh(e + f*x))^(3/2)*(c + d*x)^2, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tanh\left(e + fx\right)\right)^{\frac{3}{2}} (c + dx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(b*tanh(f*x+e))**(3/2),x)
[Out] Integral((b*tanh(e + f*x))**(3/2)*(c + d*x)**2, x)
```

3.22 $\int (c+dx)^2 \sqrt{b \tanh(e+fx)}\, dx$

Optimal. Leaf size=23

$$\text{Int}\left((c+dx)^2\sqrt{b\tanh(e+fx)}, x\right)$$

```
[Out] Unintegrable((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^2*Sqrt[b*Tanh[e + f*x]],x]
```

```
[Out] Defer[Int][(c + d*x)^2*Sqrt[b*Tanh[e + f*x]], x]
```

Rubi steps

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\, dx = \int (c+dx)^2\sqrt{b\tanh(e+fx)}\, dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^2*Sqrt[b*Tanh[e + f*x]],x]
```

```
[Out] $Aborted
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\sqrt{b\tanh\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sqrt(b*tanh(f*x + e)), x)
```

maple [A] time = 0.29, size = 0, normalized size = 0.00

$$\int (dx+c)^2\sqrt{b\tanh\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
[Out] int((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \sqrt{b \tanh\left(fx+e\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^2*sqrt(b*tanh(f*x + e)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{b \tanh\left(e+f\,x\right)}\, (c+d\,x)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(e + f*x))^(1/2)*(c + d*x)^2,x)
[Out] int((b*tanh(e + f*x))^(1/2)*(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh\left(e+fx\right)}\, (c+dx)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(b*tanh(f*x+e))**(1/2),x)
[Out] Integral(sqrt(b*tanh(e + f*x))*(c + d*x)**2, x)
```

3.23 $\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}},x\right)$$

```
[Out] Unintegrable((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^2/Sqrt[b*Tanh[e + f*x]],x]
```

```
[Out] Defer[Int][(c + d*x)^2/Sqrt[b*Tanh[e + f*x]], x]
```

Rubi steps

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^2/Sqrt[b*Tanh[e + f*x]],x]
```

```
[Out] $Aborted
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/sqrt(b*tanh(f*x + e)), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
[Out] int((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x + c)^2/sqrt(b*tanh(f*x + e)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^2}{\sqrt{b\tanh\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(b*tanh(e + f*x))^(1/2),x)
[Out] int((c + d*x)^2/(b*tanh(e + f*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*tanh(f*x+e))**(1/2),x)
[Out] Integral((c + d*x)**2/sqrt(b*tanh(e + f*x)), x)
```

3.24 $\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=1343

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2 d^2}{(-b)^{3/2}f^3}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2 d^2}{b^{3/2}f^3}-\frac{4\tanh^{-1}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)d^2}{b^{3/2}f^3}+\frac{4\,ta}{}$$

```
[Out] 4*d*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/(-b)^(3/2)/f^2+2*d^2*
arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/(-b)^(3/2)/f^3+4*d*(d*x+c)*arct
anh((b*tanh(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f^2+2*d^2*arctanh((b*tanh(f*x+e)
)^(1/2)/b^(1/2))^2/b^(3/2)/f^3-4*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))
*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3+4*d^2*arctanh((b
*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/
b^(3/2)/f^3-2*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)
^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))
^(1/2)))/b^(3/2)/f^3-2*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1
/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tan
h(f*x+e))^(1/2)))/b^(3/2)/f^3-4*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2
))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^3+2*d^2*arctanh(
(b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b
)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^3+2*d^2
*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(-2*(b^(1/2)+(b*tanh(f*x+e))^(
1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)
/f^3+4*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1+(b*tanh(f*x+e)
)^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^3-2*d^2*polylog(2,1-2*b^(1/2)/(b^(1/2)-(b
*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3-2*d^2*polylog(2,1-2*b^(1/2)/(b^(1/2)+(b*t
anh(f*x+e))^(1/2)))/b^(3/2)/f^3+d^2*polylog(2,1-2*b^(1/2)*((-b)^(1/2)-(b*ta
nh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(
3/2)/f^3+d^2*polylog(2,1-2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)
^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3-2*d^2*polylog(
2,1-2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^3+d^2*polylog(2,1-
2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(
1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^3+d^2*polylog(2,1+2*(b^(1/2)+(b*tanh(f*x+e))
^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/
2)/f^3-2*d^2*polylog(2,1-2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)
/f^3-2*(d*x+c)^2/b/f/(b*tanh(f*x+e))^(1/2)+Unintegrable((d*x+c)^2*(b*tanh(f
*x+e))^(1/2),x)/b^2
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^2/(b*Tanh[e + f*x])^(3/2),x]
```

```
[Out] (-2*(c + d*x)^2)/(b*f*Sqrt[b*Tanh[e + f*x]]) + (4*d*Defer[Int][(c + d*x)/Sq
rt[b*Tanh[e + f*x]], x])/(b*f) + Defer[Int][(c + d*x)^2*Sqrt[b*Tanh[e + f*x
]], x]/b^2
```

Rubi steps

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = -\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}} + \frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx}{b^2} + \frac{(4d)\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx}{bf}$$

Mathematica [A] time = 35.59, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^2/(b*Tanh[e + f*x])^(3/2),x]
```

```
[Out] Integrate[(c + d*x)^2/(b*Tanh[e + f*x])^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*tanh(f*x + e))^(3/2), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x)
```

```
[Out] int((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2/(b*tanh(f*x + e))^(3/2), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{\left(b\tanh\left(e+f\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(b*tanh(e + f*x))^(3/2),x)
```

```
[Out] int((c + d*x)^2/(b*tanh(e + f*x))^(3/2), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{\left(b\tanh\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*tanh(f*x+e))**(3/2),x)
```

```
[Out] Integral((c + d*x)**2/(b*tanh(e + f*x))**(3/2), x)
```

3.25 $\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(b\tanh(e+fx))^{3/2}}{c+dx}, x\right)$$

```
[Out] Unintegrable((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(b*Tanh[e + f*x])^(3/2)/(c + d*x),x]
```

```
[Out] Defer[Int][(b*Tanh[e + f*x])^(3/2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$$

Mathematica [A] time = 28.75, size = 0, normalized size = 0.00

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(b*Tanh[e + f*x])^(3/2)/(c + d*x),x]
```

```
[Out] Integrate[(b*Tanh[e + f*x])^(3/2)/(c + d*x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(f*x+e))^(3/2)/(d*x+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(f*x+e))^(3/2)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*tanh(f*x + e))^(3/2)/(d*x + c), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(b \tanh\left(fx+e\right)\right)^{\frac{3}{2}}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
[Out] int((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \tanh\left(fx+e\right)\right)^{\frac{3}{2}}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(f*x+e))^(3/2)/(d*x+c),x, algorithm="maxima")
[Out] integrate((b*tanh(f*x + e))^(3/2)/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b \tanh\left(e+f\,x\right)\right)^{3/2}}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(e + f*x))^(3/2)/(c + d*x),x)
[Out] int((b*tanh(e + f*x))^(3/2)/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \tanh\left(e+fx\right)\right)^{\frac{3}{2}}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(f*x+e))**(3/2)/(d*x+c),x)
[Out] Integral((b*tanh(e + f*x))**(3/2)/(c + d*x), x)
```

3.26 $\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\sqrt{b \tanh(e+fx)}}{c+dx}, x\right)$$

```
[Out] Unintegrable((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[b*Tanh[e + f*x]]/(c + d*x),x]
```

```
[Out] Defer[Int][Sqrt[b*Tanh[e + f*x]]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx$$

Mathematica [A] time = 2.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[b*Tanh[e + f*x]]/(c + d*x),x]
```

```
[Out] Integrate[Sqrt[b*Tanh[e + f*x]]/(c + d*x), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(f*x+e))^(1/2)/(d*x+c),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \tanh\left(fx+e\right)}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(f*x+e))^(1/2)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*tanh(f*x + e))/(d*x + c), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \tanh\left(fx+e\right)}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
[Out] int((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \tanh\left(fx+e\right)}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(f*x+e))^(1/2)/(d*x+c),x, algorithm="maxima")
[Out] integrate(sqrt(b*tanh(f*x + e))/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{b \tanh\left(e+f\,x\right)}}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*tanh(e + f*x))^(1/2)/(c + d*x),x)
[Out] int((b*tanh(e + f*x))^(1/2)/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b \tanh\left(e+fx\right)}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*tanh(f*x+e))**(1/2)/(d*x+c),x)
[Out] Integral(sqrt(b*tanh(e + f*x))/(c + d*x), x)
```

3.27 $\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]),x]
```

```
[Out] Defer[Int][1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$$

Mathematica [A] time = 2.46, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]),x]
```

```
[Out] Integrate[1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*sqrt(b*tanh(f*x + e))), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
[Out] int(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((d*x + c)*sqrt(b*tanh(f*x + e))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{b\tanh\left(e+f\,x\right)}\ (c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tanh(e + f*x))^(1/2)*(c + d*x)),x)
[Out] int(1/((b*tanh(e + f*x))^(1/2)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\tanh\left(e+fx\right)}\ (c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*tanh(f*x+e))**(1/2),x)
[Out] Integral(1/(sqrt(b*tanh(e + f*x))*(c + d*x)), x)
```

3.28 $\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$$

Mathematica [A] time = 26.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)),x]
```

```
[Out] Integrate[1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*tanh(f*x + e))^(3/2)), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
[Out] int(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x + c)*(b*tanh(f*x + e))^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(b\tanh\left(e+f\,x\right)\right)^{3/2}\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*tanh(e + f*x))^(3/2)*(c + d*x)),x)
[Out] int(1/((b*tanh(e + f*x))^(3/2)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\tanh\left(e+fx\right)\right)^{\frac{3}{2}}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*tanh(f*x+e))**(3/2),x)
[Out] Integral(1/((b*tanh(e + f*x))**(3/2)*(c + d*x)), x)
```

3.29 $\int x^m \tanh^3(a+bx)\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(x^m \tanh^3(a+bx), x\right)$$

```
[Out] Unintegrable(x^m*tanh(b*x+a)^3,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \tanh^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Tanh[a + b*x]^3,x]
```

```
[Out] Defer[Int][x^m*Tanh[a + b*x]^3, x]
```

Rubi steps

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh^3(a+bx)\,dx$$

Mathematica [A] time = 146.54, size = 0, normalized size = 0.00

$$\int x^m \tanh^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Tanh[a + b*x]^3,x]
```

```
[Out] Integrate[x^m*Tanh[a + b*x]^3, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \tanh\left(bx+a\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*tanh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral(x^m*tanh(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \tanh\left(bx+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*tanh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^m*tanh(b*x + a)^3, x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int x^m \left(\tanh^3\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*tanh(b*x+a)^3,x)
```

```
[Out] int(x^m*tanh(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x e^{\left(6\,bx+m\log(x)+6\,a\right)}}{(m+1)e^{(6\,bx+6\,a)}+3\,(m+1)e^{(4\,bx+4\,a)}+3\,(m+1)e^{(2\,bx+2\,a)}+m+1}-\int \frac{\left(3\left(2\,bxe^{(6\,a)}+(m+1)e^{(6\,a)}\right)e^{(6\,bx}\right.}{(m+1)e^{(8\,bx+8\,a)}+4\,(m+1)e^{(6\,bx+6\,a)}+6\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*tanh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)*e^(4*b*x
+ 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate((3*(2*b*x*e^(6*a) +
 (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1)*x^m/((m +
1)*e^(8*b*x + 8*a) + 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4*a)
+ 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int x^m \tanh\left(a+b\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*tanh(a + b*x)^3,x)
```

```
[Out] int(x^m*tanh(a + b*x)^3, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \tanh^3\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*tanh(b*x+a)**3,x)
```

```
[Out] Integral(x**m*tanh(a + b*x)**3, x)
```

3.30 $\int x^m \tanh^2(a+bx)\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(x^m \tanh^2(a+bx), x\right)$$

```
[Out] Unintegrable(x^m*tanh(b*x+a)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Tanh[a + b*x]^2,x]
```

```
[Out] Defer[Int][x^m*Tanh[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh^2(a+bx)\,dx$$

Mathematica [A] time = 9.92, size = 0, normalized size = 0.00

$$\int x^m \tanh^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Tanh[a + b*x]^2,x]
```

```
[Out] Integrate[x^m*Tanh[a + b*x]^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \tanh\left(bx+a\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*tanh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral(x^m*tanh(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \tanh\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*tanh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*tanh(b*x + a)^2, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int x^m \left(\tanh^2\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*tanh(b*x+a)^2,x)
```

```
[Out] int(x^m*tanh(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x e^{\left(4\,bx+m\log(x)+4\,a\right)}}{(m+1)e^{(4\,bx+4\,a)}+2\,(m+1)e^{(2\,bx+2\,a)}+m+1}-\int \frac{\left(2\left(2\,bxe^{(4\,a)}+(m+1)e^{(4\,a)}\right)e^{(4\,bx)}+(m+1)e^{(2\,bx+2\,a)}-m-1\right)x}{(m+1)e^{(6\,bx+6\,a)}+3\,(m+1)e^{(4\,bx+4\,a)}+3\,(m+1)e^{(2\,bx+2\,a)}+m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*tanh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) + 2*(m + 1)*e^(2*b*x
+ 2*a) + m + 1) - integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x)
+ (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)
*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int x^m \tanh(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*tanh(a + b*x)^2,x)
```

```
[Out] int(x^m*tanh(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \tanh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*tanh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*tanh(a + b*x)**2, x)
```

3.31 $\int x^m \tanh(a+bx)\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(x^m \tanh(a+bx), x\right)$$

```
[Out] Unintegrable(x^m*tanh(b*x+a),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^m \tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Tanh[a + b*x],x]
```

```
[Out] Defer[Int][x^m*Tanh[a + b*x], x]
```

Rubi steps

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+bx)\,dx$$

Mathematica [A] time = 7.62, size = 0, normalized size = 0.00

$$\int x^m \tanh(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Tanh[a + b*x],x]
```

```
[Out] Integrate[x^m*Tanh[a + b*x], x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \tanh\left(bx+a\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*tanh(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(x^m*tanh(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \tanh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*tanh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^m*tanh(b*x + a), x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int x^m \tanh\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*tanh(b*x+a),x)
```

```
[Out] int(x^m*tanh(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{x e^{\left(2\,bx+m\log(x)+2\,a\right)}}{(m+1)e^{(2\,bx+2\,a)}+m+1}-\int\frac{\left(\left(2\,bxe^{(2\,a)}+(m+1)e^{(2\,a)}\right)e^{(2\,bx)}+m+1\right)x^m}{(m+1)e^{(4\,bx+4\,a)}+2\,(m+1)e^{(2\,bx+2\,a)}+m+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*tanh(b*x+a),x, algorithm="maxima")
```

```
[Out] x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate(
((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) + m + 1)*x^m/((m + 1)*e^(4*b*x
 + 4*a) + 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.08

$$\int x^m \tanh\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*tanh(a + b*x),x)
```

```
[Out] int(x^m*tanh(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \tanh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*tanh(b*x+a),x)
```

```
[Out] Integral(x**m*tanh(a + b*x), x)
```

3.32 $\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx$

Optimal. Leaf size=169

$$-\frac{3d^2(c+dx)}{4f^3(a\tanh(e+fx)+a)}-\frac{3d(c+dx)^2}{4f^2(a\tanh(e+fx)+a)}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{3d(c+dx)^2}{8af^2}+\frac{(c+dx)^3}{4af}+\frac{(c+d$$

```
[Out] 3/8*d^3*x/a/f^3+3/8*d*(d*x+c)^2/a/f^2+1/4*(d*x+c)^3/a/f+1/8*(d*x+c)^4/a/d-3
/8*d^3/f^4/(a+a*tanh(f*x+e))-3/4*d^2*(d*x+c)/f^3/(a+a*tanh(f*x+e))-3/4*d*(d
*x+c)^2/f^2/(a+a*tanh(f*x+e))-1/2*(d*x+c)^3/f/(a+a*tanh(f*x+e))
```

Rubi [A] time = 0.19, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3723, 3479, 8}

$$-\frac{3d^2(c+dx)}{4f^3(a\tanh(e+fx)+a)}-\frac{3d(c+dx)^2}{4f^2(a\tanh(e+fx)+a)}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{3d(c+dx)^2}{8af^2}+\frac{(c+dx)^3}{4af}+\frac{(c+d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Tanh[e + f*x]),x]
```

```
[Out] (3*d^3*x)/(8*a*f^3) + (3*d*(c + d*x)^2)/(8*a*f^2) + (c + d*x)^3/(4*a*f) + (
c + d*x)^4/(8*a*d) - (3*d^3)/(8*f^4*(a + a*Tanh[e + f*x])) - (3*d^2*(c + d*
x))/(4*f^3*(a + a*Tanh[e + f*x])) - (3*d*(c + d*x)^2)/(4*f^2*(a + a*Tanh[e
+ f*x])) - (c + d*x)^3/(2*f*(a + a*Tanh[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3723

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[(a*d*m)/(2*b*f)
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[(a*(c + d*x)^m)
/(2*b*f*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx = \frac{(c+dx)^4}{8ad} - \frac{(c+dx)^3}{2f(a+a\tanh(e+fx))} + \frac{(3d)\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx}{2f}$$

$$= \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d(c+dx)^2}{4f^2(a+a\tanh(e+fx))} - \frac{(c+dx)^3}{2f(a+a\tanh(e+fx))} + \frac{\left(3d^2\right)\int$$

$$= \frac{3d(c+dx)^2}{8af^2} + \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^2(c+dx)}{4f^3(a+a\tanh(e+fx))} - \frac{3d(c+dx)^2}{4f^2(a+a\tanh(e+$$

$$= \frac{3d(c+dx)^2}{8af^2} + \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+a\tanh(e+fx))} - \frac{3d^2(c+dx)}{4f^3(a+a\tanh(e+$$

$$= \frac{3d^3x}{8af^3} + \frac{3d(c+dx)^2}{8af^2} + \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+a\tanh(e+fx))} - \frac{3d^2($$

Mathematica [A] time = 0.44, size = 244, normalized size = 1.44

$$\frac{\operatorname{sech}(e+fx)(\sinh(fx)+\cosh(fx))\left(2f^4x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)(\sinh(e)+\cosh(e))+(\sinh(e)-\cosh($$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Tanh[e + f*x]),x]
```

```
[Out] (Sech[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2*f*x
) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*f^3*
x^3))*Cosh[2*f*x]*(-Cosh[e] + Sinh[e]) + 2*f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d
^2*x^2 + d^3*x^3)*(Cosh[e] + Sinh[e]) + (4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2*f*x
) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*f^3*
x^3))*(Cosh[e] - Sinh[e])*Sinh[2*f*x]))/(16*a*f^4*(1 + Tanh[e + f*x]))
```

fricas [A] time = 0.75, size = 304, normalized size = 1.80

$$\left(2\,d^3f^4x^4 - 4\,c^3f^3 - 6\,c^2df^2 - 6\,cd^2f + 4\left(2\,cd^2f^4 - d^3f^3\right)x^3 - 3\,d^3 + 6\left(2\,c^2df^4 - 2\,cd^2f^3 - d^3f^2\right)x^2 + 2\left(4\,c^3f^4\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/16*((2*d^3*f^4*x^4 - 4*c^3*f^3 - 6*c^2*d*f^2 - 6*c*d^2*f + 4*(2*c*d^2*f^4
 - d^3*f^3)*x^3 - 3*d^3 + 6*(2*c^2*d*f^4 - 2*c*d^2*f^3 - d^3*f^2)*x^2 + 2*(
4*c^3*f^4 - 6*c^2*d*f^3 - 6*c*d^2*f^2 - 3*d^3*f)*x)*cosh(f*x + e) + (2*d^3*
f^4*x^4 + 4*c^3*f^3 + 6*c^2*d*f^2 + 6*c*d^2*f + 4*(2*c*d^2*f^4 + d^3*f^3)*x
^3 + 3*d^3 + 6*(2*c^2*d*f^4 + 2*c*d^2*f^3 + d^3*f^2)*x^2 + 2*(4*c^3*f^4 + 6
*c^2*d*f^3 + 6*c*d^2*f^2 + 3*d^3*f)*x)*sinh(f*x + e))/(a*f^4*cosh(f*x + e)
+ a*f^4*sinh(f*x + e))
```

giac [A] time = 0.14, size = 193, normalized size = 1.14

$$\frac{\left(2\,d^3f^4x^4e^{(2fx+2e)} + 8\,cd^2f^4x^3e^{(2fx+2e)} + 12\,c^2df^4x^2e^{(2fx+2e)} - 4\,d^3f^3x^3 + 8\,c^3f^4xe^{(2fx+2e)} - 12\,cd^2f^3x^2 - 12\,c^2$$
$$16\,af^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/16*(2*d^3*f^4*x^4*e^(2*f*x + 2*e) + 8*c*d^2*f^4*x^3*e^(2*f*x + 2*e) + 12*
c^2*d*f^4*x^2*e^(2*f*x + 2*e) - 4*d^3*f^3*x^3 + 8*c^3*f^4*x*e^(2*f*x + 2*e)
 - 12*c*d^2*f^3*x^2 - 12*c^2*d*f^3*x - 6*d^3*f^2*x^2 - 4*c^3*f^3 - 12*c*d^2
*f^2*x - 6*c^2*d*f^2 - 6*d^3*f*x - 6*c*d^2*f - 3*d^3)*e^(-2*f*x - 2*e)/(a*f
^4)
```

maple [B] time = 0.20, size = 929, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*tanh(f*x+e)),x)
```

```
[Out] 1/f^4/a*(-d^3*(1/2*(f*x+e)^3*cosh(f*x+e)^2-3/4*(f*x+e)^2*cosh(f*x+e)*sinh(f
*x+e)-1/4*(f*x+e)^3+3/4*(f*x+e)*cosh(f*x+e)^2-3/8*cosh(f*x+e)*sinh(f*x+e)-3
/8*f*x-3/8*e)-3*c*d^2*f*(1/2*(f*x+e)^2*cosh(f*x+e)^2-1/2*(f*x+e)*cosh(f*x+e
)*sinh(f*x+e)-1/4*(f*x+e)^2+1/4*cosh(f*x+e)^2)+3*d^3*e*(1/2*(f*x+e)^2*cosh(
f*x+e)^2-1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2+1/4*cosh(f*x+e)^
2)-3*c^2*d*f^2*(1/2*(f*x+e)*cosh(f*x+e)^2-1/4*cosh(f*x+e)*sinh(f*x+e)-1/4*f
*x-1/4*e)+6*c*d^2*e*f*(1/2*(f*x+e)*cosh(f*x+e)^2-1/4*cosh(f*x+e)*sinh(f*x+e
)-1/4*f*x-1/4*e)-3*d^3*e^2*(1/2*(f*x+e)*cosh(f*x+e)^2-1/4*cosh(f*x+e)*sinh(
f*x+e)-1/4*f*x-1/4*e)-1/2*cosh(f*x+e)^2*c^3*f^3+3/2*cosh(f*x+e)^2*c^2*d*e*f
^2-3/2*cosh(f*x+e)^2*c*d^2*e^2*f+1/2*cosh(f*x+e)^2*d^3*e^3+d^3*(1/2*(f*x+e)
^3*cosh(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)^4-3/4*(f*x+e)^2*cosh(f*x+e)^2+3/4*(f
*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*cosh(f*x+e)^2)+3*c*d^2*f*(1
/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^
2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-3*d^3*e*(1/2*(f*x+e)^2*cosh(f*
x+e)*sinh(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*si
nh(f*x+e)+1/4*f*x+1/4*e)+3*c^2*d*f^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1
/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-6*c*d^2*e*f*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f
*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+3*d^3*e^2*(1/2*(f*x+e)*cosh(f*x+e)*s
inh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+c^3*f^3*(1/2*cosh(f*x+e)*sinh(f
*x+e)+1/2*f*x+1/2*e)-3*c^2*d*e*f^2*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2
*e)+3*c*d^2*e^2*f*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2*e)-d^3*e^3*(1/2*
cosh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.48, size = 186, normalized size = 1.10

$$\frac{1}{4}c^3\left(\frac{2\left(fx+e\right)}{af}-\frac{e^{\left(-2fx-2e\right)}}{af}\right)+\frac{3\left(2f^2x^2e^{(2e)}-\left(2fx+1\right)e^{\left(-2fx\right)}\right)c^2de^{(-2e)}}{8af^2}+\frac{\left(4f^3x^3e^{(2e)}-3\left(2f^2x^2+2fx+\right.\right.}{8af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*c^3*(2*(f*x + e)/(a*f) - e^(-2*f*x - 2*e)/(a*f)) + 3/8*(2*f^2*x^2*e^(2*
e) - (2*f*x + 1)*e^(-2*f*x))*c^2*d*e^(-2*e)/(a*f^2) + 1/8*(4*f^3*x^3*e^(2*e
) - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*c*d^2*e^(-2*e)/(a*f^3) + 1/16*(2*
f^4*x^4*e^(2*e) - (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x))*d^3*e^(-2
*e)/(a*f^4)
```

mupad [B] time = 1.31, size = 273, normalized size = 1.62

$$\frac{\mathrm{e}^{-2e-2fx}\left(8c^3x\,\mathrm{e}^{2e+2fx}+2d^3x^4\,\mathrm{e}^{2e+2fx}+12c^2dx^2\,\mathrm{e}^{2e+2fx}+8cd^2x^3\,\mathrm{e}^{2e+2fx}\right)}{16a}-\frac{\frac{\mathrm{e}^{-2e-2fx}\left(3d^3-3d^3\,\mathrm{e}^{2e+2fx}\right)}{16}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*tanh(e + f*x)),x)
```

```
[Out] (exp(- 2*e - 2*f*x)*(8*c^3*x*exp(2*e + 2*f*x) + 2*d^3*x^4*exp(2*e + 2*f*x)
+ 12*c^2*d*x^2*exp(2*e + 2*f*x) + 8*c*d^2*x^3*exp(2*e + 2*f*x)))/(16*a) - (
(exp(- 2*e - 2*f*x)*(3*d^3 - 3*d^3*exp(2*e + 2*f*x)))/16 + (f^2*exp(- 2*e -
 2*f*x)*(6*c^2*d + 6*d^3*x^2 - 6*c^2*d*exp(2*e + 2*f*x) + 12*c*d^2*x))/16 +
 (f*exp(- 2*e - 2*f*x)*(6*c*d^2 + 6*d^3*x - 6*c*d^2*exp(2*e + 2*f*x)))/16 +
 (f^3*exp(- 2*e - 2*f*x)*(4*c^3 - 4*c^3*exp(2*e + 2*f*x) + 4*d^3*x^3 + 12*c
*d^2*x^2 + 12*c^2*d*x))/16)/(a*f^4)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3}{\tanh\left(e+fx\right)+1}\,dx + \int \frac{d^3x^3}{\tanh\left(e+fx\right)+1}\,dx + \int \frac{3cd^2x^2}{\tanh\left(e+fx\right)+1}\,dx + \int \frac{3c^2dx}{\tanh\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*tanh(f*x+e)),x)
```

```
[Out] (Integral(c**3/(tanh(e + f*x) + 1), x) + Integral(d**3*x**3/(tanh(e + f*x)
+ 1), x) + Integral(3*c*d**2*x**2/(tanh(e + f*x) + 1), x) + Integral(3*c**2
*d*x/(tanh(e + f*x) + 1), x))/a
```

3.33 $\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$

Optimal. Leaf size=122

$$-\frac{d(c+dx)}{2f^2(a\tanh(e+fx)+a)}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{d^2}{4f^3(a\tanh(e+fx)+a)}+\frac{d^2x}{4af^2}$$

```
[Out] 1/4*d^2*x/a/f^2+1/4*(d*x+c)^2/a/f+1/6*(d*x+c)^3/a/d-1/4*d^2/f^3/(a+a*tanh(f
*x+e))-1/2*d*(d*x+c)/f^2/(a+a*tanh(f*x+e))-1/2*(d*x+c)^2/f/(a+a*tanh(f*x+e)
)
```

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3723, 3479, 8}

$$-\frac{d(c+dx)}{2f^2(a\tanh(e+fx)+a)}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{d^2}{4f^3(a\tanh(e+fx)+a)}+\frac{d^2x}{4af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Tanh[e + f*x]),x]
```

```
[Out] (d^2*x)/(4*a*f^2) + (c + d*x)^2/(4*a*f) + (c + d*x)^3/(6*a*d) - d^2/(4*f^3*
(a + a*Tanh[e + f*x])) - (d*(c + d*x))/(2*f^2*(a + a*Tanh[e + f*x])) - (c +
 d*x)^2/(2*f*(a + a*Tanh[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3723

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[(a*d*m)/(2*b*f)
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[(a*(c + d*x)^m)
/(2*b*f*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx = \frac{(c+dx)^3}{6ad}-\frac{(c+dx)^2}{2f(a+a\tanh(e+fx))}+\frac{d\int\frac{c+dx}{a+a\tanh(e+fx)}\,dx}{f}$$

$$= \frac{(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{d(c+dx)}{2f^2(a+a\tanh(e+fx))}-\frac{(c+dx)^2}{2f(a+a\tanh(e+fx))}+\frac{d^2\int}{}$$

$$= \frac{(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{d^2}{4f^3(a+a\tanh(e+fx))}-\frac{d(c+dx)}{2f^2(a+a\tanh(e+fx))}-\frac{}{2f(}$$

$$= \frac{d^2x}{4af^2}+\frac{(c+dx)^2}{4af}+\frac{(c+dx)^3}{6ad}-\frac{d^2}{4f^3(a+a\tanh(e+fx))}-\frac{d(c+dx)}{2f^2(a+a\tanh(e+fx}$$

Mathematica [A] time = 0.29, size = 169, normalized size = 1.39

$$\frac{\operatorname{sech}(e+fx)(\sinh(fx)+\cosh(fx))\left(\frac{4}{3}f^3x\left(3c^2+3cdx+d^2x^2\right)(\sinh(e)+\cosh(e))+(\sinh(e)-\cosh(e))\cosh(2f$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Tanh[e + f*x]),x]
```

```
[Out] (Sech[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) +
d^2*(1 + 2*f*x + 2*f^2*x^2))*Cosh[2*f*x]*(-Cosh[e] + Sinh[e]) + (4*f^3*x*(3
*c^2 + 3*c*d*x + d^2*x^2)*(Cosh[e] + Sinh[e]))/3 + (2*c^2*f^2 + 2*c*d*f*(1
+ 2*f*x) + d^2*(1 + 2*f*x + 2*f^2*x^2))*(Cosh[e] - Sinh[e])*Sinh[2*f*x]))/(
8*a*f^3*(1 + Tanh[e + f*x]))
```

fricas [A] time = 0.48, size = 192, normalized size = 1.57

$$\frac{\left(4\,d^2f^3x^3-6\,c^2f^2-6\,cdf+6\left(2\,cdf^3-d^2f^2\right)x^2-3\,d^2+6\left(2\,c^2f^3-2\,cdf^2-d^2f\right)x\right)\cosh\left(fx+e\right)+\left(4\,d^2f^3x^3\right.}{24\left(af^3\cosh\left(fx+e\right)+af^3\sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/24*((4*d^2*f^3*x^3 - 6*c^2*f^2 - 6*c*d*f + 6*(2*c*d*f^3 - d^2*f^2)*x^2 -
3*d^2 + 6*(2*c^2*f^3 - 2*c*d*f^2 - d^2*f)*x)*cosh(f*x + e) + (4*d^2*f^3*x^3
 + 6*c^2*f^2 + 6*c*d*f + 6*(2*c*d*f^3 + d^2*f^2)*x^2 + 3*d^2 + 6*(2*c^2*f^3
 + 2*c*d*f^2 + d^2*f)*x)*sinh(f*x + e))/(a*f^3*cosh(f*x + e) + a*f^3*sinh(f
*x + e))
```

giac [A] time = 0.13, size = 123, normalized size = 1.01

$$\frac{\left(4\,d^2f^3x^3e^{(2fx+2e)}+12\,cdf^3x^2e^{(2fx+2e)}+12\,c^2f^3xe^{(2fx+2e)}-6\,d^2f^2x^2-12\,cdf^2x-6\,c^2f^2-6\,d^2fx-6\,cdf-3\right.}{24\,af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/24*(4*d^2*f^3*x^3*e^(2*f*x + 2*e) + 12*c*d*f^3*x^2*e^(2*f*x + 2*e) + 12*c
^2*f^3*x*e^(2*f*x + 2*e) - 6*d^2*f^2*x^2 - 12*c*d*f^2*x - 6*c^2*f^2 - 6*d^2
*f*x - 6*c*d*f - 3*d^2)*e^(-2*f*x - 2*e)/(a*f^3)
```

maple [B] time = 0.16, size = 446, normalized size = 3.66

$$\frac{-d^2\left(\frac{\left(fx+e\right)^2\left(\cosh^2\left(fx+e\right)\right)}{2}-\frac{\left(fx+e\right)\cosh\left(fx+e\right)\sinh\left(fx+e\right)}{2}-\frac{\left(fx+e\right)^2}{4}+\frac{\left(\cosh^2\left(fx+e\right)\right)}{4}\right)-2cdf\left(\frac{\left(fx+e\right)\left(\cosh^2\left(fx+e\right)\right)}{2}-\frac{\cosh\left(fx+e\right.}{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*tanh(f*x+e)),x)
```

```
[Out] 1/f^3/a*(-d^2*(1/2*(f*x+e)^2*cosh(f*x+e)^2-1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x
+e)-1/4*(f*x+e)^2+1/4*cosh(f*x+e)^2)-2*c*d*f*(1/2*(f*x+e)*cosh(f*x+e)^2-1/4
*cosh(f*x+e)*sinh(f*x+e)-1/4*f*x-1/4*e)+2*d^2*e*(1/2*(f*x+e)*cosh(f*x+e)^2-
1/4*cosh(f*x+e)*sinh(f*x+e)-1/4*f*x-1/4*e)-1/2*cosh(f*x+e)^2*c^2*f^2+cosh(f
*x+e)^2*c*d*f*e-1/2*cosh(f*x+e)^2*d^2*e^2+d^2*(1/2*(f*x+e)^2*cosh(f*x+e)*si
nh(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+
e)+1/4*f*x+1/4*e)+2*c*d*f*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^
```

```
2-1/4*cosh(f*x+e)^2)-2*d^2*e*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+
e)^2-1/4*cosh(f*x+e)^2)+c^2*f^2*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2*e)
-2*c*d*f*e*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2*e)+d^2*e^2*(1/2*cosh(f*
x+e)*sinh(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.40, size = 126, normalized size = 1.03

$$\frac{1}{4}c^2\left(\frac{2\left(fx+e\right)}{af}-\frac{e^{\left(-2fx-2e\right)}}{af}\right)+\frac{\left(2f^2x^2e^{(2e)}-\left(2fx+1\right)e^{\left(-2fx\right)}\right)cde^{(-2e)}}{4af^2}+\frac{\left(4f^3x^3e^{(2e)}-3\left(2f^2x^2+2fx+1\right)e\right.}{24af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*c^2*(2*(f*x + e)/(a*f) - e^(-2*f*x - 2*e)/(a*f)) + 1/4*(2*f^2*x^2*e^(2*
e) - (2*f*x + 1)*e^(-2*f*x))*c*d*e^(-2*e)/(a*f^2) + 1/24*(4*f^3*x^3*e^(2*e)
 - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*d^2*e^(-2*e)/(a*f^3)
```

mupad [B] time = 1.18, size = 187, normalized size = 1.53

$$\frac{\mathrm{e}^{-2e-2fx}\left(12c^2x\,\mathrm{e}^{2e+2fx}+4d^2x^3\,\mathrm{e}^{2e+2fx}+12cdx^2\,\mathrm{e}^{2e+2fx}\right)}{24a}-\frac{\frac{\mathrm{e}^{-2e-2fx}\left(3d^2-3d^2\,\mathrm{e}^{2e+2fx}\right)}{24}+\frac{f\,\mathrm{e}^{-2e-2fx}\left(6cd+6d^2x\right.}{24}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*tanh(e + f*x)),x)
```

```
[Out] (exp(- 2*e - 2*f*x)*(12*c^2*x*exp(2*e + 2*f*x) + 4*d^2*x^3*exp(2*e + 2*f*x)
 + 12*c*d*x^2*exp(2*e + 2*f*x)))/(24*a) - ((exp(- 2*e - 2*f*x)*(3*d^2 - 3*d
^2*exp(2*e + 2*f*x)))/24 + (f*exp(- 2*e - 2*f*x)*(6*c*d + 6*d^2*x - 6*c*d*e
xp(2*e + 2*f*x)))/24 + (f^2*exp(- 2*e - 2*f*x)*(6*c^2 - 6*c^2*exp(2*e + 2*f
*x) + 6*d^2*x^2 + 12*c*d*x))/24)/(a*f^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c^2}{\tanh\left(e+fx\right)+1}\,dx+\int\frac{d^2x^2}{\tanh\left(e+fx\right)+1}\,dx+\int\frac{2cdx}{\tanh\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*tanh(f*x+e)),x)
```

```
[Out] (Integral(c**2/(tanh(e + f*x) + 1), x) + Integral(d**2*x**2/(tanh(e + f*x)
+ 1), x) + Integral(2*c*d*x/(tanh(e + f*x) + 1), x))/a
```

3.34 $\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx$

Optimal. Leaf size=74

$$-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}-\frac{d}{4f^2(a\tanh(e+fx)+a)}+\frac{dx}{4af}$$

```
[Out] 1/4*d*x/a/f+1/4*(d*x+c)^2/a/d-1/4*d/f^2/(a+a*tanh(f*x+e))+1/2*(-d*x-c)/f/(a
+a*tanh(f*x+e))
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3723, 3479, 8}

$$-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}-\frac{d}{4f^2(a\tanh(e+fx)+a)}+\frac{dx}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Tanh[e + f*x]),x]
```

```
[Out] (d*x)/(4*a*f) + (c + d*x)^2/(4*a*d) - d/(4*f^2*(a + a*Tanh[e + f*x])) - (c
+ d*x)/(2*f*(a + a*Tanh[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3723

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Dist[(a*d*m)/(2*b*f)
, Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[(a*(c + d*x)^m)
/(2*b*f*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a
^2 + b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx &= \frac{(c+dx)^2}{4ad}-\frac{c+dx}{2f(a+a\tanh(e+fx))}+\frac{d\int \frac{1}{a+a\tanh(e+fx)}\,dx}{2f}\\ &= \frac{(c+dx)^2}{4ad}-\frac{d}{4f^2(a+a\tanh(e+fx))}-\frac{c+dx}{2f(a+a\tanh(e+fx))}+\frac{d\int 1\,dx}{4af}\\ &= \frac{dx}{4af}+\frac{(c+dx)^2}{4ad}-\frac{d}{4f^2(a+a\tanh(e+fx))}-\frac{c+dx}{2f(a+a\tanh(e+fx))}\end{aligned}$$

Mathematica [A] time = 0.31, size = 81, normalized size = 1.09

$$\frac{\left(2cf(2fx+1)+d\left(2f^2x^2+2fx+1\right)\right)\tanh(e+fx)+2cf(2fx-1)+d\left(2f^2x^2-2fx-1\right)}{8af^2(\tanh(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Tanh[e + f*x]),x]
```

```
[Out] (2*c*f*(-1 + 2*f*x) + d*(-1 - 2*f*x + 2*f^2*x^2) + (2*c*f*(1 + 2*f*x) + d*(
1 + 2*f*x + 2*f^2*x^2))*Tanh[e + f*x])/(8*a*f^2*(1 + Tanh[e + f*x]))
```

fricas [A] time = 0.42, size = 101, normalized size = 1.36

$$\frac{\left(2\,df^2x^2-2\,cf+2\left(2\,cf^2-df\right)x-d\right)\cosh\left(fx+e\right)+\left(2\,df^2x^2+2\,cf+2\left(2\,cf^2+df\right)x+d\right)\sinh\left(fx+e\right)}{8\left(af^2\cosh\left(fx+e\right)+af^2\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/8*((2*d*f^2*x^2 - 2*c*f + 2*(2*c*f^2 - d*f)*x - d)*cosh(f*x + e) + (2*d*f
^2*x^2 + 2*c*f + 2*(2*c*f^2 + d*f)*x + d)*sinh(f*x + e))/(a*f^2*cosh(f*x +
e) + a*f^2*sinh(f*x + e))
```

giac [A] time = 0.16, size = 67, normalized size = 0.91

$$\frac{\left(2\,df^2x^2e^{\left(2fx+2e\right)}+4\,cf^2xe^{\left(2fx+2e\right)}-2\,dfx-2\,cf-d\right)e^{\left(-2fx-2e\right)}}{8\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/8*(2*d*f^2*x^2*e^(2*f*x + 2*e) + 4*c*f^2*x*e^(2*f*x + 2*e) - 2*d*f*x - 2*
c*f - d)*e^(-2*f*x - 2*e)/(a*f^2)
```

maple [B] time = 0.16, size = 165, normalized size = 2.23

$$\frac{-d\left(\frac{\left(fx+e\right)\left(\cosh^2\left(fx+e\right)\right)}{2}-\frac{\cosh\left(fx+e\right)\sinh\left(fx+e\right)}{4}-\frac{fx}{4}-\frac{e}{4}\right)-\frac{\left(\cosh^2\left(fx+e\right)\right)cf}{2}+\frac{\left(\cosh^2\left(fx+e\right)\right)de}{2}+d\left(\frac{\left(fx+e\right)\cosh\left(fx+e\right)\sinh(}{2}\right.}{f^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*tanh(f*x+e)),x)
```

```
[Out] 1/f^2/a*(-d*(1/2*(f*x+e)*cosh(f*x+e)^2-1/4*cosh(f*x+e)*sinh(f*x+e)-1/4*f*x-
1/4*e)-1/2*cosh(f*x+e)^2*c*f+1/2*cosh(f*x+e)^2*d*e+d*(1/2*(f*x+e)*cosh(f*x+
e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+c*f*(1/2*cosh(f*x+e)*sinh(f
*x+e)+1/2*f*x+1/2*e)-d*e*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.34, size = 74, normalized size = 1.00

$$\frac{1}{4}\,c\left(\frac{2\left(fx+e\right)}{af}-\frac{e^{\left(-2fx-2e\right)}}{af}\right)+\frac{\left(2\,f^2x^2e^{\left(2e\right)}-\left(2\,fx+1\right)e^{\left(-2fx\right)}\right)de^{\left(-2e\right)}}{8\,af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*c*(2*(f*x + e)/(a*f) - e^(-2*f*x - 2*e)/(a*f)) + 1/8*(2*f^2*x^2*e^(2*e)
 - (2*f*x + 1)*e^(-2*f*x))*d*e^(-2*e)/(a*f^2)
```

mupad [B] time = 1.13, size = 76, normalized size = 1.03

$$\frac{\frac{d\,x^2}{4}+\left(\frac{c}{2}+\frac{d}{4\,f}\right)\,x}{a}-\frac{\frac{\frac{d}{4}+\frac{c\,f}{2}}{f^2}-x\,\left(\frac{c}{2}-\frac{d}{4\,f}\right)+x\,\left(\frac{c}{2}+\frac{d}{4\,f}\right)}{a+a\,\tanh\left(e+f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*tanh(e + f*x)),x)
```

```
[Out] (x*(c/2 + d/(4*f)) + (d*x^2)/4)/a - ((d/4 + (c*f)/2)/f^2 - x*(c/2 - d/(4*f)
) + x*(c/2 + d/(4*f)))/(a + a*tanh(e + f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c}{\tanh\left(e+f x\right)+1}\,dx+\int \frac{dx}{\tanh\left(e+f x\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e)),x)
```

```
[Out] (Integral(c/(tanh(e + f*x) + 1), x) + Integral(d*x/(tanh(e + f*x) + 1), x))
/a
```

3.35 $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx$

Optimal. Leaf size=157

$$-\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2ad}+\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2ad}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2ad}-\frac{\cosh($$

```
[Out] 1/2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a/d+1/2*ln(d*x+c)/a/d-1/2*cosh(-2
*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d+1/2*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d
)/a/d-1/2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d
```

Rubi [A] time = 0.29, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3726, 3303, 3298, 3301}

$$-\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2ad}+\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2ad}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{2ad}-\frac{\cosh($$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + a*Tanh[e + f*x])),x]
```

```
[Out] (Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*a*d) + Log[c + d
*x]/(2*a*d) - (CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(2*a*
d) - (Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a*d) + (Sin
h[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3726

```
Int[1/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Symb
ol] :> Simp[Log[c + d*x]/(2*a*d), x] + (Dist[1/(2*a), Int[Cos[2*e + 2*f*x]/
(c + d*x), x], x] + Dist[1/(2*b), Int[Sin[2*e + 2*f*x]/(c + d*x), x], x]) /
; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\log(c+dx)}{2ad} + \frac{\int \frac{\cosh(2e+2fx)}{c+dx}\,dx}{2a} - \frac{\int \frac{\sinh(2e+2fx)}{c+dx}\,dx}{2a}$$

$$= \frac{\log(c+dx)}{2ad} + \frac{\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{2a} - \frac{\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sinh\left(\frac{2}{c+}\right.}{}}{2a}$$

$$= \frac{\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2ad} + \frac{\log(c+dx)}{2ad} - \frac{\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e\right.}{2ad}$$

Mathematica [A] time = 0.27, size = 122, normalized size = 0.78

$$\frac{\mathrm{sech}(e+fx)(\sinh(fx)+\cosh(fx))\left(\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\left(\cosh\left(e-\frac{2cf}{d}\right)-\sinh\left(e-\frac{2cf}{d}\right)\right)+\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)\left(\sinh\left(e-\right.\right.\right.}{2ad(\tanh(e+fx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + a*Tanh[e + f*x])),x]
```

```
[Out] (Sech[e + f*x]*(Cosh[f*x] + Sinh[f*x])*(Log[f*(c + d*x)]*(Cosh[e] + Sinh[e]
) + CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[e - (2*c*f)/d] - Sinh[e - (2*c*f)
/d]) + (-Cosh[e - (2*c*f)/d] + Sinh[e - (2*c*f)/d])*SinhIntegral[(2*f*(c +
d*x))/d]))/(2*a*d*(1 + Tanh[e + f*x]))
```

fricas [A] time = 0.42, size = 73, normalized size = 0.46

$$\frac{\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\cosh\left(-\frac{2\left(de-cf\right)}{d}\right)+\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\sinh\left(-\frac{2\left(de-cf\right)}{d}\right)+\log\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + Ei(-2*(d*f*x + c*f)/d)
*sinh(-2*(d*e - c*f)/d) + log(d*x + c))/(a*d)
```

giac [A] time = 0.15, size = 49, normalized size = 0.31

$$\frac{\left(\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2cf}{d}\right)}+e^{(2e)}\log\left(dx+c\right)\right)e^{(-2e)}}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*(Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + e^(2*e)*log(d*x + c))*e^(-2*e)/(a
*d)
```

maple [A] time = 0.65, size = 61, normalized size = 0.39

$$\frac{\ln\left(dx+c\right)}{2da} - \frac{\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}\left(1,2fx+2e+\frac{2cf-2de}{d}\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*tanh(f*x+e)),x)
```

```
[Out] 1/2*ln(d*x+c)/d/a-1/2/a/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

maxima [A] time = 0.53, size = 48, normalized size = 0.31

$$-\frac{e^{\left(-2\,e+\frac{2\,c\,f}{d}\right)}\,E_1\left(\frac{2\,(d\,x+c)\,f}{d}\right)}{2\,a\,d}+\frac{\log\left(d\,x+c\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a*d) + 1/2*log(
d*x + c)/(a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\tanh\left(e+f\,x\right)\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tanh(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a + a*tanh(e + f*x))*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\tanh\left(e+f\,x\right)+c+d\,x\tanh\left(e+f\,x\right)+d\,x}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x)
```

```
[Out] Integral(1/(c*tanh(e + f*x) + c + d*x*tanh(e + f*x) + d*x), x)/a
```

3.36 $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx$

Optimal. Leaf size=159

$$\frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^2}-\frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{ad^2}-\frac{f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{ad^2}+\frac{f\cosh}{}$$

```
[Out] -f*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a/d^2+f*cosh(-2*e+2*c*f/d)*Shi(2*c
*f/d+2*f*x)/a/d^2-f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d^2+f*Shi(2*c*f
/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d^2-1/d/(d*x+c)/(a+a*tanh(f*x+e))
```

Rubi [A] time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3724, 3303, 3298, 3301}

$$\frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^2}-\frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{ad^2}-\frac{f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{ad^2}+\frac{f\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + a*Tanh[e + f*x])),x]
```

```
[Out] -((f*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(a*d^2)) + (f*C
oshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(a*d^2) + (f*Cosh[2*e
 - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) - (f*Sinh[2*e - (2*c
*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a*d^2) - 1/(d*(c + d*x)*(a + a*Tan
h[e + f*x]))
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3724

```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d)
, Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*
x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = -\frac{1}{d(c+dx)(a+a\tanh(e+fx))} - \frac{f\int \frac{\cosh(2e+2fx)}{c+dx}\,dx}{ad} + \frac{f\int \frac{\sinh(2e+2fx)}{c+dx}\,dx}{ad}$$

$$= -\frac{1}{d(c+dx)(a+a\tanh(e+fx))} - \frac{\left(f\cosh\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cosh\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{ad}$$

$$= -\frac{f\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{ad^2} + \frac{f\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right.}{ad^2}$$

Mathematica [A] time = 0.80, size = 206, normalized size = 1.30

$$-\frac{\mathrm{sech}(e+fx)\left(\sinh\left(\frac{cf}{d}\right)+\cosh\left(\frac{cf}{d}\right)\right)\left(2f(c+dx)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\left(\cosh\left(e-\frac{f(c+dx)}{d}\right)-\sinh\left(e-\frac{f(c+dx)}{d}\right)\right)+2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Tanh[e + f*x])),x]
```

```
[Out] -1/2*(Sech[e + f*x]*(Cosh[(c*f)/d] + Sinh[(c*f)/d])*(d*(Cosh[e + f*(-(c/d)
+ x)] + Cosh[e + f*(c/d + x)] + Sinh[e + f*(-(c/d) + x)] - Sinh[e + f*(c/d
+ x)]) + 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[e - (f*(c + d*
x))/d] - Sinh[e - (f*(c + d*x))/d]) + 2*f*(c + d*x)*(-Cosh[e - (f*(c + d*x)
)/d] + Sinh[e - (f*(c + d*x))/d])*SinhIntegral[(2*f*(c + d*x))/d]))/(a*d^2*
(c + d*x)*(1 + Tanh[e + f*x]))
```

fricas [A] time = 0.46, size = 217, normalized size = 1.36

$$-\frac{\left(dfx+cf\right)\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\cosh\left(fx+e\right)\sinh\left(-\frac{2\left(de-cf\right)}{d}\right)+\left(\left(dfx+cf\right)\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\cosh\left(-\frac{2\left(de-cf\right)}{d}\right)+\right.}{\left(ad^3x+acd^2\right)\cosh\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f)/d)
 + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + d)*cosh(f
*x + e) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + (d
*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d))*sinh(f*x + e))/(
(a*d^3*x + a*c*d^2)*cosh(f*x + e) + (a*d^3*x + a*c*d^2)*sinh(f*x + e))
```

giac [B] time = 0.19, size = 348, normalized size = 2.19

$$-\frac{\left(\begin{array}{l} 2\,(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)f^2\mathrm{Ei}\left(\frac{2\left((dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de\right)}{d}\right)e^{\left(\frac{2\left(cf-de\right)}{d}\right)}-2\,cf^3\mathrm{Ei}\left(\frac{2\left((dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de\right)}{d}\right. \end{array}\right)}{2\left((dx+c)ad^4\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(2*((d*x + c)*(
c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(2*(c*f - d*e)/d) - 2*
```

```
c*f^3*Ei(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e
^(2*(c*f - d*e)/d) + 2*d*f^2*Ei(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x
+ c)) - c*f + d*e)/d)*e^(2*(c*f - d*e)/d + 1) - d*f^2*e^(2*(d*x + c)*(c*f/(
d*x + c) - f - d*e/(d*x + c))/d) - d*f^2)*d^2/(((d*x + c)*a*d^4*(c*f/(d*x +
 c) - f - d*e/(d*x + c)) - a*c*d^4*f + a*d^5*e)*f)
```

maple [A] time = 0.62, size = 90, normalized size = 0.57

$$-\frac{1}{2da\,(dx+c)}-\frac{f\,\mathrm{e}^{-2fx-2e}}{2ad\left(dfx+cf\right)}+\frac{f\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}\left(1,2fx+2e+\frac{2cf-2de}{d}\right)}{a\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x)
```

```
[Out] -1/2/d/a/(d*x+c)-1/2/a*f*exp(-2*f*x-2*e)/d/(d*f*x+c*f)+1/a*f/d^2*exp(2*(c*f
-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

maxima [A] time = 0.74, size = 56, normalized size = 0.35

$$-\frac{1}{2\left(ad^2x+acd\right)}-\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{2\,(dx+c)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2/(a*d^2*x + a*c*d) - 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x +
c)*f/d)/((d*x + c)*a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+a\tanh\left(e+f\,x\right)\right)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tanh(e + f*x))*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*tanh(e + f*x))*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\tanh\left(e+fx\right)+c^2+2cdx\tanh\left(e+fx\right)+2cdx+d^2x^2\tanh\left(e+fx\right)+d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*tanh(f*x+e)),x)
```

```
[Out] Integral(1/(c**2*tanh(e + f*x) + c**2 + 2*c*d*x*tanh(e + f*x) + 2*c*d*x + d
**2*x**2*tanh(e + f*x) + d**2*x**2), x)/a
```

3.37 $\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$

Optimal. Leaf size=211

$$-\frac{f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^3}+\frac{f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{ad^3}+\frac{f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{ad^3}$$

```
[Out] -1/2*f/a/d^2/(d*x+c)+f^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a/d^3-f^2*co
sh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d^3+f^2*Chi(2*c*f/d+2*f*x)*sinh(-2*e+
2*c*f/d)/a/d^3-f^2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d^3-1/2/d/(d*x+c
)^2/(a+a*tanh(f*x+e))+f/d^2/(d*x+c)/(a+a*tanh(f*x+e))
```

Rubi [A] time = 0.32, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3725, 3724, 3303, 3298, 3301}

$$-\frac{f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^3}+\frac{f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{ad^3}+\frac{f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^3*(a + a*Tanh[e + f*x])),x]
```

```
[Out] -f/(2*a*d^2*(c + d*x)) + (f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d
+ 2*f*x])/(a*d^3) - (f^2*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)
/d])/(a*d^3) - (f^2*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/
(a*d^3) + (f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a*d^
3) - 1/(2*d*(c + d*x)^2*(a + a*Tanh[e + f*x])) + f/(d^2*(c + d*x)*(a + a*Ta
nh[e + f*x]))
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3724

```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Dist[f/(a*d)
, Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Dist[f/(b*d), Int[Cos[2*e + 2*f*
x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Rule 3725

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[(f*(c + d*x)^(m + 2))/(b*d^2*(m + 1)*(m + 2)), x] + (Dist[(2*b
*f)/(a*d*(m + 1)), Int[(c + d*x)^(m + 1)/(a + b*Tan[e + f*x]), x], x] + Sim
p[(c + d*x)^(m + 1)/(d*(m + 1)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c
, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx = -\frac{f}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+a\tanh(e+fx))} - \frac{f\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx}}{d}$$

$$= -\frac{f}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+a\tanh(e+fx))} + \frac{f}{d^2(c+dx)(a+a\tanh}$$

$$= -\frac{f}{2ad^2(c+dx)} - \frac{1}{2d(c+dx)^2(a+a\tanh(e+fx))} + \frac{f}{d^2(c+dx)(a+a\tanh}$$

$$= -\frac{f}{2ad^2(c+dx)} + \frac{f^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{ad^3} - \frac{f^2\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{ad^3}$$

Mathematica [A] time = 1.15, size = 264, normalized size = 1.25

$$-\frac{\mathrm{sech}(e+fx)\left(\sinh\left(\frac{cf}{d}\right)+\cosh\left(\frac{cf}{d}\right)\right)\left(4f^2(c+dx)^2\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\left(\sinh\left(e-\frac{f(c+dx)}{d}\right)-\cosh\left(e-\frac{f(c+dx)}{d}\right)\right)+4f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^3*(a + a*Tanh[e + f*x])),x]
```

```
[Out] -1/4*(Sech[e + f*x]*(Cosh[(c*f)/d] + Sinh[(c*f)/d])*(d*(d*Cosh[e + f*(-(c/d
) + x)] + (d - 2*c*f - 2*d*f*x)*Cosh[e + f*(c/d + x)] + d*Sinh[e + f*(-(c/d
) + x)] - d*Sinh[e + f*(c/d + x)] + 2*c*f*Sinh[e + f*(c/d + x)] + 2*d*f*x*S
inh[e + f*(c/d + x)]) + 4*f^2*(c + d*x)^2*CoshIntegral[(2*f*(c + d*x))/d]*(
-Cosh[e - (f*(c + d*x))/d] + Sinh[e - (f*(c + d*x))/d]) + 4*f^2*(c + d*x)^2
*(Cosh[e - (f*(c + d*x))/d] - Sinh[e - (f*(c + d*x))/d])*SinhIntegral[(2*f*
(c + d*x))/d]))/(a*d^3*(c + d*x)^2*(1 + Tanh[e + f*x]))
```

fricas [A] time = 0.44, size = 342, normalized size = 1.62

$$2\left(d^2f^2x^2+2cdf^2x+c^2f^2\right)\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\cosh\left(fx+e\right)\sinh\left(-\frac{2\left(de-cf\right)}{d}\right)+\left(d^2fx+cdf+2\left(d^2f^2x^2+2cdf^2x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*
x + e)*sinh(-2*(d*e - c*f)/d) + (d^2*f*x + c*d*f + 2*(d^2*f^2*x^2 + 2*c*d*f
^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - d^2)*cosh(f
*x + e) - (d^2*f*x + c*d*f - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*
(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^
2*f^2)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d))*sinh(f*x + e))/((a*d^
5*x^2 + 2*a*c*d^4*x + a*c^2*d^3)*cosh(f*x + e) + (a*d^5*x^2 + 2*a*c*d^4*x +
 a*c^2*d^3)*sinh(f*x + e))
```

giac [A] time = 0.15, size = 180, normalized size = 0.85

$$\frac{4\,d^2f^2x^2\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2cf}{d}\right)}+8\,cdf^2x\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2cf}{d}\right)}+4\,c^2f^2\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2cf}{d}\right)}+2\,d^2fxe^{\left(-2fx\right)}+}{4\left(ad^5x^2e^{(2e)}+2\,acd^4xe^{(2e)}+ac^2d^3e^{(2e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*(4*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + 8*c*d*f^2*x*Ei(-2*(
d*f*x + c*f)/d)*e^(2*c*f/d) + 4*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d)
+ 2*d^2*f*x*e^(-2*f*x) + 2*c*d*f*e^(-2*f*x) - d^2*e^(-2*f*x) - d^2*e^(2*e))
/(a*d^5*x^2*e^(2*e) + 2*a*c*d^4*x*e^(2*e) + a*c^2*d^3*e^(2*e))
```

maple [A] time = 0.65, size = 211, normalized size = 1.00

$$-\frac{1}{4da\left(dx+c\right)^2}+\frac{f^3\mathrm{e}^{-2fx-2e}x}{2ad\left(d^2f^2x^2+2cd\,f^2x+c^2f^2\right)}+\frac{f^3\mathrm{e}^{-2fx-2e}c}{2a\,d^2\left(d^2f^2x^2+2cd\,f^2x+c^2f^2\right)}-\frac{f^2\mathrm{e}^{-2fx-2e}}{4ad\left(d^2f^2x^2+2cd\,f^2x+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x)
```

```
[Out] -1/4/d/a/(d*x+c)^2+1/2/a*f^3*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2
*f^2)*x+1/2/a*f^3*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1
/4/a*f^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/a*f^2/d^3*ex
p(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

maxima [A] time = 0.96, size = 68, normalized size = 0.32

$$-\frac{1}{4\left(ad^3x^2+2\,acd^2x+ac^2d\right)}-\frac{e^{\left(-2\,e+\frac{2cf}{d}\right)}E_3\left(\frac{2\left(dx+c\right)f}{d}\right)}{2\left(dx+c\right)^2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/4/(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d) - 1/2*e^(-2*e + 2*c*f/d)*exp_integ
ral_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*a*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\tanh\left(e+f\,x\right)\right)\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tanh(e + f*x))*(c + d*x)^3),x)
```

```
[Out] int(1/((a + a*tanh(e + f*x))*(c + d*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^3\tanh\left(e+fx\right)+c^3+3c^2dx\tanh\left(e+fx\right)+3c^2dx+3cd^2x^2\tanh\left(e+fx\right)+3cd^2x^2+d^3x^3\tanh\left(e+fx\right)+d^3x^3}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(a+a*tanh(f*x+e)),x)

[Out] Integral(1/(c**3*tanh(e + f*x) + c**3 + 3*c**2*d*x*tanh(e + f*x) + 3*c**2*d
*x + 3*c*d**2*x**2*tanh(e + f*x) + 3*c*d**2*x**2 + d**3*x**3*tanh(e + f*x)
+ d**3*x**3), x)/a
```

3.38 $\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=230

$$-\frac{3d^2(c+dx)e^{-4e-4fx}}{128a^2f^3}-\frac{3d^2(c+dx)e^{-2e-2fx}}{8a^2f^3}-\frac{3d(c+dx)^2e^{-4e-4fx}}{64a^2f^2}-\frac{3d(c+dx)^2e^{-2e-2fx}}{8a^2f^2}-\frac{(c+dx)^3e^{-4e-4fx}}{16a^2f}-\frac{(c+$$

```
[Out] -3/512*d^3*exp(-4*f*x-4*e)/a^2/f^4-3/16*d^3*exp(-2*f*x-2*e)/a^2/f^4-3/128*d
^2*exp(-4*f*x-4*e)*(d*x+c)/a^2/f^3-3/8*d^2*exp(-2*f*x-2*e)*(d*x+c)/a^2/f^3-
3/64*d*exp(-4*f*x-4*e)*(d*x+c)^2/a^2/f^2-3/8*d*exp(-2*f*x-2*e)*(d*x+c)^2/a^
2/f^2-1/16*exp(-4*f*x-4*e)*(d*x+c)^3/a^2/f-1/4*exp(-2*f*x-2*e)*(d*x+c)^3/a^
2/f+1/16*(d*x+c)^4/a^2/d
```

Rubi [A] time = 0.28, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3729, 2176, 2194}

$$-\frac{3d^2(c+dx)e^{-4e-4fx}}{128a^2f^3}-\frac{3d^2(c+dx)e^{-2e-2fx}}{8a^2f^3}-\frac{3d(c+dx)^2e^{-4e-4fx}}{64a^2f^2}-\frac{3d(c+dx)^2e^{-2e-2fx}}{8a^2f^2}-\frac{(c+dx)^3e^{-4e-4fx}}{16a^2f}-\frac{(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] (-3*d^3*E^(-4*e - 4*f*x))/(512*a^2*f^4) - (3*d^3*E^(-2*e - 2*f*x))/(16*a^2*
f^4) - (3*d^2*E^(-4*e - 4*f*x)*(c + d*x))/(128*a^2*f^3) - (3*d^2*E^(-2*e -
2*f*x)*(c + d*x))/(8*a^2*f^3) - (3*d*E^(-4*e - 4*f*x)*(c + d*x)^2)/(64*a^2*
f^2) - (3*d*E^(-2*e - 2*f*x)*(c + d*x)^2)/(8*a^2*f^2) - (E^(-4*e - 4*f*x)*(
c + d*x)^3)/(16*a^2*f) - (E^(-2*e - 2*f*x)*(c + d*x)^3)/(4*a^2*f) + (c + d*
x)^4/(16*a^2*d)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^3}{4a^2} + \frac{e^{-4e-4fx}(c+dx)^3}{4a^2} + \frac{e^{-2e-2fx}(c+dx)^3}{2a^2}\right) dx$$
$$= \frac{(c+dx)^4}{16a^2d} + \frac{\int e^{-4e-4fx}(c+dx)^3\,dx}{4a^2} + \frac{\int e^{-2e-2fx}(c+dx)^3\,dx}{2a^2}$$
$$= -\frac{e^{-4e-4fx}(c+dx)^3}{16a^2f} - \frac{e^{-2e-2fx}(c+dx)^3}{4a^2f} + \frac{(c+dx)^4}{16a^2d} + \frac{(3d)\int e^{-4e-4fx}(c+dx)^2\,dx}{16a^2f} +$$
$$= -\frac{3de^{-4e-4fx}(c+dx)^2}{64a^2f^2} - \frac{3de^{-2e-2fx}(c+dx)^2}{8a^2f^2} - \frac{e^{-4e-4fx}(c+dx)^3}{16a^2f} - \frac{e^{-2e-2fx}(c+dx)}{4a^2f}$$
$$= -\frac{3d^2e^{-4e-4fx}(c+dx)}{128a^2f^3} - \frac{3d^2e^{-2e-2fx}(c+dx)}{8a^2f^3} - \frac{3de^{-4e-4fx}(c+dx)^2}{64a^2f^2} - \frac{3de^{-2e-2fx}(c+dx)}{8a^2f^2}$$
$$= -\frac{3d^3e^{-4e-4fx}}{512a^2f^4} - \frac{3d^3e^{-2e-2fx}}{16a^2f^4} - \frac{3d^2e^{-4e-4fx}(c+dx)}{128a^2f^3} - \frac{3d^2e^{-2e-2fx}(c+dx)}{8a^2f^3} - \frac{3de^{-4e-4fx}}{6}$$

Mathematica [A] time = 1.14, size = 420, normalized size = 1.83

$$\frac{\operatorname{sech}^2(e+fx)(\sinh(fx)+\cosh(fx))^2\left(f^4x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)(\sinh(2e)+\cosh(2e)) + \frac{1}{32}(\sinh(2e)-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] (Sech[e + f*x]^2*(Cosh[f*x] + Sinh[f*x])^2*(-((4*c^3*f^3 + 6*c^2*d*f^2*(1 +
 2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 +
4*f^3*x^3))*Cosh[2*f*x]) + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 12*c*d
^2*f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*x^3))*
Cosh[4*f*x]*(-Cosh[2*e] + Sinh[2*e]))/32 + f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d
^2*x^2 + d^3*x^3)*(Cosh[2*e] + Sinh[2*e]) + (4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2
*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*
f^3*x^3))*Sinh[2*f*x] + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 12*c*d^2*
f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*x^3))*(Co
sh[2*e] - Sinh[2*e])*Sinh[4*f*x])/32))/(16*a^2*f^4*(1 + Tanh[e + f*x])^2)
```

fricas [B] time = 0.52, size = 573, normalized size = 2.49

$$-\frac{128\,d^3f^3x^3 + 128\,c^3f^3 + 192\,c^2df^2 + 192\,cd^2f + 96\,d^3 + 192\left(2\,cd^2f^3 + d^3f^2\right)x^2 - \left(32\,d^3f^4x^4 - 32\,c^3f^3 - 24\,c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/512*(128*d^3*f^3*x^3 + 128*c^3*f^3 + 192*c^2*d*f^2 + 192*c*d^2*f + 96*d^
3 + 192*(2*c*d^2*f^3 + d^3*f^2)*x^2 - (32*d^3*f^4*x^4 - 32*c^3*f^3 - 24*c^2
*d*f^2 - 12*c*d^2*f + 32*(4*c*d^2*f^4 - d^3*f^3)*x^3 - 3*d^3 + 24*(8*c^2*d*
f^4 - 4*c*d^2*f^3 - d^3*f^2)*x^2 + 4*(32*c^3*f^4 - 24*c^2*d*f^3 - 12*c*d^2*
f^2 - 3*d^3*f)*x)*cosh(f*x + e)^2 - 2*(32*d^3*f^4*x^4 + 32*c^3*f^3 + 24*c^2
*d*f^2 + 12*c*d^2*f + 32*(4*c*d^2*f^4 + d^3*f^3)*x^3 + 3*d^3 + 24*(8*c^2*d*
f^4 + 4*c*d^2*f^3 + d^3*f^2)*x^2 + 4*(32*c^3*f^4 + 24*c^2*d*f^3 + 12*c*d^2*
f^2 + 3*d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) - (32*d^3*f^4*x^4 - 32*c^3*f^
3 - 24*c^2*d*f^2 - 12*c*d^2*f + 32*(4*c*d^2*f^4 - d^3*f^3)*x^3 - 3*d^3 + 24
*(8*c^2*d*f^4 - 4*c*d^2*f^3 - d^3*f^2)*x^2 + 4*(32*c^3*f^4 - 24*c^2*d*f^3 -
 12*c*d^2*f^2 - 3*d^3*f)*x)*sinh(f*x + e)^2 + 192*(2*c^2*d*f^3 + 2*c*d^2*f^
```

```
2 + d^3*f)*x)/(a^2*f^4*cosh(f*x + e)^2 + 2*a^2*f^4*cosh(f*x + e)*sinh(f*x +
 e) + a^2*f^4*sinh(f*x + e)^2)
```

giac [A] time = 0.14, size = 383, normalized size = 1.67

$$\left(32\,d^3f^4x^4e^{(4fx+4e)} + 128\,cd^2f^4x^3e^{(4fx+4e)} + 192\,c^2df^4x^2e^{(4fx+4e)} - 128\,d^3f^3x^3e^{(2fx+2e)} - 32\,d^3f^3x^3 + 128\,c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/512*(32*d^3*f^4*x^4*e^(4*f*x + 4*e) + 128*c*d^2*f^4*x^3*e^(4*f*x + 4*e) +
 192*c^2*d*f^4*x^2*e^(4*f*x + 4*e) - 128*d^3*f^3*x^3*e^(2*f*x + 2*e) - 32*d
^3*f^3*x^3 + 128*c^3*f^4*x*e^(4*f*x + 4*e) - 384*c*d^2*f^3*x^2*e^(2*f*x + 2
*e) - 96*c*d^2*f^3*x^2 - 384*c^2*d*f^3*x*e^(2*f*x + 2*e) - 192*d^3*f^2*x^2*
e^(2*f*x + 2*e) - 96*c^2*d*f^3*x - 24*d^3*f^2*x^2 - 128*c^3*f^3*e^(2*f*x +
2*e) - 384*c*d^2*f^2*x*e^(2*f*x + 2*e) - 32*c^3*f^3 - 48*c*d^2*f^2*x - 192*
c^2*d*f^2*e^(2*f*x + 2*e) - 192*d^3*f*x*e^(2*f*x + 2*e) - 24*c^2*d*f^2 - 12
*d^3*f*x - 192*c*d^2*f*e^(2*f*x + 2*e) - 12*c*d^2*f - 96*d^3*e^(2*f*x + 2*e
) - 3*d^3)*e^(-4*f*x - 4*e)/(a^2*f^4)
```

maple [B] time = 0.26, size = 1918, normalized size = 8.34

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*tanh(f*x+e))^2,x)
```

```
[Out] 1/f^4/a^2*(-2*d^3*(1/4*(f*x+e)^3*cosh(f*x+e)^4-3/16*(f*x+e)^2*sinh(f*x+e)*c
osh(f*x+e)^3-9/32*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-3/32*(f*x+e)^3+3/32*(f*
x+e)*cosh(f*x+e)^4-3/128*cosh(f*x+e)^3*sinh(f*x+e)-45/256*cosh(f*x+e)*sinh(
f*x+e)-45/256*f*x-45/256*e+9/32*(f*x+e)*cosh(f*x+e)^2)+2*d^3*(1/4*(f*x+e)^3
*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)+3/32*(f*x+
e)^4-3/16*(f*x+e)^2*cosh(f*x+e)^4+3/32*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+45
/64*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+45/128*(f*x+e)^2-3/128*cosh(f*x+e)^4-45
/128*cosh(f*x+e)^2-9/16*(f*x+e)^2*cosh(f*x+e)^2)+1/2*cosh(f*x+e)^4*d^3*e^3-
1/2*cosh(f*x+e)^4*c^3*f^3+6*d^3*e^2*(1/4*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+
3/8*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/16*(f*x+e)^2-1/16*cosh(f*x+e)^4-3/16*
cosh(f*x+e)^2)+2*c^3*f^3*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x+e)+3
/8*f*x+3/8*e)-6*d^3*e*(1/4*(f*x+e)^2*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)^
2*cosh(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)^3-1/8*(f*x+e)*cosh(f*x+e)^4+1/32*cosh
(f*x+e)^3*sinh(f*x+e)+15/64*cosh(f*x+e)*sinh(f*x+e)+15/64*f*x+15/64*e-3/8*(
f*x+e)*cosh(f*x+e)^2)+6*d^3*e*(1/4*(f*x+e)^2*cosh(f*x+e)^4-1/8*(f*x+e)*sinh
(f*x+e)*cosh(f*x+e)^3-3/16*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-3/32*(f*x+e)^2+1
/32*cosh(f*x+e)^4+3/32*cosh(f*x+e)^2)-6*d^3*e^2*(1/4*(f*x+e)*cosh(f*x+e)^4-
1/16*cosh(f*x+e)^3*sinh(f*x+e)-3/32*cosh(f*x+e)*sinh(f*x+e)-3/32*f*x-3/32*e
)-2*d^3*e^3*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)
+3*d^3*e*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*c
osh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-3*d^3*e^2*(1/2*(f*x
+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-c^3*f^3*(1/2*c
osh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2*e)+d^3*e^3*(1/2*cosh(f*x+e)*sinh(f*x+e)+
1/2*f*x+1/2*e)-6*c^2*d*f^2*(1/4*(f*x+e)*cosh(f*x+e)^4-1/16*cosh(f*x+e)^3*si
nh(f*x+e)-3/32*cosh(f*x+e)*sinh(f*x+e)-3/32*f*x-3/32*e)+6*c*d^2*f*(1/4*(f*x
+e)^2*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/8*(
f*x+e)^3-1/8*(f*x+e)*cosh(f*x+e)^4+1/32*cosh(f*x+e)^3*sinh(f*x+e)+15/64*cos
h(f*x+e)*sinh(f*x+e)+15/64*f*x+15/64*e-3/8*(f*x+e)*cosh(f*x+e)^2)+6*c^2*d*f
^2*(1/4*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)*cosh(f*x+e)*sinh(f*x+
e)+3/16*(f*x+e)^2-1/16*cosh(f*x+e)^4-3/16*cosh(f*x+e)^2)-6*c*d^2*f*(1/4*(f*
```

```
x+e)^2*cosh(f*x+e)^4-1/8*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3-3/16*(f*x+e)*cos
h(f*x+e)*sinh(f*x+e)-3/32*(f*x+e)^2+1/32*cosh(f*x+e)^4+3/32*cosh(f*x+e)^2)-
d^3*(1/2*(f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)^4-3/4*(f*x+e)^2*cosh
(f*x+e)^2+3/4*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*cosh(f*x+e)
^2)-3*c*d^2*f*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x
+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-3*c^2*d*f^2*(1
/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+12*c*d^
2*e*f*(1/4*(f*x+e)*cosh(f*x+e)^4-1/16*cosh(f*x+e)^3*sinh(f*x+e)-3/32*cosh(f
*x+e)*sinh(f*x+e)-3/32*f*x-3/32*e)-6*c^2*d*e*f^2*((1/4*cosh(f*x+e)^3+3/8*co
sh(f*x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)+6*c*d^2*e^2*f*((1/4*cosh(f*x+e)^3+3/8
*cosh(f*x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)+3/2*cosh(f*x+e)^4*c^2*d*e*f^2-3/2*
cosh(f*x+e)^4*c*d^2*e^2*f-12*c*d^2*e*f*(1/4*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)
^3+3/8*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/16*(f*x+e)^2-1/16*cosh(f*x+e)^4-3/
16*cosh(f*x+e)^2)+6*c*d^2*e*f*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x
+e)^2-1/4*cosh(f*x+e)^2)+3*c^2*d*e*f^2*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/2*f*x
+1/2*e)-3*c*d^2*e^2*f*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.79, size = 296, normalized size = 1.29

$$\frac{1}{16}c^3\left(\frac{4\left(fx+e\right)}{a^2f}-\frac{4\,e^{\left(-2\,fx-2\,e\right)}+e^{\left(-4\,fx-4\,e\right)}}{a^2f}\right)+\frac{3\left(8\,f^2x^2e^{(4\,e)}-8\left(2\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{\left(-2\,fx\right)}-\left(4\,fx+1\right)e^{\left(-4\,fx\right)}\right)c^2d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*c^3*(4*(f*x + e)/(a^2*f) - (4*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e))/(a^
2*f)) + 3/64*(8*f^2*x^2*e^(4*e) - 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) -
(4*f*x + 1)*e^(-4*f*x))*c^2*d*e^(-4*e)/(a^2*f^2) + 1/128*(32*f^3*x^3*e^(4*e
) - 48*(2*f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - 3*(8*f^2*
x^2 + 4*f*x + 1)*e^(-4*f*x))*c*d^2*e^(-4*e)/(a^2*f^3) + 1/512*(32*f^4*x^4*e
^(4*e) - 32*(4*f^3*x^3*e^(2*e) + 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) + 3*e^(2
*e))*e^(-2*f*x) - (32*f^3*x^3 + 24*f^2*x^2 + 12*f*x + 3)*e^(-4*f*x))*d^3*e^
(-4*e)/(a^2*f^4)
```

mupad [B] time = 1.25, size = 267, normalized size = 1.16

$$\frac{c^3\,x}{4\,a^2}-\mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{32\,c^3\,f^3+24\,c^2\,d\,f^2+12\,c\,d^2\,f+3\,d^3}{512\,a^2\,f^4}+\frac{d^3\,x^3}{16\,a^2\,f}+\frac{3\,d\,x\left(8\,c^2\,f^2+4\,c\,d\,f+d^2\right)}{128\,a^2\,f^3}+\frac{3\,d^2\,x^2\left(d+4\right.}{64\,a^2\,f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*tanh(e + f*x))^2,x)
```

```
[Out] (c^3*x)/(4*a^2) - exp(- 4*e - 4*f*x)*((3*d^3 + 32*c^3*f^3 + 24*c^2*d*f^2 +
12*c*d^2*f)/(512*a^2*f^4) + (d^3*x^3)/(16*a^2*f) + (3*d*x*(d^2 + 8*c^2*f^2
+ 4*c*d*f))/(128*a^2*f^3) + (3*d^2*x^2*(d + 4*c*f))/(64*a^2*f^2)) - exp(- 2
*e - 2*f*x)*((3*d^3 + 4*c^3*f^3 + 6*c^2*d*f^2 + 6*c*d^2*f)/(16*a^2*f^4) + (
d^3*x^3)/(4*a^2*f) + (3*d*x*(d^2 + 2*c^2*f^2 + 2*c*d*f))/(8*a^2*f^3) + (3*d
^2*x^2*(d + 2*c*f))/(8*a^2*f^2)) + (d^3*x^4)/(16*a^2) + (3*c^2*d*x^2)/(8*a^
2) + (c*d^2*x^3)/(4*a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c^3}{\tanh^2\left(e+fx\right)+2\tanh\left(e+fx\right)+1}\,dx+\int\frac{d^3x^3}{\tanh^2\left(e+fx\right)+2\tanh\left(e+fx\right)+1}\,dx+\int\frac{3cd^2x^2}{\tanh^2\left(e+fx\right)+2\tanh\left(e+fx\right)+1}\,dx+\int\frac{3c^2d}{\tanh^2\left(e+fx\right)+2\,\mathrm{ta}}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*tanh(f*x+e))**2,x)
```

```
[Out] (Integral(c**3/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(d**3
*x**3/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(3*c*d**2*x**2
/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(3*c**2*d*x/(tanh(e
 + f*x)**2 + 2*tanh(e + f*x) + 1), x))/a**2
```

3.39 $\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=170

$$-\frac{d(c+dx)e^{-4e-4fx}}{32a^2f^2}-\frac{d(c+dx)e^{-2e-2fx}}{4a^2f^2}-\frac{(c+dx)^2e^{-4e-4fx}}{16a^2f}-\frac{(c+dx)^2e^{-2e-2fx}}{4a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{d^2e^{-4e-4fx}}{128a^2f^3}-\frac{d^2e^{-2e-2fx}}{8a^2f^3}$$

```
[Out] -1/128*d^2*exp(-4*f*x-4*e)/a^2/f^3-1/8*d^2*exp(-2*f*x-2*e)/a^2/f^3-1/32*d*e
xp(-4*f*x-4*e)*(d*x+c)/a^2/f^2-1/4*d*exp(-2*f*x-2*e)*(d*x+c)/a^2/f^2-1/16*e
xp(-4*f*x-4*e)*(d*x+c)^2/a^2/f-1/4*exp(-2*f*x-2*e)*(d*x+c)^2/a^2/f+1/12*(d*
x+c)^3/a^2/d
```

Rubi [A] time = 0.19, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3729, 2176, 2194}

$$-\frac{d(c+dx)e^{-4e-4fx}}{32a^2f^2}-\frac{d(c+dx)e^{-2e-2fx}}{4a^2f^2}-\frac{(c+dx)^2e^{-4e-4fx}}{16a^2f}-\frac{(c+dx)^2e^{-2e-2fx}}{4a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{d^2e^{-4e-4fx}}{128a^2f^3}-\frac{d^2e^{-2e-2fx}}{8a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] -(d^2*E^(-4*e - 4*f*x))/(128*a^2*f^3) - (d^2*E^(-2*e - 2*f*x))/(8*a^2*f^3)
- (d*E^(-4*e - 4*f*x)*(c + d*x))/(32*a^2*f^2) - (d*E^(-2*e - 2*f*x)*(c + d*
x))/(4*a^2*f^2) - (E^(-4*e - 4*f*x)*(c + d*x)^2)/(16*a^2*f) - (E^(-2*e - 2*
f*x)*(c + d*x)^2)/(4*a^2*f) + (c + d*x)^3/(12*a^2*d)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^2}{4a^2} + \frac{e^{-4e-4fx}(c+dx)^2}{4a^2} + \frac{e^{-2e-2fx}(c+dx)^2}{2a^2}\right) dx$$
$$= \frac{(c+dx)^3}{12a^2d} + \frac{\int e^{-4e-4fx}(c+dx)^2\,dx}{4a^2} + \frac{\int e^{-2e-2fx}(c+dx)^2\,dx}{2a^2}$$
$$= -\frac{e^{-4e-4fx}(c+dx)^2}{16a^2f} - \frac{e^{-2e-2fx}(c+dx)^2}{4a^2f} + \frac{(c+dx)^3}{12a^2d} + \frac{d\int e^{-4e-4fx}(c+dx)\,dx}{8a^2f} +$$
$$= -\frac{de^{-4e-4fx}(c+dx)}{32a^2f^2} - \frac{de^{-2e-2fx}(c+dx)}{4a^2f^2} - \frac{e^{-4e-4fx}(c+dx)^2}{16a^2f} - \frac{e^{-2e-2fx}(c+dx)^2}{4a^2f}$$
$$= -\frac{d^2e^{-4e-4fx}}{128a^2f^3} - \frac{d^2e^{-2e-2fx}}{8a^2f^3} - \frac{de^{-4e-4fx}(c+dx)}{32a^2f^2} - \frac{de^{-2e-2fx}(c+dx)}{4a^2f^2} - \frac{e^{-4e-4fx}(c+$$

Mathematica [A] time = 0.99, size = 207, normalized size = 1.22

$$\frac{\operatorname{sech}^2(e+fx)\left(\left(24c^2f^2(4fx+1)+12cdf\left(8f^2x^2+4fx+1\right)+d^2\left(32f^3x^3+24f^2x^2+12fx+3\right)\right)\sinh(2(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] (Sech[e + f*x]^2*(-48*(2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) + d^2*(1 + 2*f*x + 2
*f^2*x^2)) + (24*c^2*f^2*(-1 + 4*f*x) + 12*c*d*f*(-1 - 4*f*x + 8*f^2*x^2) +
 d^2*(-3 - 12*f*x - 24*f^2*x^2 + 32*f^3*x^3))*Cosh[2*(e + f*x)] + (24*c^2*f
^2*(1 + 4*f*x) + 12*c*d*f*(1 + 4*f*x + 8*f^2*x^2) + d^2*(3 + 12*f*x + 24*f^
2*x^2 + 32*f^3*x^3))*Sinh[2*(e + f*x)]))/(384*a^2*f^3*(1 + Tanh[e + f*x])^2
)
```

fricas [B] time = 0.42, size = 361, normalized size = 2.12

$$-\frac{96\,d^2f^2x^2 + 96\,c^2f^2 + 96\,cdf - \left(32\,d^2f^3x^3 - 24\,c^2f^2 - 12\,cdf + 24\left(4\,cdf^3 - d^2f^2\right)x^2 - 3\,d^2 + 12\left(8\,c^2f^3 - 4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(96*d^2*f^2*x^2 + 96*c^2*f^2 + 96*c*d*f - (32*d^2*f^3*x^3 - 24*c^2*f
^2 - 12*c*d*f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*c*
d*f^2 - d^2*f)*x)*cosh(f*x + e)^2 - 2*(32*d^2*f^3*x^3 + 24*c^2*f^2 + 12*c*d
*f + 24*(4*c*d*f^3 + d^2*f^2)*x^2 + 3*d^2 + 12*(8*c^2*f^3 + 4*c*d*f^2 + d^2
*f)*x)*cosh(f*x + e)*sinh(f*x + e) - (32*d^2*f^3*x^3 - 24*c^2*f^2 - 12*c*d*
f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*c*d*f^2 - d^2*
f)*x)*sinh(f*x + e)^2 + 48*d^2 + 96*(2*c*d*f^2 + d^2*f)*x)/(a^2*f^3*cosh(f*
x + e)^2 + 2*a^2*f^3*cosh(f*x + e)*sinh(f*x + e) + a^2*f^3*sinh(f*x + e)^2)
```

giac [A] time = 0.15, size = 227, normalized size = 1.34

$$\frac{\left(32\,d^2f^3x^3e^{(4fx+4e)} + 96\,cdf^3x^2e^{(4fx+4e)} + 96\,c^2f^3xe^{(4fx+4e)} - 96\,d^2f^2x^2e^{(2fx+2e)} - 24\,d^2f^2x^2 - 192\,cdf^2xe^{(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/384*(32*d^2*f^3*x^3*e^(4*f*x + 4*e) + 96*c*d*f^3*x^2*e^(4*f*x + 4*e) + 96
*c^2*f^3*x*e^(4*f*x + 4*e) - 96*d^2*f^2*x^2*e^(2*f*x + 2*e) - 24*d^2*f^2*x^
2 - 192*c*d*f^2*x*e^(2*f*x + 2*e) - 48*c*d*f^2*x - 96*c^2*f^2*e^(2*f*x + 2*
e) - 96*d^2*f*x*e^(2*f*x + 2*e) - 24*c^2*f^2 - 12*d^2*f*x - 96*c*d*f*e^(2*f
*x + 2*e) - 12*c*d*f - 48*d^2*e^(2*f*x + 2*e) - 3*d^2)*e^(-4*f*x - 4*e)/(a^
2*f^3)
```

maple [B] time = 0.24, size = 913, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*tanh(f*x+e))^2,x)
```

```
[Out] 1/f^3/a^2*(2*d^2*(1/4*(f*x+e)^2*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)^2*cos
h(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)^3-1/8*(f*x+e)*cosh(f*x+e)^4+1/32*cosh(f*x+
e)^3*sinh(f*x+e)+15/64*cosh(f*x+e)*sinh(f*x+e)+15/64*f*x+15/64*e-3/8*(f*x+e
)*cosh(f*x+e)^2)+4*c*d*f*(1/4*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)
*cosh(f*x+e)*sinh(f*x+e)+3/16*(f*x+e)^2-1/16*cosh(f*x+e)^4-3/16*cosh(f*x+e)
^2)-4*e*d^2*(1/4*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)*cosh(f*x+e)*
sinh(f*x+e)+3/16*(f*x+e)^2-1/16*cosh(f*x+e)^4-3/16*cosh(f*x+e)^2)+2*c^2*f^2
*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)-4*c*d*f*e*
((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)+2*d^2*e^2*(
(1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)-2*d^2*(1/4*(
f*x+e)^2*cosh(f*x+e)^4-1/8*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3-3/16*(f*x+e)*c
osh(f*x+e)*sinh(f*x+e)-3/32*(f*x+e)^2+1/32*cosh(f*x+e)^4+3/32*cosh(f*x+e)^2
)-4*c*d*f*(1/4*(f*x+e)*cosh(f*x+e)^4-1/16*cosh(f*x+e)^3*sinh(f*x+e)-3/32*co
sh(f*x+e)*sinh(f*x+e)-3/32*f*x-3/32*e)+4*e*d^2*(1/4*(f*x+e)*cosh(f*x+e)^4-1
/16*cosh(f*x+e)^3*sinh(f*x+e)-3/32*cosh(f*x+e)*sinh(f*x+e)-3/32*f*x-3/32*e)
-1/2*cosh(f*x+e)^4*c^2*f^2+cosh(f*x+e)^4*c*d*f*e-1/2*cosh(f*x+e)^4*d^2*e^2-
d^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f
*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-2*c*d*f*(1/2*(f*x+e)*cos
h(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+2*e*d^2*(1/2*(f*x+e)*
cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-c^2*f^2*(1/2*cosh(
f*x+e)*sinh(f*x+e)+1/2*f*x+1/2*e)+2*c*d*f*e*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/
2*f*x+1/2*e)-d^2*e^2*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.63, size = 190, normalized size = 1.12

$$\frac{1}{16}c^2\left(\frac{4\left(fx+e\right)}{a^2f}-\frac{4\,e^{\left(-2\,fx-2\,e\right)}+e^{\left(-4\,fx-4\,e\right)}}{a^2f}\right)+\frac{\left(8\,f^2x^2e^{(4\,e)}-8\left(2\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{\left(-2\,fx\right)}-\left(4\,fx+1\right)e^{\left(-4\,fx\right)}\right)cde^{(-}}{32\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*c^2*(4*(f*x + e)/(a^2*f) - (4*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e))/(a^
2*f)) + 1/32*(8*f^2*x^2*e^(4*e) - 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) -
(4*f*x + 1)*e^(-4*f*x))*c*d*e^(-4*e)/(a^2*f^2) + 1/384*(32*f^3*x^3*e^(4*e)
- 48*(2*f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - 3*(8*f^2*x^
2 + 4*f*x + 1)*e^(-4*f*x))*d^2*e^(-4*e)/(a^2*f^3)
```

mupad [B] time = 1.18, size = 165, normalized size = 0.97

$$\frac{c^2\,x}{4\,a^2}-\mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{8\,c^2\,f^2+4\,c\,d\,f+d^2}{128\,a^2\,f^3}+\frac{d^2\,x^2}{16\,a^2\,f}+\frac{d\,x\,\left(d+4\,c\,f\right)}{32\,a^2\,f^2}\right)-\mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{2\,c^2\,f^2+2\,c\,d\,f+d^2}{8\,a^2\,f^3}+\frac{d^2\,x^2}{4\,a^2\,f}+\frac{d}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*tanh(e + f*x))^2,x)
```

```
[Out] (c^2*x)/(4*a^2) - exp(- 4*e - 4*f*x)*((d^2 + 8*c^2*f^2 + 4*c*d*f)/(128*a^2*
f^3) + (d^2*x^2)/(16*a^2*f) + (d*x*(d + 4*c*f))/(32*a^2*f^2)) - exp(- 2*e -
 2*f*x)*((d^2 + 2*c^2*f^2 + 2*c*d*f)/(8*a^2*f^3) + (d^2*x^2)/(4*a^2*f) + (d
*x*(d + 2*c*f))/(4*a^2*f^2)) + (d^2*x^3)/(12*a^2) + (c*d*x^2)/(4*a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2}{\tanh^2\left(e+fx\right)+2\tanh\left(e+fx\right)+1}\,dx + \int \frac{d^2x^2}{\tanh^2\left(e+fx\right)+2\tanh\left(e+fx\right)+1}\,dx + \int \frac{2cdx}{\tanh^2\left(e+fx\right)+2\tanh\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*tanh(f*x+e))**2,x)
```

```
[Out] (Integral(c**2/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(d**2
*x**2/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(2*c*d*x/(tanh
(e + f*x)**2 + 2*tanh(e + f*x) + 1), x))/a**2
```

3.40 $\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=133

$$-\frac{c+dx}{4f\left(a^2\tanh(e+fx)+a^2\right)}+\frac{x(c+dx)}{4a^2}-\frac{3d}{16f^2\left(a^2\tanh(e+fx)+a^2\right)}+\frac{3dx}{16a^2f}-\frac{dx^2}{8a^2}-\frac{c+dx}{4f(a\tanh(e+fx)+a)^2}-\frac{16}{}$$

```
[Out] 3/16*d*x/a^2/f-1/8*d*x^2/a^2+1/4*x*(d*x+c)/a^2-1/16*d/f^2/(a+a*tanh(f*x+e))
^2+1/4*(-d*x-c)/f/(a+a*tanh(f*x+e))^2-3/16*d/f^2/(a^2+a^2*tanh(f*x+e))+1/4*
(-d*x-c)/f/(a^2+a^2*tanh(f*x+e))
```

Rubi [A] time = 0.13, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3479, 8, 3730}

$$-\frac{c+dx}{4f\left(a^2\tanh(e+fx)+a^2\right)}+\frac{x(c+dx)}{4a^2}-\frac{3d}{16f^2\left(a^2\tanh(e+fx)+a^2\right)}+\frac{3dx}{16a^2f}-\frac{dx^2}{8a^2}-\frac{c+dx}{4f(a\tanh(e+fx)+a)^2}-\frac{16}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] (3*d*x)/(16*a^2*f) - (d*x^2)/(8*a^2) + (x*(c + d*x))/(4*a^2) - d/(16*f^2*(a
 + a*Tanh[e + f*x])^2) - (c + d*x)/(4*f*(a + a*Tanh[e + f*x])^2) - (3*d)/(1
6*f^2*(a^2 + a^2*Tanh[e + f*x])) - (c + d*x)/(4*f*(a^2 + a^2*Tanh[e + f*x])
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3730

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{x(c+dx)}{4a^2} - \frac{c+dx}{4f(a+a\tanh(e+fx))^2} - \frac{c+dx}{4f\left(a^2+a^2\tanh(e+fx)\right)} - d\int\left(\frac{x}{4a^2}-$$

$$= -\frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} - \frac{c+dx}{4f(a+a\tanh(e+fx))^2} - \frac{c+dx}{4f\left(a^2+a^2\tanh(e+fx)\right)} + \frac{d}{}$$

$$= -\frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} - \frac{d}{16f^2(a+a\tanh(e+fx))^2} - \frac{c+dx}{4f(a+a\tanh(e+fx))^2} - \frac{}{8f}$$

$$= \frac{dx}{8a^2f} - \frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} - \frac{d}{16f^2(a+a\tanh(e+fx))^2} - \frac{c+dx}{4f(a+a\tanh(e+fx))}$$

$$= \frac{3dx}{16a^2f} - \frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} - \frac{d}{16f^2(a+a\tanh(e+fx))^2} - \frac{c+dx}{4f(a+a\tanh(e+fx)}$$

Mathematica [A] time = 0.54, size = 114, normalized size = 0.86

$$\frac{\operatorname{sech}^2(e+fx)\left(\left(4cf(4fx+1)+d\left(8f^2x^2+4fx+1\right)\right)\sinh(2(e+fx))+\left(4cf(4fx-1)+d\left(8f^2x^2-4fx-1\right)\right)\right.}{64a^2f^2(\tanh(e+fx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] (Sech[e + f*x]^2*(-8*(d + 2*c*f + 2*d*f*x) + (4*c*f*(-1 + 4*f*x) + d*(-1 -
4*f*x + 8*f^2*x^2))*Cosh[2*(e + f*x)] + (4*c*f*(1 + 4*f*x) + d*(1 + 4*f*x +
 8*f^2*x^2))*Sinh[2*(e + f*x)]))/(64*a^2*f^2*(1 + Tanh[e + f*x])^2)
```

fricas [A] time = 0.48, size = 192, normalized size = 1.44

$$-\frac{16\,dfx-\left(8\,df^2x^2-4\,cf+4\left(4\,cf^2-df\right)x-d\right)\cosh\left(fx+e\right)^2-2\left(8\,df^2x^2+4\,cf+4\left(4\,cf^2+df\right)x+d\right)\cos}{64\left(a^2f^2\cosh\left(fx+e\right)^2+2\,a^2f^2\cosh\left(fx+e\right)\sin\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/64*(16*d*f*x - (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*cosh(f*x
+ e)^2 - 2*(8*d*f^2*x^2 + 4*c*f + 4*(4*c*f^2 + d*f)*x + d)*cosh(f*x + e)*si
nh(f*x + e) - (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*sinh(f*x + e)
^2 + 16*c*f + 8*d)/(a^2*f^2*cosh(f*x + e)^2 + 2*a^2*f^2*cosh(f*x + e)*sinh(
f*x + e) + a^2*f^2*sinh(f*x + e)^2)
```

giac [A] time = 0.15, size = 109, normalized size = 0.82

$$\frac{\left(8\,df^2x^2e^{\left(4fx+4e\right)}+16\,cf^2xe^{\left(4fx+4e\right)}-16\,dfxe^{\left(2fx+2e\right)}-4\,dfx-16\,cfe^{\left(2fx+2e\right)}-4\,cf-8\,de^{\left(2fx+2e\right)}-d\right)e^{(-4}}{64\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/64*(8*d*f^2*x^2*e^(4*f*x + 4*e) + 16*c*f^2*x*e^(4*f*x + 4*e) - 16*d*f*x*e
^(2*f*x + 2*e) - 4*d*f*x - 16*c*f*e^(2*f*x + 2*e) - 4*c*f - 8*d*e^(2*f*x +
2*e) - d)*e^(-4*f*x - 4*e)/(a^2*f^2)
```

maple [B] time = 0.24, size = 331, normalized size = 2.49

$$2d\left(\frac{(fx+e)\sinh(fx+e)(\cosh^3(fx+e))}{4}+\frac{3(fx+e)\cosh(fx+e)\sinh(fx+e)}{8}+\frac{3(fx+e)^2}{16}-\frac{(\cosh^4(fx+e))}{16}-\frac{3(\cosh^2(fx+e))}{16}\right)+2cf\left(\left(\frac{(\cosh}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*tanh(f*x+e))^2,x)
```

```
[Out] 1/f^2/a^2*(2*d*(1/4*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)*cosh(f*x+
e)*sinh(f*x+e)+3/16*(f*x+e)^2-1/16*cosh(f*x+e)^4-3/16*cosh(f*x+e)^2)+2*c*f*
((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)-2*d*e*((1/4
*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)-2*d*(1/4*(f*x+e)
*cosh(f*x+e)^4-1/16*cosh(f*x+e)^3*sinh(f*x+e)-3/32*cosh(f*x+e)*sinh(f*x+e)-
3/32*f*x-3/32*e)-1/2*cosh(f*x+e)^4*c*f+1/2*cosh(f*x+e)^4*d*e-d*(1/2*(f*x+e)
*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-c*f*(1/2*cosh(f*x
+e)*sinh(f*x+e)+1/2*f*x+1/2*e)+d*e*(1/2*cosh(f*x+e)*sinh(f*x+e)+1/2*f*x+1/2
*e))
```

maxima [A] time = 1.45, size = 106, normalized size = 0.80

$$\frac{1}{16}c\left(\frac{4\left(fx+e\right)}{a^2f}-\frac{4\,e^{\left(-2\,fx-2\,e\right)}+e^{\left(-4\,fx-4\,e\right)}}{a^2f}\right)+\frac{\left(8\,f^2x^2e^{(4\,e)}-8\left(2\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{\left(-2\,fx\right)}-\left(4\,fx+1\right)e^{\left(-4\,fx\right)}\right)de^{(-4\,e}}{64\,a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/16*c*(4*(f*x + e)/(a^2*f) - (4*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e))/(a^2*
f)) + 1/64*(8*f^2*x^2*e^(4*e) - 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - (4
*f*x + 1)*e^(-4*f*x))*d*e^(-4*e)/(a^2*f^2)
```

mupad [B] time = 1.14, size = 89, normalized size = 0.67

$$\frac{d\,x^2}{8\,a^2}-\mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{d+4\,c\,f}{64\,a^2\,f^2}+\frac{d\,x}{16\,a^2\,f}\right)-\mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{d+2\,c\,f}{8\,a^2\,f^2}+\frac{d\,x}{4\,a^2\,f}\right)+\frac{c\,x}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*tanh(e + f*x))^2,x)
```

```
[Out] (d*x^2)/(8*a^2) - exp(- 4*e - 4*f*x)*((d + 4*c*f)/(64*a^2*f^2) + (d*x)/(16*
a^2*f)) - exp(- 2*e - 2*f*x)*((d + 2*c*f)/(8*a^2*f^2) + (d*x)/(4*a^2*f)) +
(c*x)/(4*a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c}{\tanh^2\left(e+fx\right)+2\tanh\left(e+fx\right)+1}\,dx+\int \frac{dx}{\tanh^2\left(e+fx\right)+2\tanh\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e))**2,x)
```

```
[Out] (Integral(c/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(d*x/(ta
nh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x))/a**2
```

3.41 $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=297

$$-\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2a^2d}-\frac{\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{4a^2d}+\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{\text{Chi}\left($$

```
[Out] 1/4*Chi(4*c*f/d+4*f*x)*cosh(-4*e+4*c*f/d)/a^2/d+1/2*Chi(2*c*f/d+2*f*x)*cosh
(-2*e+2*c*f/d)/a^2/d+1/4*ln(d*x+c)/a^2/d-1/2*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d
+2*f*x)/a^2/d-1/4*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d+1/4*Chi(4*c*f
/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^2/d-1/4*Shi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d
)/a^2/d+1/2*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^2/d-1/2*Shi(2*c*f/d+2*f
*x)*sinh(-2*e+2*c*f/d)/a^2/d
```

Rubi [A] time = 0.77, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3728, 3303, 3298, 3301, 3312}

$$-\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2a^2d}-\frac{\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{4a^2d}+\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2a^2d}+\frac{\text{Chi}\left($$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + a*Tanh[e + f*x])^2),x]
```

```
[Out] (Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (Cosh[4
*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + Log[c + d*x]/(
4*a^2*d) - (CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(4*a^2*d
) - (CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(2*a^2*d) - (Co
sh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (Sinh[2*e
- (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) - (Cosh[4*e - (4*c*
f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + (Sinh[4*e - (4*c*f)/d]*S
inhIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3728

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = \int\left(\frac{1}{4a^2(c+dx)} + \frac{\cosh(2e+2fx)}{2a^2(c+dx)} + \frac{\cosh^2(2e+2fx)}{4a^2(c+dx)} - \frac{\sinh(2e+2fx)}{2a^2(c+dx)} -$$

$$= \frac{\log(c+dx)}{4a^2d} + \frac{\int \frac{\cosh^2(2e+2fx)}{c+dx}\,dx}{4a^2} + \frac{\int \frac{\sinh^2(2e+2fx)}{c+dx}\,dx}{4a^2} - \frac{\int \frac{\sinh(4e+4fx)}{c+dx}\,dx}{4a^2} +$$

$$= \frac{\log(c+dx)}{4a^2d} - \frac{\int \left(\frac{1}{2(c+dx)} - \frac{\cosh(4e+4fx)}{2(c+dx)}\right)dx}{4a^2} + \frac{\int \left(\frac{1}{2(c+dx)} + \frac{\cosh(4e+4fx)}{2(c+dx)}\right)dx}{4a^2}$$

$$= \frac{\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{\mathrm{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4\right.}{4a^2d}$$

$$= \frac{\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{\mathrm{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4\right.}{4a^2d}$$

$$= \frac{\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} + \frac{\log(c+dx)}{4a^2d} - \frac{\mathrm{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4\right.}{4a^2d}$$

Mathematica [A] time = 0.47, size = 199, normalized size = 0.67

$$\frac{\left(\cosh\left(2e-\frac{2cf}{d}\right)-\sinh\left(2e-\frac{2cf}{d}\right)\right)\left(\mathrm{Chi}\left(\frac{4f(c+dx)}{d}\right)\left(\cosh\left(2e-\frac{2cf}{d}\right)-\sinh\left(2e-\frac{2cf}{d}\right)\right)+2\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)+\sinh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + a*Tanh[e + f*x])^2),x]
```

```
[Out] ((Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d])*(2*CoshIntegral[(2*f*(c +
d*x))/d] + Cosh[2*e - (2*c*f)/d]*Log[f*(c + d*x)] + CoshIntegral[(4*f*(c +
d*x))/d]*(Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d]) + Log[f*(c + d*x)]
*Sinh[2*e - (2*c*f)/d] - 2*SinhIntegral[(2*f*(c + d*x))/d] - Cosh[2*e - (2*
c*f)/d]*SinhIntegral[(4*f*(c + d*x))/d] + Sinh[2*e - (2*c*f)/d]*SinhIntegra
l[(4*f*(c + d*x))/d]))/(4*a^2*d)
```

fricas [A] time = 0.45, size = 133, normalized size = 0.45

$$\frac{2\,\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\cosh\left(-\frac{2\left(de-cf\right)}{d}\right)+\mathrm{Ei}\left(-\frac{4\left(dfx+cf\right)}{d}\right)\cosh\left(-\frac{4\left(de-cf\right)}{d}\right)+2\,\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\sinh\left(-\frac{2\left(de-cf\right)}{d}\right)+\mathrm{Ei}\left(\right.}{4\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + Ei(-4*(d*f*x + c*f)/
d)*cosh(-4*(d*e - c*f)/d) + 2*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d)
 + Ei(-4*(d*f*x + c*f)/d)*sinh(-4*(d*e - c*f)/d) + log(d*x + c))/(a^2*d)
```

giac [A] time = 0.13, size = 78, normalized size = 0.26

$$\frac{\left(\mathrm{Ei}\left(-\frac{4\left(dfx+cf\right)}{d}\right)e^{\left(\frac{4cf}{d}\right)}+2\,\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2cf}{d}+2e\right)}+e^{(4e)}\log\left(dx+c\right)\right)e^{(-4e)}}{4\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*(Ei(-4*(d*f*x + c*f)/d)*e^(4*c*f/d) + 2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f
/d + 2*e) + e^(4*e)*log(d*x + c))*e^(-4*e)/(a^2*d)
```

maple [A] time = 0.79, size = 106, normalized size = 0.36

$$\frac{\ln\left(dx+c\right)}{4d\,a^2}-\frac{\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{Ei}\left(1,4fx+4e+\frac{4cf-4de}{d}\right)}{4a^2d}-\frac{\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}\left(1,2fx+2e+\frac{2cf-2de}{d}\right)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x)
```

```
[Out] 1/4*ln(d*x+c)/d/a^2-1/4/a^2/d*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)
/d)-1/2/a^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

maxima [A] time = 1.07, size = 81, normalized size = 0.27

$$-\frac{e^{\left(-4e+\frac{4cf}{d}\right)}E_1\left(\frac{4\left(dx+c\right)f}{d}\right)}{4\,a^2d}-\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2\left(dx+c\right)f}{d}\right)}{2\,a^2d}+\frac{\log\left(dx+c\right)}{4\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^(-4*e + 4*c*f/d)*exp_integral_e(1, 4*(d*x + c)*f/d)/(a^2*d) - 1/2*e^
(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a^2*d) + 1/4*log(d*x +
 c)/(a^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{\left(a+a\tanh\left(e+f\,x\right)\right)^2\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tanh(e + f*x))^2*(c + d*x)),x)
```

```
[Out] int(1/((a + a*tanh(e + f*x))^2*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{1}{c\tanh^2\left(e+fx\right)+2c\tanh\left(e+fx\right)+c+dx\tanh^2\left(e+fx\right)+2dx\tanh\left(e+fx\right)+dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e))**2,x)
```

```
[Out] Integral(1/(c*tanh(e + f*x)**2 + 2*c*tanh(e + f*x) + c + d*x*tanh(e + f*x)*
*2 + 2*d*x*tanh(e + f*x) + d*x), x)/a**2
```

3.42 $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=420

$$\frac{f\mathrm{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{a^2d^2}+\frac{f\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\mathrm{Chi}($$

```
[Out] -1/4/a^2/d/(d*x+c)-f*Chi(4*c*f/d+4*f*x)*cosh(-4*e+4*c*f/d)/a^2/d^2-f*Chi(2*
c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a^2/d^2-1/2*cosh(2*f*x+2*e)/a^2/d/(d*x+c)-1
/4*cosh(2*f*x+2*e)^2/a^2/d/(d*x+c)+f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/
a^2/d^2+f*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d^2-f*Chi(4*c*f/d+4*f*x
)*sinh(-4*e+4*c*f/d)/a^2/d^2+f*Shi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^2/d^
2-f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^2/d^2+f*Shi(2*c*f/d+2*f*x)*sinh
(-2*e+2*c*f/d)/a^2/d^2+1/2*sinh(2*f*x+2*e)/a^2/d/(d*x+c)-1/4*sinh(2*f*x+2*e
)^2/a^2/d/(d*x+c)+1/4*sinh(4*f*x+4*e)/a^2/d/(d*x+c)
```

Rubi [A] time = 0.79, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3728, 3297, 3303, 3298, 3301, 3313, 12}

$$\frac{f\mathrm{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{a^2d^2}+\frac{f\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{a^2d^2}-\frac{f\mathrm{Chi}($$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^2),x]
```

```
[Out] -1/(4*a^2*d*(c + d*x)) - Cosh[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Cosh[2*e +
 2*f*x]^2/(4*a^2*d*(c + d*x)) - (f*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*
f)/d + 2*f*x])/(a^2*d^2) - (f*Cosh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d
 + 4*f*x])/(a^2*d^2) + (f*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)
/d])/(a^2*d^2) + (f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/
(a^2*d^2) + Sinh[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Sinh[2*e + 2*f*x]^2/(4*
a^2*d*(c + d*x)) + Sinh[4*e + 4*f*x]/(4*a^2*d*(c + d*x)) + (f*Cosh[2*e - (2
*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*Sinh[2*e - (2*c*f)
/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (f*Cosh[4*e - (4*c*f)/d]*S
inhIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (f*Sinh[4*e - (4*c*f)/d]*SinhIn
tegral[(4*c*f)/d + 4*f*x])/(a^2*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3728

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = \int \left(\frac{1}{4a^2(c+dx)^2} + \frac{\cosh(2e+2fx)}{2a^2(c+dx)^2} + \frac{\cosh^2(2e+2fx)}{4a^2(c+dx)^2} - \frac{\sinh(2e+2}{2a^2(c+dx}\right.$$

$$= -\frac{1}{4a^2d(c+dx)} + \frac{\int \frac{\cosh^2(2e+2fx)}{(c+dx)^2}\,dx}{4a^2} + \frac{\int \frac{\sinh^2(2e+2fx)}{(c+dx)^2}\,dx}{4a^2} - \frac{\int \frac{\sinh(4e+4fx)}{(c+dx)^2}}{4a^2}$$

$$= -\frac{1}{4a^2d(c+dx)} - \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} + \frac{\sinh(2e+2fx}{2a^2d(c+dx)}$$

$$= -\frac{1}{4a^2d(c+dx)} - \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} + \frac{\sinh(2e+2fx}{2a^2d(c+dx)}$$

$$= -\frac{1}{4a^2d(c+dx)} - \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} - \frac{f\cosh\left(2e-\frac{2}{}\right.}{}$$

$$= -\frac{1}{4a^2d(c+dx)} - \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} - \frac{f\cosh\left(2e-\frac{2}{}\right.}{}$$

Mathematica [A] time = 1.42, size = 442, normalized size = 1.05

$$\frac{\left(\sinh\left(2\left(f\left(x-\frac{c}{d}\right)+e\right)\right)-\cosh\left(2\left(f\left(x-\frac{c}{d}\right)+e\right)\right)\right)\left(4f(c+dx)\mathrm{Chi}\left(\frac{4f(c+dx)}{d}\right)\left(\cosh\left(2e-\frac{2f(c+dx)}{d}\right)-\sinh\left(2\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^2),x]
```

```
[Out] ((-Cosh[2*(e + f*(-(c/d) + x))] + Sinh[2*(e + f*(-(c/d) + x))])*(2*d*Cosh[(
2*c*f)/d] + d*Cosh[2*(e + f*(-(c/d) + x))] + d*Cosh[2*(e + f*(c/d + x))] -
2*d*Sinh[(2*c*f)/d] + 4*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[2
*f*x] + Sinh[2*f*x]) + d*Sinh[2*(e + f*(-(c/d) + x))] - d*Sinh[2*(e + f*(c/
d + x))] + 4*f*(c + d*x)*CoshIntegral[(4*f*(c + d*x))/d]*(Cosh[2*e - (2*f*(
c + d*x))/d] - Sinh[2*e - (2*f*(c + d*x))/d]) - 4*c*f*Cosh[2*f*x]*SinhInteg
ral[(2*f*(c + d*x))/d] - 4*d*f*x*Cosh[2*f*x]*SinhIntegral[(2*f*(c + d*x))/d
] - 4*c*f*Sinh[2*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 4*d*f*x*Sinh[2*f*x]
*SinhIntegral[(2*f*(c + d*x))/d] - 4*c*f*Cosh[2*e - (2*f*(c + d*x))/d]*Sinh
Integral[(4*f*(c + d*x))/d] - 4*d*f*x*Cosh[2*e - (2*f*(c + d*x))/d]*SinhInt
egral[(4*f*(c + d*x))/d] + 4*c*f*Sinh[2*e - (2*f*(c + d*x))/d]*SinhIntegral
[(4*f*(c + d*x))/d] + 4*d*f*x*Sinh[2*e - (2*f*(c + d*x))/d]*SinhIntegral[(4
*f*(c + d*x))/d]))/(4*a^2*d^2*(c + d*x))
```

fricas **[A]** time = 0.50, size = 609, normalized size = 1.45

$$-\frac{2\left(dfx+cf\right)\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\cosh\left(fx+e\right)^2\sinh\left(-\frac{2\left(de-cf\right)}{d}\right)+2\left(dfx+cf\right)\mathrm{Ei}\left(-\frac{4\left(dfx+cf\right)}{d}\right)\cosh\left(fx+e\right)^2\sinh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-2*(d*e -
 c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-4*(
d*e - c*f)/d) + (2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)
/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + d)*co
sh(f*x + e)^2 + (2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)
/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + 2*(d*
f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) + 2*(d*f*x + c*f)*
Ei(-4*(d*f*x + c*f)/d)*sinh(-4*(d*e - c*f)/d) + d)*sinh(f*x + e)^2 + 4*((d*
f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f)/d) + (d
*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-4*(d*e - c*f)/d) + (
(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + (d*f*x + c*f)
*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d))*cosh(f*x + e))*sinh(f*x + e
) + d)/((a^2*d^3*x + a^2*c*d^2)*cosh(f*x + e)^2 + 2*(a^2*d^3*x + a^2*c*d^2)
*cosh(f*x + e)*sinh(f*x + e) + (a^2*d^3*x + a^2*c*d^2)*sinh(f*x + e)^2)
```

giac **[A]** time = 0.20, size = 633, normalized size = 1.51

$$-\left(\begin{array}{l}4\left(dx+c\right)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)f^2\mathrm{Ei}\left(\frac{4\left(\left(dx+c\right)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de\right)}{d}\right)e^{\left(\frac{4\left(cf-de\right)}{d}\right)}-4\,cf^3\mathrm{Ei}\left(\frac{4\left(\left(dx+c\right)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de\right)}{d}\right)e\end{array}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/4*(4*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(4*((d*x + c)*(
c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(4*(c*f - d*e)/d) - 4*
c*f^3*Ei(4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e
^(4*(c*f - d*e)/d) + 4*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei
(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(2*(c*f
 - d*e)/d) - 4*c*f^3*Ei(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) -
```

```
c*f + d*e)/d)*e^(2*(c*f - d*e)/d) + 4*d*f^2*Ei(4*((d*x + c)*(c*f/(d*x + c)
- f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(4*(c*f - d*e)/d + 1) + 4*d*f^2*Ei(2
*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(2*(c*f -
 d*e)/d + 1) - d*f^2*e^(4*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d)
- 2*d*f^2*e^(2*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d) - d*f^2)*d^
2/(((d*x + c)*a^2*d^4*(c*f/(d*x + c) - f - d*e/(d*x + c)) - a^2*c*d^4*f + a
^2*d^5*e)*f)
```

maple [A] time = 0.91, size = 163, normalized size = 0.39

$$-\frac{1}{4a^2d\,(dx+c)}-\frac{f\,\mathrm{e}^{-4fx-4e}}{4a^2d\left(dfx+cf\right)}+\frac{f\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{Ei}\left(1,4fx+4e+\frac{4cf-4de}{d}\right)}{a^2d^2}-\frac{f\,\mathrm{e}^{-2fx-2e}}{2a^2d\left(dfx+cf\right)}+\frac{f\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}\left(1,2fx\right.}{a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x)
```

```
[Out] -1/4/a^2/d/(d*x+c)-1/4*f/a^2*exp(-4*f*x-4*e)/d/(d*f*x+c*f)+f/a^2/d^2*exp(4*
(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)-1/2*f/a^2*exp(-2*f*x-2*e)/d/(d*f
*x+c*f)+f/a^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

maxima [A] time = 1.95, size = 100, normalized size = 0.24

$$-\frac{1}{4\left(a^2d^2x+a^2cd\right)}-\frac{e^{\left(-4\,e+\frac{4\,cf}{d}\right)}E_2\left(\frac{4\,(dx+c)f}{d}\right)}{4\,(dx+c)a^2d}-\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{2\,(dx+c)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/4/(a^2*d^2*x + a^2*c*d) - 1/4*e^(-4*e + 4*c*f/d)*exp_integral_e(2, 4*(d*
x + c)*f/d)/((d*x + c)*a^2*d) - 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*
(d*x + c)*f/d)/((d*x + c)*a^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\tanh\left(e+f\,x\right)\right)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tanh(e + f*x))^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*tanh(e + f*x))^2*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\tanh^2\left(e+fx\right)+2c^2\tanh\left(e+fx\right)+c^2+2cdx\tanh^2\left(e+fx\right)+4cdx\tanh\left(e+fx\right)+2cdx+d^2x^2\tanh^2\left(e+fx\right)+2d^2x^2\tanh\left(e+fx\right)+d^2x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*tanh(f*x+e))**2,x)
```

```
[Out] Integral(1/(c**2*tanh(e + f*x)**2 + 2*c**2*tanh(e + f*x) + c**2 + 2*c*d*x*t
anh(e + f*x)**2 + 4*c*d*x*tanh(e + f*x) + 2*c*d*x + d**2*x**2*tanh(e + f*x)
**2 + 2*d**2*x**2*tanh(e + f*x) + d**2*x**2), x)/a**2
```

3.43 $\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$

Optimal. Leaf size=336

$$-\frac{d^2(c+dx)e^{-6e-6fx}}{288a^3f^3}-\frac{9d^2(c+dx)e^{-4e-4fx}}{256a^3f^3}-\frac{9d^2(c+dx)e^{-2e-2fx}}{32a^3f^3}-\frac{d(c+dx)^2e^{-6e-6fx}}{96a^3f^2}-\frac{9d(c+dx)^2e^{-4e-4fx}}{128a^3f^2}-\frac{9d(c+d}{3}$$

```
[Out] -1/1728*d^3*exp(-6*f*x-6*e)/a^3/f^4-9/1024*d^3*exp(-4*f*x-4*e)/a^3/f^4-9/64
*d^3*exp(-2*f*x-2*e)/a^3/f^4-1/288*d^2*exp(-6*f*x-6*e)*(d*x+c)/a^3/f^3-9/25
6*d^2*exp(-4*f*x-4*e)*(d*x+c)/a^3/f^3-9/32*d^2*exp(-2*f*x-2*e)*(d*x+c)/a^3/
f^3-1/96*d*exp(-6*f*x-6*e)*(d*x+c)^2/a^3/f^2-9/128*d*exp(-4*f*x-4*e)*(d*x+c
)^2/a^3/f^2-9/32*d*exp(-2*f*x-2*e)*(d*x+c)^2/a^3/f^2-1/48*exp(-6*f*x-6*e)*(
d*x+c)^3/a^3/f-3/32*exp(-4*f*x-4*e)*(d*x+c)^3/a^3/f-3/16*exp(-2*f*x-2*e)*(d
*x+c)^3/a^3/f+1/32*(d*x+c)^4/a^3/d
```

Rubi [A] time = 0.38, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3729, 2176, 2194}

$$-\frac{d^2(c+dx)e^{-6e-6fx}}{288a^3f^3}-\frac{9d^2(c+dx)e^{-4e-4fx}}{256a^3f^3}-\frac{9d^2(c+dx)e^{-2e-2fx}}{32a^3f^3}-\frac{d(c+dx)^2e^{-6e-6fx}}{96a^3f^2}-\frac{9d(c+dx)^2e^{-4e-4fx}}{128a^3f^2}-\frac{9d(c+d}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Tanh[e + f*x])^3,x]
```

```
[Out] -(d^3*E^(-6*e - 6*f*x))/(1728*a^3*f^4) - (9*d^3*E^(-4*e - 4*f*x))/(1024*a^3
*f^4) - (9*d^3*E^(-2*e - 2*f*x))/(64*a^3*f^4) - (d^2*E^(-6*e - 6*f*x)*(c +
d*x))/(288*a^3*f^3) - (9*d^2*E^(-4*e - 4*f*x)*(c + d*x))/(256*a^3*f^3) - (9
*d^2*E^(-2*e - 2*f*x)*(c + d*x))/(32*a^3*f^3) - (d*E^(-6*e - 6*f*x)*(c + d*
x)^2)/(96*a^3*f^2) - (9*d*E^(-4*e - 4*f*x)*(c + d*x)^2)/(128*a^3*f^2) - (9*
d*E^(-2*e - 2*f*x)*(c + d*x)^2)/(32*a^3*f^2) - (E^(-6*e - 6*f*x)*(c + d*x)^
3)/(48*a^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^3)/(32*a^3*f) - (3*E^(-2*e -
2*f*x)*(c + d*x)^3)/(16*a^3*f) + (c + d*x)^4/(32*a^3*d)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx = \int \left(\frac{(c+dx)^3}{8a^3} + \frac{e^{-6e-6fx}(c+dx)^3}{8a^3} + \frac{3e^{-4e-4fx}(c+dx)^3}{8a^3} + \frac{3e^{-2e-2fx}(c+dx)^3}{8a^3}\right) d$$

$$= \frac{(c+dx)^4}{32a^3d} + \frac{\int e^{-6e-6fx}(c+dx)^3\,dx}{8a^3} + \frac{3\int e^{-4e-4fx}(c+dx)^3\,dx}{8a^3} + \frac{3\int e^{-2e-2fx}(c+}{8a^3}$$

$$= -\frac{e^{-6e-6fx}(c+dx)^3}{48a^3f} - \frac{3e^{-4e-4fx}(c+dx)^3}{32a^3f} - \frac{3e^{-2e-2fx}(c+dx)^3}{16a^3f} + \frac{(c+dx)^4}{32a^3d} + \frac{d\int}{}$$

$$= -\frac{de^{-6e-6fx}(c+dx)^2}{96a^3f^2} - \frac{9de^{-4e-4fx}(c+dx)^2}{128a^3f^2} - \frac{9de^{-2e-2fx}(c+dx)^2}{32a^3f^2} - \frac{e^{-6e-6fx}(c+}{48a^3f}$$

$$= -\frac{d^2e^{-6e-6fx}(c+dx)}{288a^3f^3} - \frac{9d^2e^{-4e-4fx}(c+dx)}{256a^3f^3} - \frac{9d^2e^{-2e-2fx}(c+dx)}{32a^3f^3} - \frac{de^{-6e-6fx}(c}{96a^3f}$$

$$= -\frac{d^3e^{-6e-6fx}}{1728a^3f^4} - \frac{9d^3e^{-4e-4fx}}{1024a^3f^4} - \frac{9d^3e^{-2e-2fx}}{64a^3f^4} - \frac{d^2e^{-6e-6fx}(c+dx)}{288a^3f^3} - \frac{9d^2e^{-4e-4fx}(c-}{256a^3f^3}$$

Mathematica [A] time = 2.85, size = 615, normalized size = 1.83

$$\frac{\operatorname{sech}^3(e+fx)\left(3456c^3f^4x\sinh(3(e+fx)) - 2592c^3f^3\sinh(e+fx) + 576c^3f^3\sinh(3(e+fx)) + 5184c^2df^4x^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Tanh[e + f*x])^3,x]
```

```
[Out] (Sech[e + f*x]^3*(-243*(32*c^3*f^3 + 8*c^2*d*f^2*(5 + 12*f*x) + 4*c*d^2*f*(
9 + 20*f*x + 24*f^2*x^2) + d^3*(17 + 36*f*x + 40*f^2*x^2 + 32*f^3*x^3))*Cos
h[e + f*x] + 16*(36*c^3*f^3*(-1 + 6*f*x) + 18*c^2*d*f^2*(-1 - 6*f*x + 18*f^
2*x^2) + 6*c*d^2*f*(-1 - 6*f*x - 18*f^2*x^2 + 36*f^3*x^3) + d^3*(-1 - 6*f*x
 - 18*f^2*x^2 - 36*f^3*x^3 + 54*f^4*x^4))*Cosh[3*(e + f*x)] - 3645*d^3*Sinh
[e + f*x] - 6804*c*d^2*f*Sinh[e + f*x] - 5832*c^2*d*f^2*Sinh[e + f*x] - 259
2*c^3*f^3*Sinh[e + f*x] - 6804*d^3*f*x*Sinh[e + f*x] - 11664*c*d^2*f^2*x*Si
nh[e + f*x] - 7776*c^2*d*f^3*x*Sinh[e + f*x] - 5832*d^3*f^2*x^2*Sinh[e + f*
x] - 7776*c*d^2*f^3*x^2*Sinh[e + f*x] - 2592*d^3*f^3*x^3*Sinh[e + f*x] + 16
*d^3*Sinh[3*(e + f*x)] + 96*c*d^2*f*Sinh[3*(e + f*x)] + 288*c^2*d*f^2*Sinh[
3*(e + f*x)] + 576*c^3*f^3*Sinh[3*(e + f*x)] + 96*d^3*f*x*Sinh[3*(e + f*x)]
 + 576*c*d^2*f^2*x*Sinh[3*(e + f*x)] + 1728*c^2*d*f^3*x*Sinh[3*(e + f*x)] +
 3456*c^3*f^4*x*Sinh[3*(e + f*x)] + 288*d^3*f^2*x^2*Sinh[3*(e + f*x)] + 172
8*c*d^2*f^3*x^2*Sinh[3*(e + f*x)] + 5184*c^2*d*f^4*x^2*Sinh[3*(e + f*x)] +
576*d^3*f^3*x^3*Sinh[3*(e + f*x)] + 3456*c*d^2*f^4*x^3*Sinh[3*(e + f*x)] +
864*d^3*f^4*x^4*Sinh[3*(e + f*x)]))/(27648*a^3*f^4*(1 + Tanh[e + f*x])^3)
```

fricas [B] time = 0.53, size = 844, normalized size = 2.51

$$\frac{16\left(54\,d^3f^4x^4 - 36\,c^3f^3 - 18\,c^2df^2 - 6\,cd^2f + 36\left(6\,cd^2f^4 - d^3f^3\right)x^3 - d^3 + 18\left(18\,c^2df^4 - 6\,cd^2f^3 - d^3f^2\right)x^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/27648*(16*(54*d^3*f^4*x^4 - 36*c^3*f^3 - 18*c^2*d*f^2 - 6*c*d^2*f + 36*(6
*c*d^2*f^4 - d^3*f^3)*x^3 - d^3 + 18*(18*c^2*d*f^4 - 6*c*d^2*f^3 - d^3*f^2)
*x^2 + 6*(36*c^3*f^4 - 18*c^2*d*f^3 - 6*c*d^2*f^2 - d^3*f)*x)*cosh(f*x + e)
^3 + 48*(54*d^3*f^4*x^4 - 36*c^3*f^3 - 18*c^2*d*f^2 - 6*c*d^2*f + 36*(6*c*d
^2*f^4 - d^3*f^3)*x^3 - d^3 + 18*(18*c^2*d*f^4 - 6*c*d^2*f^3 - d^3*f^2)*x^2
```

```
 + 6*(36*c^3*f^4 - 18*c^2*d*f^3 - 6*c*d^2*f^2 - d^3*f)*x)*cosh(f*x + e)*sin
h(f*x + e)^2 + 16*(54*d^3*f^4*x^4 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*d^2*f +
 36*(6*c*d^2*f^4 + d^3*f^3)*x^3 + d^3 + 18*(18*c^2*d*f^4 + 6*c*d^2*f^3 + d^
3*f^2)*x^2 + 6*(36*c^3*f^4 + 18*c^2*d*f^3 + 6*c*d^2*f^2 + d^3*f)*x)*sinh(f*
x + e)^3 - 243*(32*d^3*f^3*x^3 + 32*c^3*f^3 + 40*c^2*d*f^2 + 36*c*d^2*f + 1
7*d^3 + 8*(12*c*d^2*f^3 + 5*d^3*f^2)*x^2 + 4*(24*c^2*d*f^3 + 20*c*d^2*f^2 +
 9*d^3*f)*x)*cosh(f*x + e) - 3*(864*d^3*f^3*x^3 + 864*c^3*f^3 + 1944*c^2*d*
f^2 + 2268*c*d^2*f + 1215*d^3 + 648*(4*c*d^2*f^3 + 3*d^3*f^2)*x^2 - 16*(54*
d^3*f^4*x^4 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*d^2*f + 36*(6*c*d^2*f^4 + d^3
*f^3)*x^3 + d^3 + 18*(18*c^2*d*f^4 + 6*c*d^2*f^3 + d^3*f^2)*x^2 + 6*(36*c^3
*f^4 + 18*c^2*d*f^3 + 6*c*d^2*f^2 + d^3*f)*x)*cosh(f*x + e)^2 + 324*(8*c^2*
d*f^3 + 12*c*d^2*f^2 + 7*d^3*f)*x)*sinh(f*x + e))/(a^3*f^4*cosh(f*x + e)^3
+ 3*a^3*f^4*cosh(f*x + e)^2*sinh(f*x + e) + 3*a^3*f^4*cosh(f*x + e)*sinh(f*
x + e)^2 + a^3*f^4*sinh(f*x + e)^3)
```

giac [A] time = 0.16, size = 573, normalized size = 1.71

$$\left(864\,d^3f^4x^4e^{(6fx+6e)} + 3456\,cd^2f^4x^3e^{(6fx+6e)} + 5184\,c^2df^4x^2e^{(6fx+6e)} - 5184\,d^3f^3x^3e^{(4fx+4e)} - 2592\,d^3f^3x^3e^{(2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/27648*(864*d^3*f^4*x^4*e^(6*f*x + 6*e) + 3456*c*d^2*f^4*x^3*e^(6*f*x + 6*
e) + 5184*c^2*d*f^4*x^2*e^(6*f*x + 6*e) - 5184*d^3*f^3*x^3*e^(4*f*x + 4*e)
- 2592*d^3*f^3*x^3*e^(2*f*x + 2*e) - 576*d^3*f^3*x^3 + 3456*c^3*f^4*x*e^(6*
f*x + 6*e) - 15552*c*d^2*f^3*x^2*e^(4*f*x + 4*e) - 7776*c*d^2*f^3*x^2*e^(2*
f*x + 2*e) - 1728*c*d^2*f^3*x^2 - 15552*c^2*d*f^3*x*e^(4*f*x + 4*e) - 7776*
d^3*f^2*x^2*e^(4*f*x + 4*e) - 7776*c^2*d*f^3*x*e^(2*f*x + 2*e) - 1944*d^3*f
^2*x^2*e^(2*f*x + 2*e) - 1728*c^2*d*f^3*x - 288*d^3*f^2*x^2 - 5184*c^3*f^3*
e^(4*f*x + 4*e) - 15552*c*d^2*f^2*x*e^(4*f*x + 4*e) - 2592*c^3*f^3*e^(2*f*x
 + 2*e) - 3888*c*d^2*f^2*x*e^(2*f*x + 2*e) - 576*c^3*f^3 - 576*c*d^2*f^2*x
- 7776*c^2*d*f^2*e^(4*f*x + 4*e) - 7776*d^3*f*x*e^(4*f*x + 4*e) - 1944*c^2*
d*f^2*e^(2*f*x + 2*e) - 972*d^3*f*x*e^(2*f*x + 2*e) - 288*c^2*d*f^2 - 96*d^
3*f*x - 7776*c*d^2*f*e^(4*f*x + 4*e) - 972*c*d^2*f*e^(2*f*x + 2*e) - 96*c*d
^2*f - 3888*d^3*e^(4*f*x + 4*e) - 243*d^3*e^(2*f*x + 2*e) - 16*d^3)*e^(-6*f
*x - 6*e)/(a^3*f^4)
```

maple [B] time = 0.34, size = 3296, normalized size = 9.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*tanh(f*x+e))^3,x)
```

```
[Out] 1/f^4/a^3*(d^3*(1/4*(f*x+e)^3*cosh(f*x+e)^4-3/16*(f*x+e)^2*sinh(f*x+e)*cosh
(f*x+e)^3-9/32*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-3/32*(f*x+e)^3+3/32*(f*x+e
)*cosh(f*x+e)^4-3/128*cosh(f*x+e)^3*sinh(f*x+e)-45/256*cosh(f*x+e)*sinh(f*x
+e)-45/256*f*x-45/256*e+9/32*(f*x+e)*cosh(f*x+e)^2)-3*d^3*(1/4*(f*x+e)^3*si
nh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)+3/32*(f*x+e)^
4-3/16*(f*x+e)^2*cosh(f*x+e)^4+3/32*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+45/64
*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+45/128*(f*x+e)^2-3/128*cosh(f*x+e)^4-45/12
8*cosh(f*x+e)^2-9/16*(f*x+e)^2*cosh(f*x+e)^2)+12*c*d^2*f*(1/6*(f*x+e)^2*sin
h(f*x+e)*cosh(f*x+e)^5+5/24*(f*x+e)^2*sinh(f*x+e)*cosh(f*x+e)^3+5/16*(f*x+e
)^2*cosh(f*x+e)*sinh(f*x+e)+5/48*(f*x+e)^3-1/18*(f*x+e)*cosh(f*x+e)^6+1/108
*cosh(f*x+e)^5*sinh(f*x+e)+65/1728*cosh(f*x+e)^3*sinh(f*x+e)+245/1152*cosh(
f*x+e)*sinh(f*x+e)+245/1152*f*x+245/1152*e-5/48*(f*x+e)*cosh(f*x+e)^4-5/16*
(f*x+e)*cosh(f*x+e)^2)+12*c^2*d*f^2*(1/6*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^5+
```

```
5/24*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+5/16*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)
+5/32*(f*x+e)^2-1/36*cosh(f*x+e)^6-5/96*cosh(f*x+e)^4-5/32*cosh(f*x+e)^2)-1
2*c*d^2*f*(1/6*(f*x+e)^2*cosh(f*x+e)^6-1/18*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)
^5-5/72*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3-5/48*(f*x+e)*cosh(f*x+e)*sinh(f*x
+e)-5/96*(f*x+e)^2+1/108*cosh(f*x+e)^6+5/288*cosh(f*x+e)^4+5/96*cosh(f*x+e)
^2)-12*c^2*d*f^2*(1/6*(f*x+e)*cosh(f*x+e)^6-1/36*cosh(f*x+e)^5*sinh(f*x+e)-
5/144*cosh(f*x+e)^3*sinh(f*x+e)-5/96*cosh(f*x+e)*sinh(f*x+e)-5/96*f*x-5/96*
e)-12*d^3*e^2*(1/6*(f*x+e)*cosh(f*x+e)^6-1/36*cosh(f*x+e)^5*sinh(f*x+e)-5/1
44*cosh(f*x+e)^3*sinh(f*x+e)-5/96*cosh(f*x+e)*sinh(f*x+e)-5/96*f*x-5/96*e)-
2/3*cosh(f*x+e)^6*c^3*f^3+2/3*cosh(f*x+e)^6*d^3*e^3+4*c^3*f^3*((1/6*cosh(f*
x+e)^5+5/24*cosh(f*x+e)^3+5/16*cosh(f*x+e))*sinh(f*x+e)+5/16*f*x+5/16*e)-4*
d^3*e^3*((1/6*cosh(f*x+e)^5+5/24*cosh(f*x+e)^3+5/16*cosh(f*x+e))*sinh(f*x+e
)+5/16*f*x+5/16*e)-12*d^3*e*(1/6*(f*x+e)^2*sinh(f*x+e)*cosh(f*x+e)^5+5/24*(
f*x+e)^2*sinh(f*x+e)*cosh(f*x+e)^3+5/16*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+5
/48*(f*x+e)^3-1/18*(f*x+e)*cosh(f*x+e)^6+1/108*cosh(f*x+e)^5*sinh(f*x+e)+65
/1728*cosh(f*x+e)^3*sinh(f*x+e)+245/1152*cosh(f*x+e)*sinh(f*x+e)+245/1152*f
*x+245/1152*e-5/48*(f*x+e)*cosh(f*x+e)^4-5/16*(f*x+e)*cosh(f*x+e)^2)+12*d^3
*e^2*(1/6*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^5+5/24*(f*x+e)*sinh(f*x+e)*cosh(f
*x+e)^3+5/16*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+5/32*(f*x+e)^2-1/36*cosh(f*x+e
)^6-5/96*cosh(f*x+e)^4-5/32*cosh(f*x+e)^2)+12*d^3*e*(1/6*(f*x+e)^2*cosh(f*x
+e)^6-1/18*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^5-5/72*(f*x+e)*sinh(f*x+e)*cosh(
f*x+e)^3-5/48*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-5/96*(f*x+e)^2+1/108*cosh(f*x
+e)^6+5/288*cosh(f*x+e)^4+5/96*cosh(f*x+e)^2)-1/4*cosh(f*x+e)^4*d^3*e^3+1/4
*cosh(f*x+e)^4*c^3*f^3-9*d^3*e^2*(1/4*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+3/8
*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/16*(f*x+e)^2-1/16*cosh(f*x+e)^4-3/16*cos
h(f*x+e)^2)-3*c^3*f^3*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x+e)+3/8*
f*x+3/8*e)+9*d^3*e*(1/4*(f*x+e)^2*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)^2*c
osh(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)^3-1/8*(f*x+e)*cosh(f*x+e)^4+1/32*cosh(f*
x+e)^3*sinh(f*x+e)+15/64*cosh(f*x+e)*sinh(f*x+e)+15/64*f*x+15/64*e-3/8*(f*x
+e)*cosh(f*x+e)^2)-3*d^3*e*(1/4*(f*x+e)^2*cosh(f*x+e)^4-1/8*(f*x+e)*sinh(f*
x+e)*cosh(f*x+e)^3-3/16*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-3/32*(f*x+e)^2+1/32
*cosh(f*x+e)^4+3/32*cosh(f*x+e)^2)+3*d^3*e^2*(1/4*(f*x+e)*cosh(f*x+e)^4-1/1
6*cosh(f*x+e)^3*sinh(f*x+e)-3/32*cosh(f*x+e)*sinh(f*x+e)-3/32*f*x-3/32*e)+3
*d^3*e^3*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)+3*
c^2*d*f^2*(1/4*(f*x+e)*cosh(f*x+e)^4-1/16*cosh(f*x+e)^3*sinh(f*x+e)-3/32*co
sh(f*x+e)*sinh(f*x+e)-3/32*f*x-3/32*e)-9*c*d^2*f*(1/4*(f*x+e)^2*sinh(f*x+e)
*cosh(f*x+e)^3+3/8*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)^3-1/8*(f*x
+e)*cosh(f*x+e)^4+1/32*cosh(f*x+e)^3*sinh(f*x+e)+15/64*cosh(f*x+e)*sinh(f*x
+e)+15/64*f*x+15/64*e-3/8*(f*x+e)*cosh(f*x+e)^2)-9*c^2*d*f^2*(1/4*(f*x+e)*s
inh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/16*(f*x+e)^2
-1/16*cosh(f*x+e)^4-3/16*cosh(f*x+e)^2)+3*c*d^2*f*(1/4*(f*x+e)^2*cosh(f*x+e
)^4-1/8*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3-3/16*(f*x+e)*cosh(f*x+e)*sinh(f*x
+e)-3/32*(f*x+e)^2+1/32*cosh(f*x+e)^4+3/32*cosh(f*x+e)^2)+4*d^3*(1/6*(f*x+e
)^3*sinh(f*x+e)*cosh(f*x+e)^5+5/24*(f*x+e)^3*sinh(f*x+e)*cosh(f*x+e)^3+5/16
*(f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)+5/64*(f*x+e)^4-1/12*(f*x+e)^2*cosh(f*x+e
)^6+1/36*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^5+65/576*(f*x+e)*sinh(f*x+e)*cosh(
f*x+e)^3+245/384*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+245/768*(f*x+e)^2-1/216*co
sh(f*x+e)^6-65/2304*cosh(f*x+e)^4-245/768*cosh(f*x+e)^2-5/32*(f*x+e)^2*cosh
(f*x+e)^4-15/32*(f*x+e)^2*cosh(f*x+e)^2)-4*d^3*(1/6*(f*x+e)^3*cosh(f*x+e)^6
-1/12*(f*x+e)^2*sinh(f*x+e)*cosh(f*x+e)^5-5/48*(f*x+e)^2*sinh(f*x+e)*cosh(f
*x+e)^3-5/32*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-5/96*(f*x+e)^3+1/36*(f*x+e)*
cosh(f*x+e)^6-1/216*cosh(f*x+e)^5*sinh(f*x+e)-65/3456*cosh(f*x+e)^3*sinh(f*
x+e)-245/2304*cosh(f*x+e)*sinh(f*x+e)-245/2304*f*x-245/2304*e+5/96*(f*x+e)*
cosh(f*x+e)^4+5/32*(f*x+e)*cosh(f*x+e)^2)-24*c*d^2*e*f*(1/6*(f*x+e)*sinh(f*
x+e)*cosh(f*x+e)^5+5/24*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+5/16*(f*x+e)*cosh
(f*x+e)*sinh(f*x+e)+5/32*(f*x+e)^2-1/36*cosh(f*x+e)^6-5/96*cosh(f*x+e)^4-5/
32*cosh(f*x+e)^2)-12*c^2*d*e*f^2*((1/6*cosh(f*x+e)^5+5/24*cosh(f*x+e)^3+5/1
6*cosh(f*x+e))*sinh(f*x+e)+5/16*f*x+5/16*e)+12*c*d^2*e^2*f*((1/6*cosh(f*x+e
)^5+5/24*cosh(f*x+e)^3+5/16*cosh(f*x+e))*sinh(f*x+e)+5/16*f*x+5/16*e)+2*cos
```

```
h(f*x+e)^6*c^2*d*e*f^2-2*cosh(f*x+e)^6*c*d^2*e^2*f+24*c*d^2*e*f*(1/6*(f*x+e
)*cosh(f*x+e)^6-1/36*cosh(f*x+e)^5*sinh(f*x+e)-5/144*cosh(f*x+e)^3*sinh(f*x
+e)-5/96*cosh(f*x+e)*sinh(f*x+e)-5/96*f*x-5/96*e)-6*c*d^2*e*f*(1/4*(f*x+e)*
cosh(f*x+e)^4-1/16*cosh(f*x+e)^3*sinh(f*x+e)-3/32*cosh(f*x+e)*sinh(f*x+e)-3
/32*f*x-3/32*e)+9*c^2*d*e*f^2*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x
+e)+3/8*f*x+3/8*e)-9*c*d^2*e^2*f*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(
f*x+e)+3/8*f*x+3/8*e)-3/4*cosh(f*x+e)^4*c^2*d*e*f^2+3/4*cosh(f*x+e)^4*c*d^2
*e^2*f+18*c*d^2*e*f*(1/4*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)*cosh
(f*x+e)*sinh(f*x+e)+3/16*(f*x+e)^2-1/16*cosh(f*x+e)^4-3/16*cosh(f*x+e)^2))
```

maxima [A] time = 2.50, size = 407, normalized size = 1.21

$$\frac{1}{96}c^3\left(\frac{12\left(fx+e\right)}{a^3f}-\frac{18\,e^{\left(-2fx-2e\right)}+9\,e^{\left(-4fx-4e\right)}+2\,e^{\left(-6fx-6e\right)}}{a^3f}\right)+\frac{\left(72\,f^2x^2e^{(6e)}-108\left(2\,fxe^{(4e)}+e^{(4e)}\right)e^{\left(-2fx\right)}-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*c^3*(12*(f*x + e)/(a^3*f) - (18*e^(-2*f*x - 2*e) + 9*e^(-4*f*x - 4*e)
+ 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/384*(72*f^2*x^2*e^(6*e) - 108*(2*f*x*e^(
4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 4*(6
*f*x + 1)*e^(-6*f*x))*c^2*d*e^(-6*e)/(a^3*f^2) + 1/2304*(288*f^3*x^3*e^(6*e
) - 648*(2*f^2*x^2*e^(4*e) + 2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 81*(8*f^
2*x^2*e^(2*e) + 4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 8*(18*f^2*x^2 + 6*f*x
 + 1)*e^(-6*f*x))*c*d^2*e^(-6*e)/(a^3*f^3) + 1/27648*(864*f^4*x^4*e^(6*e) -
 1296*(4*f^3*x^3*e^(4*e) + 6*f^2*x^2*e^(4*e) + 6*f*x*e^(4*e) + 3*e^(4*e))*e
^(-2*f*x) - 81*(32*f^3*x^3*e^(2*e) + 24*f^2*x^2*e^(2*e) + 12*f*x*e^(2*e) +
3*e^(2*e))*e^(-4*f*x) - 16*(36*f^3*x^3 + 18*f^2*x^2 + 6*f*x + 1)*e^(-6*f*x)
)*d^3*e^(-6*e)/(a^3*f^4)
```

mupad [B] time = 1.35, size = 376, normalized size = 1.12

$$\frac{c^3\,x}{8\,a^3}-e^{-4e-4fx}\left(\frac{96\,c^3\,f^3+72\,c^2\,d\,f^2+36\,c\,d^2\,f+9\,d^3}{1024\,a^3\,f^4}+\frac{3\,d^3\,x^3}{32\,a^3\,f}+\frac{9\,d\,x\left(8\,c^2\,f^2+4\,c\,d\,f+d^2\right)}{256\,a^3\,f^3}+\frac{9\,d^2\,x^2\left(d+4\right.}{128\,a^3\,f^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*tanh(e + f*x))^3,x)
```

```
[Out] (c^3*x)/(8*a^3) - exp(- 4*e - 4*f*x)*((9*d^3 + 96*c^3*f^3 + 72*c^2*d*f^2 +
36*c*d^2*f)/(1024*a^3*f^4) + (3*d^3*x^3)/(32*a^3*f) + (9*d*x*(d^2 + 8*c^2*f
^2 + 4*c*d*f))/(256*a^3*f^3) + (9*d^2*x^2*(d + 4*c*f))/(128*a^3*f^2)) - exp
(- 6*e - 6*f*x)*((d^3 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*d^2*f)/(1728*a^3*f^
4) + (d^3*x^3)/(48*a^3*f) + (d*x*(d^2 + 18*c^2*f^2 + 6*c*d*f))/(288*a^3*f^3
) + (d^2*x^2*(d + 6*c*f))/(96*a^3*f^2)) - exp(- 2*e - 2*f*x)*((9*d^3 + 12*c
^3*f^3 + 18*c^2*d*f^2 + 18*c*d^2*f)/(64*a^3*f^4) + (3*d^3*x^3)/(16*a^3*f) +
 (9*d*x*(d^2 + 2*c^2*f^2 + 2*c*d*f))/(32*a^3*f^3) + (9*d^2*x^2*(d + 2*c*f))
/(32*a^3*f^2)) + (d^3*x^4)/(32*a^3) + (3*c^2*d*x^2)/(16*a^3) + (c*d^2*x^3)/
(8*a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c^3}{\tanh^3\left(e+fx\right)+3\tanh^2\left(e+fx\right)+3\tanh\left(e+fx\right)+1}\,dx+\int\frac{d^3x^3}{\tanh^3\left(e+fx\right)+3\tanh^2\left(e+fx\right)+3\tanh\left(e+fx\right)+1}\,dx+\int\frac{3cd^2x^2}{\tanh^3\left(e+fx\right)+3\tanh^2\left(e+f\right.}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*tanh(f*x+e))**3,x)
```

```
[Out] (Integral(c**3/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) + 1
), x) + Integral(d**3*x**3/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(
e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(tanh(e + f*x)**3 + 3*tanh(e + f
*x)**2 + 3*tanh(e + f*x) + 1), x) + Integral(3*c**2*d*x/(tanh(e + f*x)**3 +
 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) + 1), x))/a**3
```

3.44 $\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$

Optimal. Leaf size=246

$$-\frac{d(c+dx)e^{-6e-6fx}}{144a^3f^2}-\frac{3d(c+dx)e^{-4e-4fx}}{64a^3f^2}-\frac{3d(c+dx)e^{-2e-2fx}}{16a^3f^2}-\frac{(c+dx)^2e^{-6e-6fx}}{48a^3f}-\frac{3(c+dx)^2e^{-4e-4fx}}{32a^3f}-\frac{3(c+dx)^2e^{-2}}{16a^3f}$$

```
[Out] -1/864*d^2*exp(-6*f*x-6*e)/a^3/f^3-3/256*d^2*exp(-4*f*x-4*e)/a^3/f^3-3/32*d
^2*exp(-2*f*x-2*e)/a^3/f^3-1/144*d*exp(-6*f*x-6*e)*(d*x+c)/a^3/f^2-3/64*d*e
xp(-4*f*x-4*e)*(d*x+c)/a^3/f^2-3/16*d*exp(-2*f*x-2*e)*(d*x+c)/a^3/f^2-1/48*
exp(-6*f*x-6*e)*(d*x+c)^2/a^3/f-3/32*exp(-4*f*x-4*e)*(d*x+c)^2/a^3/f-3/16*e
xp(-2*f*x-2*e)*(d*x+c)^2/a^3/f+1/24*(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.27, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3729, 2176, 2194}

$$-\frac{d(c+dx)e^{-6e-6fx}}{144a^3f^2}-\frac{3d(c+dx)e^{-4e-4fx}}{64a^3f^2}-\frac{3d(c+dx)e^{-2e-2fx}}{16a^3f^2}-\frac{(c+dx)^2e^{-6e-6fx}}{48a^3f}-\frac{3(c+dx)^2e^{-4e-4fx}}{32a^3f}-\frac{3(c+dx)^2e^{-2}}{16a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Tanh[e + f*x])^3,x]
```

```
[Out] -(d^2*E^(-6*e - 6*f*x))/(864*a^3*f^3) - (3*d^2*E^(-4*e - 4*f*x))/(256*a^3*f
^3) - (3*d^2*E^(-2*e - 2*f*x))/(32*a^3*f^3) - (d*E^(-6*e - 6*f*x)*(c + d*x)
)/(144*a^3*f^2) - (3*d*E^(-4*e - 4*f*x)*(c + d*x))/(64*a^3*f^2) - (3*d*E^(-
2*e - 2*f*x)*(c + d*x))/(16*a^3*f^2) - (E^(-6*e - 6*f*x)*(c + d*x)^2)/(48*a
^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^2)/(32*a^3*f) - (3*E^(-2*e - 2*f*x)*(
c + d*x)^2)/(16*a^3*f) + (c + d*x)^3/(24*a^3*d)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx = \int\left(\frac{(c+dx)^2}{8a^3}+\frac{e^{-6e-6fx}(c+dx)^2}{8a^3}+\frac{3e^{-4e-4fx}(c+dx)^2}{8a^3}+\frac{3e^{-2e-2fx}(c+dx)^2}{8a^3}\right)d$$
$$= \frac{(c+dx)^3}{24a^3d}+\frac{\int e^{-6e-6fx}(c+dx)^2\,dx}{8a^3}+\frac{3\int e^{-4e-4fx}(c+dx)^2\,dx}{8a^3}+\frac{3\int e^{-2e-2fx}(c+}{8a^3}$$
$$= -\frac{e^{-6e-6fx}(c+dx)^2}{48a^3f}-\frac{3e^{-4e-4fx}(c+dx)^2}{32a^3f}-\frac{3e^{-2e-2fx}(c+dx)^2}{16a^3f}+\frac{(c+dx)^3}{24a^3d}+\frac{d\int}{}$$
$$= -\frac{de^{-6e-6fx}(c+dx)}{144a^3f^2}-\frac{3de^{-4e-4fx}(c+dx)}{64a^3f^2}-\frac{3de^{-2e-2fx}(c+dx)}{16a^3f^2}-\frac{e^{-6e-6fx}(c+dx}{48a^3f}$$
$$= -\frac{d^2e^{-6e-6fx}}{864a^3f^3}-\frac{3d^2e^{-4e-4fx}}{256a^3f^3}-\frac{3d^2e^{-2e-2fx}}{32a^3f^3}-\frac{de^{-6e-6fx}(c+dx)}{144a^3f^2}-\frac{3de^{-4e-4fx}(c+d}{64a^3f^2}$$

Mathematica [A] time = 1.55, size = 371, normalized size = 1.51

$$\frac{\operatorname{sech}^3(e+fx)\left(-81\left(24c^2f^2+4cdf(12fx+5)+d^2\left(24f^2x^2+20fx+9\right)\right)\cosh(e+fx)+8\left(18c^2f^2(6fx-1)+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + a*Tanh[e + f*x])^3,x]
```

```
[Out] (Sech[e + f*x]^3*(-81*(24*c^2*f^2 + 4*c*d*f*(5 + 12*f*x) + d^2*(9 + 20*f*x
+ 24*f^2*x^2))*Cosh[e + f*x] + 8*(18*c^2*f^2*(-1 + 6*f*x) + 6*c*d*f*(-1 - 6
*f*x + 18*f^2*x^2) + d^2*(-1 - 6*f*x - 18*f^2*x^2 + 36*f^3*x^3))*Cosh[3*(e
+ f*x)] - 567*d^2*Sinh[e + f*x] - 972*c*d*f*Sinh[e + f*x] - 648*c^2*f^2*Sin
h[e + f*x] - 972*d^2*f*x*Sinh[e + f*x] - 1296*c*d*f^2*x*Sinh[e + f*x] - 648
*d^2*f^2*x^2*Sinh[e + f*x] + 8*d^2*Sinh[3*(e + f*x)] + 48*c*d*f*Sinh[3*(e +
 f*x)] + 144*c^2*f^2*Sinh[3*(e + f*x)] + 48*d^2*f*x*Sinh[3*(e + f*x)] + 288
*c*d*f^2*x*Sinh[3*(e + f*x)] + 864*c^2*f^3*x*Sinh[3*(e + f*x)] + 144*d^2*f^
2*x^2*Sinh[3*(e + f*x)] + 864*c*d*f^3*x^2*Sinh[3*(e + f*x)] + 288*d^2*f^3*x
^3*Sinh[3*(e + f*x)]))/(6912*a^3*f^3*(1 + Tanh[e + f*x])^3)
```

fricas [B] time = 0.45, size = 532, normalized size = 2.16

$$\frac{8\left(36\,d^2f^3x^3-18\,c^2f^2-6\,cdf+18\left(6\,cdf^3-d^2f^2\right)x^2-d^2+6\left(18\,c^2f^3-6\,cdf^2-d^2f\right)x\right)\cosh\left(fx+e\right)^3+24}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/6912*(8*(36*d^2*f^3*x^3 - 18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2)
*x^2 - d^2 + 6*(18*c^2*f^3 - 6*c*d*f^2 - d^2*f)*x)*cosh(f*x + e)^3 + 24*(36
*d^2*f^3*x^3 - 18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2)*x^2 - d^2 +
6*(18*c^2*f^3 - 6*c*d*f^2 - d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e)^2 + 8*(36
*d^2*f^3*x^3 + 18*c^2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2)*x^2 + d^2 +
6*(18*c^2*f^3 + 6*c*d*f^2 + d^2*f)*x)*sinh(f*x + e)^3 - 81*(24*d^2*f^2*x^2
+ 24*c^2*f^2 + 20*c*d*f + 9*d^2 + 4*(12*c*d*f^2 + 5*d^2*f)*x)*cosh(f*x + e)
 - 3*(216*d^2*f^2*x^2 + 216*c^2*f^2 + 324*c*d*f - 8*(36*d^2*f^3*x^3 + 18*c^
2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2)*x^2 + d^2 + 6*(18*c^2*f^3 + 6*c*
d*f^2 + d^2*f)*x)*cosh(f*x + e)^2 + 189*d^2 + 108*(4*c*d*f^2 + 3*d^2*f)*x)*
sinh(f*x + e))/(a^3*f^3*cosh(f*x + e)^3 + 3*a^3*f^3*cosh(f*x + e)^2*sinh(f*
x + e) + 3*a^3*f^3*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^3*sinh(f*x + e)^3)
```

giac [A] time = 0.15, size = 331, normalized size = 1.35

$$\left(288\,d^2f^3x^3e^{(6fx+6e)} + 864\,cdf^3x^2e^{(6fx+6e)} + 864\,c^2f^3xe^{(6fx+6e)} - 1296\,d^2f^2x^2e^{(4fx+4e)} - 648\,d^2f^2x^2e^{(2fx+2e)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/6912*(288*d^2*f^3*x^3*e^(6*f*x + 6*e) + 864*c*d*f^3*x^2*e^(6*f*x + 6*e) +
 864*c^2*f^3*x*e^(6*f*x + 6*e) - 1296*d^2*f^2*x^2*e^(4*f*x + 4*e) - 648*d^2
*f^2*x^2*e^(2*f*x + 2*e) - 144*d^2*f^2*x^2 - 2592*c*d*f^2*x*e^(4*f*x + 4*e)
 - 1296*c*d*f^2*x*e^(2*f*x + 2*e) - 288*c*d*f^2*x - 1296*c^2*f^2*e^(4*f*x +
 4*e) - 1296*d^2*f*x*e^(4*f*x + 4*e) - 648*c^2*f^2*e^(2*f*x + 2*e) - 324*d^
2*f*x*e^(2*f*x + 2*e) - 144*c^2*f^2 - 48*d^2*f*x - 1296*c*d*f*e^(4*f*x + 4*
e) - 324*c*d*f*e^(2*f*x + 2*e) - 48*c*d*f - 648*d^2*e^(4*f*x + 4*e) - 81*d^
2*e^(2*f*x + 2*e) - 8*d^2)*e^(-6*f*x - 6*e)/(a^3*f^3)
```

maple [B] time = 0.30, size = 1529, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*tanh(f*x+e))^3,x)
```

```
[Out] 1/f^3/a^3*(4*d^2*(1/6*(f*x+e)^2*sinh(f*x+e)*cosh(f*x+e)^5+5/24*(f*x+e)^2*si
nh(f*x+e)*cosh(f*x+e)^3+5/16*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+5/48*(f*x+e)
^3-1/18*(f*x+e)*cosh(f*x+e)^6+1/108*cosh(f*x+e)^5*sinh(f*x+e)+65/1728*cosh(
f*x+e)^3*sinh(f*x+e)+245/1152*cosh(f*x+e)*sinh(f*x+e)+245/1152*f*x+245/1152
*e-5/48*(f*x+e)*cosh(f*x+e)^4-5/16*(f*x+e)*cosh(f*x+e)^2)+8*c*d*f*(1/6*(f*x
+e)*sinh(f*x+e)*cosh(f*x+e)^5+5/24*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+5/16*(
f*x+e)*cosh(f*x+e)*sinh(f*x+e)+5/32*(f*x+e)^2-1/36*cosh(f*x+e)^6-5/96*cosh(
f*x+e)^4-5/32*cosh(f*x+e)^2)-8*e*d^2*(1/6*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^5
+5/24*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+5/16*(f*x+e)*cosh(f*x+e)*sinh(f*x+e
)+5/32*(f*x+e)^2-1/36*cosh(f*x+e)^6-5/96*cosh(f*x+e)^4-5/32*cosh(f*x+e)^2)+
4*c^2*f^2*((1/6*cosh(f*x+e)^5+5/24*cosh(f*x+e)^3+5/16*cosh(f*x+e))*sinh(f*x
+e)+5/16*f*x+5/16*e)-8*c*d*f*e*((1/6*cosh(f*x+e)^5+5/24*cosh(f*x+e)^3+5/16*
cosh(f*x+e))*sinh(f*x+e)+5/16*f*x+5/16*e)+4*d^2*e^2*((1/6*cosh(f*x+e)^5+5/2
4*cosh(f*x+e)^3+5/16*cosh(f*x+e))*sinh(f*x+e)+5/16*f*x+5/16*e)-4*d^2*(1/6*(
f*x+e)^2*cosh(f*x+e)^6-1/18*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^5-5/72*(f*x+e)*
sinh(f*x+e)*cosh(f*x+e)^3-5/48*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-5/96*(f*x+e)
^2+1/108*cosh(f*x+e)^6+5/288*cosh(f*x+e)^4+5/96*cosh(f*x+e)^2)-8*c*d*f*(1/6
*(f*x+e)*cosh(f*x+e)^6-1/36*cosh(f*x+e)^5*sinh(f*x+e)-5/144*cosh(f*x+e)^3*s
inh(f*x+e)-5/96*cosh(f*x+e)*sinh(f*x+e)-5/96*f*x-5/96*e)+8*e*d^2*(1/6*(f*x+
e)*cosh(f*x+e)^6-1/36*cosh(f*x+e)^5*sinh(f*x+e)-5/144*cosh(f*x+e)^3*sinh(f*
x+e)-5/96*cosh(f*x+e)*sinh(f*x+e)-5/96*f*x-5/96*e)-2/3*cosh(f*x+e)^6*c^2*f^
2+4/3*cosh(f*x+e)^6*c*d*f*e-2/3*cosh(f*x+e)^6*d^2*e^2-3*d^2*(1/4*(f*x+e)^2*
sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)
^3-1/8*(f*x+e)*cosh(f*x+e)^4+1/32*cosh(f*x+e)^3*sinh(f*x+e)+15/64*cosh(f*x+
e)*sinh(f*x+e)+15/64*f*x+15/64*e-3/8*(f*x+e)*cosh(f*x+e)^2)-6*c*d*f*(1/4*(f
*x+e)*sinh(f*x+e)*cosh(f*x+e)^3+3/8*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/16*(f
*x+e)^2-1/16*cosh(f*x+e)^4-3/16*cosh(f*x+e)^2)+6*e*d^2*(1/4*(f*x+e)*sinh(f*
x+e)*cosh(f*x+e)^3+3/8*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/16*(f*x+e)^2-1/16*
cosh(f*x+e)^4-3/16*cosh(f*x+e)^2)-3*c^2*f^2*((1/4*cosh(f*x+e)^3+3/8*cosh(f*
x+e))*sinh(f*x+e)+3/8*f*x+3/8*e)+6*c*d*f*e*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x
+e))*sinh(f*x+e)+3/8*f*x+3/8*e)-3*d^2*e^2*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+
e))*sinh(f*x+e)+3/8*f*x+3/8*e)+d^2*(1/4*(f*x+e)^2*cosh(f*x+e)^4-1/8*(f*x+e)
*sinh(f*x+e)*cosh(f*x+e)^3-3/16*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-3/32*(f*x+e
)^2+1/32*cosh(f*x+e)^4+3/32*cosh(f*x+e)^2)+2*c*d*f*(1/4*(f*x+e)*cosh(f*x+e)
```

```
^4-1/16*cosh(f*x+e)^3*sinh(f*x+e)-3/32*cosh(f*x+e)*sinh(f*x+e)-3/32*f*x-3/3
2*e)-2*e*d^2*(1/4*(f*x+e)*cosh(f*x+e)^4-1/16*cosh(f*x+e)^3*sinh(f*x+e)-3/32
*cosh(f*x+e)*sinh(f*x+e)-3/32*f*x-3/32*e)+1/4*cosh(f*x+e)^4*c^2*f^2-1/2*cos
h(f*x+e)^4*c*d*f*e+1/4*cosh(f*x+e)^4*d^2*e^2)
```

maxima [A] time = 1.76, size = 255, normalized size = 1.04

$$\frac{1}{96}c^2\left(\frac{12\left(fx+e\right)}{a^3f}-\frac{18e^{\left(-2fx-2e\right)}+9e^{\left(-4fx-4e\right)}+2e^{\left(-6fx-6e\right)}}{a^3f}\right)+\frac{\left(72f^2x^2e^{(6e)}-108\left(2fxe^{(4e)}+e^{(4e)}\right)e^{\left(-2fx\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*c^2*(12*(f*x + e)/(a^3*f) - (18*e^(-2*f*x - 2*e) + 9*e^(-4*f*x - 4*e)
+ 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/576*(72*f^2*x^2*e^(6*e) - 108*(2*f*x*e^(
4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 4*(6
*f*x + 1)*e^(-6*f*x))*c*d*e^(-6*e)/(a^3*f^2) + 1/6912*(288*f^3*x^3*e^(6*e)
- 648*(2*f^2*x^2*e^(4*e) + 2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 81*(8*f^2*
x^2*e^(2*e) + 4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 8*(18*f^2*x^2 + 6*f*x +
 1)*e^(-6*f*x))*d^2*e^(-6*e)/(a^3*f^3)
```

mupad [B] time = 1.24, size = 236, normalized size = 0.96

$$\frac{c^2x}{8a^3}-e^{-2e-2fx}\left(\frac{6c^2f^2+6cdf+3d^2}{32a^3f^3}+\frac{3d^2x^2}{16a^3f}+\frac{3dx\left(d+2cf\right)}{16a^3f^2}\right)-e^{-4e-4fx}\left(\frac{24c^2f^2+12cdf+3d^2}{256a^3f^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*tanh(e + f*x))^3,x)
```

```
[Out] (c^2*x)/(8*a^3) - exp(- 2*e - 2*f*x)*((3*d^2 + 6*c^2*f^2 + 6*c*d*f)/(32*a^3
*f^3) + (3*d^2*x^2)/(16*a^3*f) + (3*d*x*(d + 2*c*f))/(16*a^3*f^2)) - exp(-
4*e - 4*f*x)*((3*d^2 + 24*c^2*f^2 + 12*c*d*f)/(256*a^3*f^3) + (3*d^2*x^2)/(
32*a^3*f) + (3*d*x*(d + 4*c*f))/(64*a^3*f^2)) - exp(- 6*e - 6*f*x)*((d^2 +
18*c^2*f^2 + 6*c*d*f)/(864*a^3*f^3) + (d^2*x^2)/(48*a^3*f) + (d*x*(d + 6*c*
f))/(144*a^3*f^2)) + (d^2*x^3)/(24*a^3) + (c*d*x^2)/(8*a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{c^2}{\tanh^3\left(e+fx\right)+3\tanh^2\left(e+fx\right)+3\tanh\left(e+fx\right)+1}dx+\int\frac{d^2x^2}{\tanh^3\left(e+fx\right)+3\tanh^2\left(e+fx\right)+3\tanh\left(e+fx\right)+1}dx+\int\frac{2}{\tanh^3\left(e+fx\right)+3\tanh^2}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*tanh(f*x+e))**3,x)
```

```
[Out] (Integral(c**2/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) + 1
), x) + Integral(d**2*x**2/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(
e + f*x) + 1), x) + Integral(2*c*d*x/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2
 + 3*tanh(e + f*x) + 1), x))/a**3
```

3.45 $\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$

Optimal. Leaf size=183

$$-\frac{c+dx}{8f\left(a^3\tanh(e+fx)+a^3\right)}+\frac{x(c+dx)}{8a^3}-\frac{11d}{96f^2\left(a^3\tanh(e+fx)+a^3\right)}+\frac{11dx}{96a^3f}-\frac{dx^2}{16a^3}-\frac{c+dx}{8af(a\tanh(e+fx)+a)^2}-$$

```
[Out] 11/96*d*x/a^3/f-1/16*d*x^2/a^3+1/8*x*(d*x+c)/a^3-1/36*d/f^2/(a+a*tanh(f*x+e
))^3+1/6*(-d*x-c)/f/(a+a*tanh(f*x+e))^3-5/96*d/a/f^2/(a+a*tanh(f*x+e))^2+1/
8*(-d*x-c)/a/f/(a+a*tanh(f*x+e))^2-11/96*d/f^2/(a^3+a^3*tanh(f*x+e))+1/8*(-
d*x-c)/f/(a^3+a^3*tanh(f*x+e))
```

Rubi [A] time = 0.21, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3479, 8, 3730}

$$-\frac{c+dx}{8f\left(a^3\tanh(e+fx)+a^3\right)}+\frac{x(c+dx)}{8a^3}-\frac{11d}{96f^2\left(a^3\tanh(e+fx)+a^3\right)}+\frac{11dx}{96a^3f}-\frac{dx^2}{16a^3}-\frac{c+dx}{8af(a\tanh(e+fx)+a)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Tanh[e + f*x])^3,x]
```

```
[Out] (11*d*x)/(96*a^3*f) - (d*x^2)/(16*a^3) + (x*(c + d*x))/(8*a^3) - d/(36*f^2*
(a + a*Tanh[e + f*x])^3) - (c + d*x)/(6*f*(a + a*Tanh[e + f*x])^3) - (5*d)/
(96*a*f^2*(a + a*Tanh[e + f*x])^2) - (c + d*x)/(8*a*f*(a + a*Tanh[e + f*x])
^2) - (11*d)/(96*f^2*(a^3 + a^3*Tanh[e + f*x])) - (c + d*x)/(8*f*(a^3 + a^3
*Tanh[e + f*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3479

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a +
b*Tan[c + d*x])^n)/(2*b*d*n), x] + Dist[1/(2*a), Int[(a + b*Tan[c + d*x])^(
n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 3730

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Dist[(c + d*x)
^m, u, x] - Dist[d*m, Int[Dist[(c + d*x)^(m - 1), u, x], x], x]] /; FreeQ[{
a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx = \frac{x(c+dx)}{8a^3} - \frac{c+dx}{6f(a+a\tanh(e+fx))^3} - \frac{c+dx}{8af(a+a\tanh(e+fx))^2} - \frac{c}{8f\left(a^3+a^3}$$

$$= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} - \frac{c+dx}{6f(a+a\tanh(e+fx))^3} - \frac{c+dx}{8af(a+a\tanh(e+fx))^2} - \frac{}{8f}$$

$$= -\frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} - \frac{d}{36f^2(a+a\tanh(e+fx))^3} - \frac{c+dx}{6f(a+a\tanh(e+fx))^3} - \frac{}{3}$$

$$= \frac{dx}{16a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} - \frac{d}{36f^2(a+a\tanh(e+fx))^3} - \frac{c+dx}{6f(a+a\tanh(e+f$$

$$= \frac{3dx}{32a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} - \frac{d}{36f^2(a+a\tanh(e+fx))^3} - \frac{c+dx}{6f(a+a\tanh(e+f$$

$$= \frac{11dx}{96a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} - \frac{d}{36f^2(a+a\tanh(e+fx))^3} - \frac{c+dx}{6f(a+a\tanh(e+f$$

Mathematica [A] time = 0.80, size = 185, normalized size = 1.01

$$\frac{\operatorname{sech}^3(e+fx)\left(4\left(6cf(6fx-1)+d\left(18f^2x^2-6fx-1\right)\right)\cosh(3(e+fx)) - 27(12cf+d(12fx+5))\cosh(e+f$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Tanh[e + f*x])^3,x]
```

```
[Out] (Sech[e + f*x]^3*(-27*(12*c*f + d*(5 + 12*f*x))*Cosh[e + f*x] + 4*(6*c*f*(-
1 + 6*f*x) + d*(-1 - 6*f*x + 18*f^2*x^2))*Cosh[3*(e + f*x)] - 81*d*Sinh[e +
 f*x] - 108*c*f*Sinh[e + f*x] - 108*d*f*x*Sinh[e + f*x] + 4*d*Sinh[3*(e + f
*x)] + 24*c*f*Sinh[3*(e + f*x)] + 24*d*f*x*Sinh[3*(e + f*x)] + 144*c*f^2*x*
Sinh[3*(e + f*x)] + 72*d*f^2*x^2*Sinh[3*(e + f*x)]))/(1152*a^3*f^2*(1 + Tan
h[e + f*x])^3)
```

fricas [A] time = 0.42, size = 286, normalized size = 1.56

$$\frac{4\left(18\,df^2x^2-6\,cf+6\left(6\,cf^2-df\right)x-d\right)\cosh\left(fx+e\right)^3 + 12\left(18\,df^2x^2-6\,cf+6\left(6\,cf^2-df\right)x-d\right)\cosh\left(f$$

$$1152\left($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/1152*(4*(18*d*f^2*x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*cosh(f*x + e)^3
+ 12*(18*d*f^2*x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*cosh(f*x + e)*sinh(f*
x + e)^2 + 4*(18*d*f^2*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*sinh(f*x + e)
^3 - 27*(12*d*f*x + 12*c*f + 5*d)*cosh(f*x + e) - 3*(36*d*f*x - 4*(18*d*f^2
*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*cosh(f*x + e)^2 + 36*c*f + 27*d)*si
nh(f*x + e))/(a^3*f^2*cosh(f*x + e)^3 + 3*a^3*f^2*cosh(f*x + e)^2*sinh(f*x
+ e) + 3*a^3*f^2*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^2*sinh(f*x + e)^3)
```

giac [A] time = 0.15, size = 151, normalized size = 0.83

$$\frac{\left(72\,df^2x^2e^{(6fx+6e)} + 144\,cf^2xe^{(6fx+6e)} - 216\,dfxe^{(4fx+4e)} - 108\,dfxe^{(2fx+2e)} - 24\,dfx - 216\,cfe^{(4fx+4e)} - 10\right.}{1152\,a^3f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/1152*(72*d*f^2*x^2*e^(6*f*x + 6*e) + 144*c*f^2*x*e^(6*f*x + 6*e) - 216*d*
f*x*e^(4*f*x + 4*e) - 108*d*f*x*e^(2*f*x + 2*e) - 24*d*f*x - 216*c*f*e^(4*f
*x + 4*e) - 108*c*f*e^(2*f*x + 2*e) - 24*c*f - 108*d*e^(4*f*x + 4*e) - 27*d
*e^(2*f*x + 2*e) - 4*d)*e^(-6*f*x - 6*e)/(a^3*f^2)
```

maple [B] time = 0.29, size = 533, normalized size = 2.91

$$4d\left(\frac{(fx+e)\sinh(fx+e)(\cosh^5(fx+e))}{6}+\frac{5(fx+e)\sinh(fx+e)(\cosh^3(fx+e))}{24}+\frac{5(fx+e)\cosh(fx+e)\sinh(fx+e)}{16}+\frac{5(fx+e)^2}{32}-\frac{(\cosh^6(fx+e))}{36}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*tanh(f*x+e))^3,x)
```

```
[Out] 1/f^2/a^3*(4*d*(1/6*(f*x+e)*sinh(f*x+e)*cosh(f*x+e)^5+5/24*(f*x+e)*sinh(f*x
+e)*cosh(f*x+e)^3+5/16*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+5/32*(f*x+e)^2-1/36*
cosh(f*x+e)^6-5/96*cosh(f*x+e)^4-5/32*cosh(f*x+e)^2)+4*c*f*((1/6*cosh(f*x+e
)^5+5/24*cosh(f*x+e)^3+5/16*cosh(f*x+e))*sinh(f*x+e)+5/16*f*x+5/16*e)-4*d*e
*((1/6*cosh(f*x+e)^5+5/24*cosh(f*x+e)^3+5/16*cosh(f*x+e))*sinh(f*x+e)+5/16*
f*x+5/16*e)-4*d*(1/6*(f*x+e)*cosh(f*x+e)^6-1/36*cosh(f*x+e)^5*sinh(f*x+e)-5
/144*cosh(f*x+e)^3*sinh(f*x+e)-5/96*cosh(f*x+e)*sinh(f*x+e)-5/96*f*x-5/96*e
)-2/3*cosh(f*x+e)^6*c*f+2/3*cosh(f*x+e)^6*d*e-3*d*(1/4*(f*x+e)*sinh(f*x+e)*
cosh(f*x+e)^3+3/8*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/16*(f*x+e)^2-1/16*cosh(
f*x+e)^4-3/16*cosh(f*x+e)^2)-3*c*f*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sin
h(f*x+e)+3/8*f*x+3/8*e)+3*d*e*((1/4*cosh(f*x+e)^3+3/8*cosh(f*x+e))*sinh(f*x
+e)+3/8*f*x+3/8*e)+d*(1/4*(f*x+e)*cosh(f*x+e)^4-1/16*cosh(f*x+e)^3*sinh(f*x
+e)-3/32*cosh(f*x+e)*sinh(f*x+e)-3/32*f*x-3/32*e)+1/4*cosh(f*x+e)^4*c*f-1/4
*cosh(f*x+e)^4*d*e)
```

maxima [A] time = 1.05, size = 139, normalized size = 0.76

$$\frac{1}{96}c\left(\frac{12\left(fx+e\right)}{a^3f}-\frac{18\,e^{\left(-2fx-2e\right)}+9\,e^{\left(-4fx-4e\right)}+2\,e^{\left(-6fx-6e\right)}}{a^3f}\right)+\frac{\left(72\,f^2x^2e^{(6e)}-108\left(2\,fxe^{(4e)}+e^{(4e)}\right)e^{\left(-2fx\right)}-27\right.}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/96*c*(12*(f*x + e)/(a^3*f) - (18*e^(-2*f*x - 2*e) + 9*e^(-4*f*x - 4*e) +
2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/1152*(72*f^2*x^2*e^(6*e) - 108*(2*f*x*e^(4
*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 4*(6*
f*x + 1)*e^(-6*f*x))*d*e^(-6*e)/(a^3*f^2)
```

mupad [B] time = 1.13, size = 129, normalized size = 0.70

$$\frac{d\,x^2}{16\,a^3}-\mathrm{e}^{-2e-2fx}\left(\frac{3\,d+6\,c\,f}{32\,a^3\,f^2}+\frac{3\,d\,x}{16\,a^3\,f}\right)-\mathrm{e}^{-4e-4fx}\left(\frac{3\,d+12\,c\,f}{128\,a^3\,f^2}+\frac{3\,d\,x}{32\,a^3\,f}\right)-\mathrm{e}^{-6e-6fx}\left(\frac{d+6\,c\,f}{288\,a^3\,f^2}+\frac{d\,x}{48\,a^3\,f}\right)+\frac{c\,x}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*tanh(e + f*x))^3,x)
```

```
[Out] (d*x^2)/(16*a^3) - exp(- 2*e - 2*f*x)*((3*d + 6*c*f)/(32*a^3*f^2) + (3*d*x)
/(16*a^3*f)) - exp(- 4*e - 4*f*x)*((3*d + 12*c*f)/(128*a^3*f^2) + (3*d*x)/(
32*a^3*f)) - exp(- 6*e - 6*f*x)*((d + 6*c*f)/(288*a^3*f^2) + (d*x)/(48*a^3*
f)) + (c*x)/(8*a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{dx}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*tanh(f*x+e))**3,x)
```

```
[Out] (Integral(c/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) + 1),
x) + Integral(d*x/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x)
+ 1), x))/a**3
```

3.46 $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx$

Optimal. Leaf size=437

$$-\frac{3\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8a^3d}-\frac{\text{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{8a^3d}-\frac{3\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{8a^3d}+\frac{3\text{Chi}\left($$

```
[Out] 1/8*Chi(6*c*f/d+6*f*x)*cosh(-6*e+6*c*f/d)/a^3/d+3/8*Chi(4*c*f/d+4*f*x)*cosh
(-4*e+4*c*f/d)/a^3/d+3/8*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/a^3/d+1/8*ln
(d*x+c)/a^3/d-3/8*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d-3/8*cosh(-4*e
+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d-1/8*cosh(-6*e+6*c*f/d)*Shi(6*c*f/d+6*f*x
)/a^3/d+1/8*Chi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d-1/8*Shi(6*c*f/d+6*f
*x)*sinh(-6*e+6*c*f/d)/a^3/d+3/8*Chi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^3/
d-3/8*Shi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^3/d+3/8*Chi(2*c*f/d+2*f*x)*si
nh(-2*e+2*c*f/d)/a^3/d-3/8*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^3/d
```

Rubi [A] time = 1.82, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 53, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3728, 3303, 3298, 3301, 3312, 5448, 5470}

$$-\frac{3\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8a^3d}-\frac{\text{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{8a^3d}-\frac{3\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{8a^3d}+\frac{3\text{Chi}\left($$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + a*Tanh[e + f*x])^3),x]
```

```
[Out] (3*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*Co
sh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (Cosh[6*e
- (6*c*f)/d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + Log[c + d*x]/(8*a
^3*d) - (CoshIntegral[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(8*a^3*d) -
 (3*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(8*a^3*d) - (3*C
oshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(8*a^3*d) - (3*Cosh[2
*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*Sinh[2*e -
(2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (3*Cosh[4*e - (4*c*
f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (3*Sinh[4*e - (4*c*f)/d]
*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - (Cosh[6*e - (6*c*f)/d]*SinhIn
tegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + (Sinh[6*e - (6*c*f)/d]*SinhIntegral[
(6*c*f)/d + 6*f*x])/(8*a^3*d)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3728

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5470

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = \int \left(\frac{1}{8a^3(c+dx)} + \frac{3\cosh(2e+2fx)}{8a^3(c+dx)} + \frac{3\cosh^2(2e+2fx)}{8a^3(c+dx)} + \frac{\cosh^3(2e+$$

$$= \frac{\log(c+dx)}{8a^3d} + \frac{\int \frac{\cosh^3(2e+2fx)}{c+dx}\,dx}{8a^3} - \frac{\int \frac{\sinh^3(2e+2fx)}{c+dx}\,dx}{8a^3} + \frac{3\int \frac{\sinh(2e+2fx)\,\sin}{c+dx}}{16a^3}$$

$$= \frac{\log(c+dx)}{8a^3d} - \frac{i\int \left(\frac{3i\sinh(2e+2fx)}{4(c+dx)} - \frac{i\sinh(6e+6fx)}{4(c+dx)}\right)dx}{8a^3} + \frac{\int \left(\frac{3\cosh(2e+2fx)}{4(c+dx)} +}{8a}$$

$$= \frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} - \frac{3\text{Chi}\left(\frac{4cf}{d}+4fx\right)\sin}{8a^3d}$$

$$= \frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\log(c+dx)}{8a^3d} - \frac{3\text{Chi}\left(\frac{4cf}{d}+4fx\right)\sin}{8a^3d}$$

$$= \frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} + \frac{\cosh\left(6e-\frac{6cf}{d}\right)\text{Chi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d} -$$

Mathematica [A] time = 0.72, size = 312, normalized size = 0.71

$$\frac{\text{sech}^3(e+fx)(\sinh(fx)+\cosh(fx))^3\left(\left(\cosh\left(e-\frac{4cf}{d}\right)-\sinh\left(e-\frac{4cf}{d}\right)\right)\left(-\text{Chi}\left(\frac{6f(c+dx)}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)+\text{C}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(a + a*Tanh[e + f*x])^3),x]
```

```
[Out] (Sech[e + f*x]^3*(Cosh[f*x] + Sinh[f*x])^3*(Cosh[3*e]*Log[f*(c + d*x)] + Lo
g[f*(c + d*x)]*Sinh[3*e] + (Cosh[e - (4*c*f)/d] - Sinh[e - (4*c*f)/d])*(3*C
oshIntegral[(4*f*(c + d*x))/d] + Cosh[2*e - (2*c*f)/d]*CoshIntegral[(6*f*(c
 + d*x))/d] - CoshIntegral[(6*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 3*Cos
hIntegral[(2*f*(c + d*x))/d]*(Cosh[2*e - (2*c*f)/d] + Sinh[2*e - (2*c*f)/d]
) - 3*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 3*Sinh[2*e -
(2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 3*SinhIntegral[(4*f*(c + d*x))
/d] - Cosh[2*e - (2*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d] + Sinh[2*e - (2
*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d])))/(8*a^3*d*(1 + Tanh[e + f*x])^3)
```

fricas [A] time = 0.47, size = 193, normalized size = 0.44

$$\frac{3\,\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)\cosh\left(-\frac{2\left(de-cf\right)}{d}\right)+3\,\mathrm{Ei}\left(-\frac{4\left(dfx+cf\right)}{d}\right)\cosh\left(-\frac{4\left(de-cf\right)}{d}\right)+\mathrm{Ei}\left(-\frac{6\left(dfx+cf\right)}{d}\right)\cosh\left(-\frac{6\left(de-cf\right)}{d}\right)+3\,\mathrm{E}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + 3*Ei(-4*(d*f*x + c*f
)/d)*cosh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d)
 + 3*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) + 3*Ei(-4*(d*f*x + c*f)/
d)*sinh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*sinh(-6*(d*e - c*f)/d) +
 log(d*x + c))/(a^3*d)
```

giac [A] time = 0.14, size = 107, normalized size = 0.24

$$\frac{\left(\mathrm{Ei}\left(-\frac{6\left(dfx+cf\right)}{d}\right)e^{\left(\frac{6cf}{d}\right)}+3\,\mathrm{Ei}\left(-\frac{4\left(dfx+cf\right)}{d}\right)e^{\left(\frac{4cf}{d}+2e\right)}+3\,\mathrm{Ei}\left(-\frac{2\left(dfx+cf\right)}{d}\right)e^{\left(\frac{2cf}{d}+4e\right)}+e^{(6e)}\log\left(dx+c\right)\right)e^{(-6e)}}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/8*(Ei(-6*(d*f*x + c*f)/d)*e^(6*c*f/d) + 3*Ei(-4*(d*f*x + c*f)/d)*e^(4*c*f
/d + 2*e) + 3*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d + 4*e) + e^(6*e)*log(d*x +
c))*e^(-6*e)/(a^3*d)
```

maple [A] time = 1.06, size = 151, normalized size = 0.35

$$\frac{\ln\left(dx+c\right)}{8a^3d}-\frac{\mathrm{e}^{\frac{6cf-6de}{d}}\,\mathrm{Ei}\left(1,6fx+6e+\frac{6cf-6de}{d}\right)}{8a^3d}-\frac{3\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{Ei}\left(1,4fx+4e+\frac{4cf-4de}{d}\right)}{8a^3d}-\frac{3\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{Ei}\left(1,2fx+2e+\frac{2c}{}\right.}{8a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x)
```

```
[Out] 1/8*ln(d*x+c)/a^3/d-1/8/a^3/d*exp(6*(c*f-d*e)/d)*Ei(1,6*f*x+6*e+6*(c*f-d*e)
/d)-3/8/a^3/d*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)-3/8/a^3/d*ex
p(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

maxima [A] time = 4.53, size = 114, normalized size = 0.26

$$-\frac{e^{\left(-6e+\frac{6cf}{d}\right)}E_1\left(\frac{6\left(dx+c\right)f}{d}\right)}{8\,a^3d}-\frac{3\,e^{\left(-4e+\frac{4cf}{d}\right)}E_1\left(\frac{4\left(dx+c\right)f}{d}\right)}{8\,a^3d}-\frac{3\,e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2\left(dx+c\right)f}{d}\right)}{8\,a^3d}+\frac{\log\left(dx+c\right)}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(1, 6*(d*x + c)*f/d)/(a^3*d) - 3/8*e^
(-4*e + 4*c*f/d)*exp_integral_e(1, 4*(d*x + c)*f/d)/(a^3*d) - 3/8*e^(-2*e +
 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a^3*d) + 1/8*log(d*x + c)/(a^
3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a + a\tanh\left(e + f\,x\right)\right)^3\,(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tanh(e + f*x))^3*(c + d*x)),x)
```

```
[Out] int(1/((a + a*tanh(e + f*x))^3*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\tanh^3\left(e+fx\right)+3c\tanh^2\left(e+fx\right)+3c\tanh\left(e+fx\right)+c+dx\tanh^3\left(e+fx\right)+3dx\tanh^2\left(e+fx\right)+3dx\tanh\left(e+fx\right)+dx}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*tanh(f*x+e))**3,x)
```

```
[Out] Integral(1/(c*tanh(e + f*x)**3 + 3*c*tanh(e + f*x)**2 + 3*c*tanh(e + f*x) +
 c + d*x*tanh(e + f*x)**3 + 3*d*x*tanh(e + f*x)**2 + 3*d*x*tanh(e + f*x) +
d*x), x)/a**3
```

3.47 $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx$

Optimal. Leaf size=692

$$\frac{3f\text{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}+\frac{3f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f}{}$$

```
[Out] -1/8/a^3/d/(d*x+c)-3/4*f*Chi(6*c*f/d+6*f*x)*cosh(-6*e+6*c*f/d)/a^3/d^2-3/2*
f*Chi(4*c*f/d+4*f*x)*cosh(-4*e+4*c*f/d)/a^3/d^2-3/4*f*Chi(2*c*f/d+2*f*x)*co
sh(-2*e+2*c*f/d)/a^3/d^2-9/32*cosh(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*cosh(2*f*x+
2*e)^2/a^3/d/(d*x+c)-1/8*cosh(2*f*x+2*e)^3/a^3/d/(d*x+c)-3/32*cosh(6*f*x+6*
e)/a^3/d/(d*x+c)+3/4*f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d^2+3/2*f*
cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d^2+3/4*f*cosh(-6*e+6*c*f/d)*Shi(
6*c*f/d+6*f*x)/a^3/d^2-3/4*f*Chi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d^2+
3/4*f*Shi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d^2-3/2*f*Chi(4*c*f/d+4*f*x
)*sinh(-4*e+4*c*f/d)/a^3/d^2+3/2*f*Shi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^
3/d^2-3/4*f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^3/d^2+3/4*f*Shi(2*c*f/d
+2*f*x)*sinh(-2*e+2*c*f/d)/a^3/d^2+15/32*sinh(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*
sinh(2*f*x+2*e)^2/a^3/d/(d*x+c)+1/8*sinh(2*f*x+2*e)^3/a^3/d/(d*x+c)+3/8*sin
h(4*f*x+4*e)/a^3/d/(d*x+c)+3/32*sinh(6*f*x+6*e)/a^3/d/(d*x+c)
```

Rubi [A] time = 1.81, antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 60, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3728, 3297, 3303, 3298, 3301, 3313, 12, 5448, 5470}

$$\frac{3f\text{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}+\frac{3f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^3),x]
```

```
[Out] -1/(8*a^3*d*(c + d*x)) - (9*Cosh[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3*Co
sh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) - Cosh[2*e + 2*f*x]^3/(8*a^3*d*(c +
d*x)) - (3*Cosh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) - (3*f*Cosh[2*e - (2*c*f
)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) - (3*f*Cosh[4*e - (4*c*f)
/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (3*f*Cosh[6*e - (6*c*f)/
d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) + (3*f*CoshIntegral[(6*c*f)
/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(4*a^3*d^2) + (3*f*CoshIntegral[(4*c*f)/
d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(2*a^3*d^2) + (3*f*CoshIntegral[(2*c*f)/d
 + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(4*a^3*d^2) + (15*Sinh[2*e + 2*f*x])/(32*a
^3*d*(c + d*x)) - (3*Sinh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) + Sinh[2*e +
2*f*x]^3/(8*a^3*d*(c + d*x)) + (3*Sinh[4*e + 4*f*x])/(8*a^3*d*(c + d*x)) +
(3*Sinh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) + (3*f*Cosh[2*e - (2*c*f)/d]*Sin
hIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) - (3*f*Sinh[2*e - (2*c*f)/d]*Sinh
Integral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) + (3*f*Cosh[4*e - (4*c*f)/d]*SinhI
ntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (3*f*Sinh[4*e - (4*c*f)/d]*SinhIn
tegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) + (3*f*Cosh[6*e - (6*c*f)/d]*SinhInt
egral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) - (3*f*Sinh[6*e - (6*c*f)/d]*SinhInte
gral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3728

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5470

```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(p_.)*Sinh[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Sinh[a +
b*x]^p*Sinh[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0
] && IGtQ[q, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = \int\left(\frac{1}{8a^3(c+dx)^2}+\frac{3\cosh(2e+2fx)}{8a^3(c+dx)^2}+\frac{3\cosh^2(2e+2fx)}{8a^3(c+dx)^2}+\frac{\cosh^3(2e+$$

$$= -\frac{1}{8a^3d(c+dx)}+\frac{\int\frac{\cosh^3(2e+2fx)}{(c+dx)^2}\,dx}{8a^3}-\frac{\int\frac{\sinh^3(2e+2fx)}{(c+dx)^2}\,dx}{8a^3}+\frac{3\int\frac{\sinh(2e+2fx)\,s}{(c+d}}{16a}$$

$$= -\frac{1}{8a^3d(c+dx)}-\frac{3\cosh(2e+2fx)}{8a^3d(c+dx)}-\frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)}-\frac{\cosh^3(2e+2f}{8a^3d(c+dx)}$$

$$= -\frac{1}{8a^3d(c+dx)}-\frac{3\cosh(2e+2fx)}{8a^3d(c+dx)}-\frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)}-\frac{\cosh^3(2e+2f}{8a^3d(c+dx)}$$

$$= -\frac{1}{8a^3d(c+dx)}-\frac{9\cosh(2e+2fx)}{32a^3d(c+dx)}-\frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)}-\frac{\cosh^3(2e+2f}{8a^3d(c+dx)}$$

$$= -\frac{1}{8a^3d(c+dx)}-\frac{9\cosh(2e+2fx)}{32a^3d(c+dx)}-\frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)}-\frac{\cosh^3(2e+2f}{8a^3d(c+dx)}$$

$$= -\frac{1}{8a^3d(c+dx)}-\frac{9\cosh(2e+2fx)}{32a^3d(c+dx)}-\frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)}-\frac{\cosh^3(2e+2f}{8a^3d(c+dx)}$$

Mathematica [A] time = 3.13, size = 794, normalized size = 1.15

$$-\frac{\operatorname{sech}^3(e+fx)\left(\sinh\left(\frac{3cf}{d}\right)+\cosh\left(\frac{3cf}{d}\right)\right)\left(-6cf\operatorname{Chi}\left(\frac{6f(c+dx)}{d}\right)\sinh\left(3e-\frac{3f(c+dx)}{d}\right)-6dfx\operatorname{Chi}\left(\frac{6f(c+dx)}{d}\right)\sinh\left(3e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^3),x]
```

```
[Out] -1/8*(Sech[e + f*x]^3*(Cosh[(3*c*f)/d] + Sinh[(3*c*f)/d])*(3*d*Cosh[e + f*(
(-3*c)/d + x)] + d*Cosh[3*(e + f*(-(c/d) + x))] + d*Cosh[3*(e + f*(c/d + x)
)] + 3*d*Cosh[e + f*((3*c)/d + x)] + 6*c*f*Cosh[3*e - (3*f*(c + d*x))/d]*Co
shIntegral[(6*f*(c + d*x))/d] + 6*d*f*x*Cosh[3*e - (3*f*(c + d*x))/d]*CoshI
ntegral[(6*f*(c + d*x))/d] + 6*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*
(Cosh[e - (c*f)/d + 3*f*x] + Sinh[e - (c*f)/d + 3*f*x]) + 3*d*Sinh[e + f*((
-3*c)/d + x)] + d*Sinh[3*(e + f*(-(c/d) + x))] - d*Sinh[3*(e + f*(c/d + x))
] - 3*d*Sinh[e + f*((3*c)/d + x)] - 6*c*f*CoshIntegral[(6*f*(c + d*x))/d]*S
inh[3*e - (3*f*(c + d*x))/d] - 6*d*f*x*CoshIntegral[(6*f*(c + d*x))/d]*Sinh
[3*e - (3*f*(c + d*x))/d] + 12*f*(c + d*x)*CoshIntegral[(4*f*(c + d*x))/d]*
(Cosh[e - (f*(c + 3*d*x))/d] - Sinh[e - (f*(c + 3*d*x))/d]) - 6*c*f*Cosh[e
- (c*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Cosh[e - (c*f)
/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*c*f*Sinh[e - (c*f)/d + 3*f*
x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Sinh[e - (c*f)/d + 3*f*x]*Sinh
Integral[(2*f*(c + d*x))/d] - 12*c*f*Cosh[e - (f*(c + 3*d*x))/d]*SinhIntegr
al[(4*f*(c + d*x))/d] - 12*d*f*x*Cosh[e - (f*(c + 3*d*x))/d]*SinhIntegral[(
4*f*(c + d*x))/d] + 12*c*f*Sinh[e - (f*(c + 3*d*x))/d]*SinhIntegral[(4*f*(c
 + d*x))/d] + 12*d*f*x*Sinh[e - (f*(c + 3*d*x))/d]*SinhIntegral[(4*f*(c + d
*x))/d] - 6*c*f*Cosh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[(6*f*(c + d*x))/
d] - 6*d*f*x*Cosh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[(6*f*(c + d*x))/d]
+ 6*c*f*Sinh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[(6*f*(c + d*x))/d] + 6*d
```

```
*f*x*Sinh[3*e - (3*f*(c + d*x))/d]*SinhIntegral[(6*f*(c + d*x))/d]))/(a^3*d
^2*(c + d*x)*(1 + Tanh[e + f*x])^3)
```

fricas [A] time = 0.46, size = 1164, normalized size = 1.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-2*(d*e -
 c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-4*(
d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh
(-6*(d*e - c*f)/d) + (3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e -
 c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) +
3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d) + d)*cosh(f*x
 + e)^3 + 3*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) +
2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + (d*f*x + c*
f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-2*(d*f
*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d
)*sinh(-4*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*sinh(-6*(d*
e - c*f)/d))*sinh(f*x + e)^3 + 3*(3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*co
sh(f*x + e)*sinh(-2*(d*e - c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)
*cosh(f*x + e)*sinh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)
/d)*cosh(f*x + e)*sinh(-6*(d*e - c*f)/d) + (3*(d*f*x + c*f)*Ei(-2*(d*f*x +
c*f)/d)*cosh(-2*(d*e - c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cos
h(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e -
 c*f)/d) + d)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*d*cosh(f*x + e) + 9*((d*f*
x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-2*(d*e - c*f)/d) + 2*
(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-4*(d*e - c*f)/d)
 + (d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-6*(d*e - c*f)
/d) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + 2*(d*f
*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(
-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d))*cosh(f*x + e)^2)*sinh(f*x + e))
/((a^3*d^3*x + a^3*c*d^2)*cosh(f*x + e)^3 + 3*(a^3*d^3*x + a^3*c*d^2)*cosh(
f*x + e)^2*sinh(f*x + e) + 3*(a^3*d^3*x + a^3*c*d^2)*cosh(f*x + e)*sinh(f*x
 + e)^2 + (a^3*d^3*x + a^3*c*d^2)*sinh(f*x + e)^3)
```

giac [A] time = 0.23, size = 912, normalized size = 1.32

$$-\left(6\,(dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)f^2\mathrm{Ei}\left(\frac{6\left((dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de\right)}{d}\right)e^{\left(\frac{6\left(cf-de\right)}{d}\right)}-6\,cf^3\mathrm{Ei}\left(\frac{6\left((dx+c)\left(\frac{cf}{dx+c}-f-\frac{de}{dx+c}\right)-cf+de\right)}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] -1/8*(6*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*Ei(6*((d*x + c)*(
c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(6*(c*f - d*e)/d) - 6*
c*f^3*Ei(6*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e
^(6*(c*f - d*e)/d) + 12*(d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))*f^2*E
i(4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(4*(c*
f - d*e)/d) - 12*c*f^3*Ei(4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))
- c*f + d*e)/d)*e^(4*(c*f - d*e)/d) + 6*(d*x + c)*(c*f/(d*x + c) - f - d*e/
(d*x + c))*f^2*Ei(2*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f +
d*e)/d)*e^(2*(c*f - d*e)/d) - 6*c*f^3*Ei(2*((d*x + c)*(c*f/(d*x + c) - f -
```

```
d*e/(d*x + c)) - c*f + d*e)/d)*e^(2*(c*f - d*e)/d) + 6*d*f^2*Ei(6*((d*x + c
)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d*e)/d)*e^(6*(c*f - d*e)/d +
1) + 12*d*f^2*Ei(4*((d*x + c)*(c*f/(d*x + c) - f - d*e/(d*x + c)) - c*f + d
*e)/d)*e^(4*(c*f - d*e)/d + 1) + 6*d*f^2*Ei(2*((d*x + c)*(c*f/(d*x + c) - f
 - d*e/(d*x + c)) - c*f + d*e)/d)*e^(2*(c*f - d*e)/d + 1) - d*f^2*e^(6*(d*x
 + c)*(c*f/(d*x + c) - f - d*e/(d*x + c))/d) - 3*d*f^2*e^(4*(d*x + c)*(c*f/
(d*x + c) - f - d*e/(d*x + c))/d) - 3*d*f^2*e^(2*(d*x + c)*(c*f/(d*x + c) -
 f - d*e/(d*x + c))/d) - d*f^2)*d^2/(((d*x + c)*a^3*d^4*(c*f/(d*x + c) - f
- d*e/(d*x + c)) - a^3*c*d^4*f + a^3*d^5*e)*f)
```

maple [A] time = 2.11, size = 239, normalized size = 0.35

$$-\frac{1}{8a^3d\,(dx+c)}-\frac{f\,\mathrm{e}^{-6fx-6e}}{8a^3d\left(dfx+cf\right)}+\frac{3f\,\mathrm{e}^{\frac{6cf-6de}{d}}\,\mathrm{Ei}\left(1,6fx+6e+\frac{6cf-6de}{d}\right)}{4a^3d^2}-\frac{3f\,\mathrm{e}^{-4fx-4e}}{8a^3d\left(dfx+cf\right)}+\frac{3f\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{Ei}\left(1,4fx-\right.}{2a^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x)
```

```
[Out] -1/8/a^3/d/(d*x+c)-1/8*f/a^3*exp(-6*f*x-6*e)/d/(d*f*x+c*f)+3/4*f/a^3/d^2*ex
p(6*(c*f-d*e)/d)*Ei(1,6*f*x+6*e+6*(c*f-d*e)/d)-3/8*f/a^3*exp(-4*f*x-4*e)/d/
(d*f*x+c*f)+3/2*f/a^3/d^2*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)-
3/8*f/a^3*exp(-2*f*x-2*e)/d/(d*f*x+c*f)+3/4*f/a^3/d^2*exp(2*(c*f-d*e)/d)*Ei
(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

maxima [A] time = 7.71, size = 140, normalized size = 0.20

$$-\frac{1}{8\left(a^3d^2x+a^3cd\right)}-\frac{e^{\left(-6e+\frac{6cf}{d}\right)}E_2\left(\frac{6(dx+c)f}{d}\right)}{8\,(dx+c)a^3d}-\frac{3\,e^{\left(-4e+\frac{4cf}{d}\right)}E_2\left(\frac{4(dx+c)f}{d}\right)}{8\,(dx+c)a^3d}-\frac{3\,e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{8\,(dx+c)a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8/(a^3*d^2*x + a^3*c*d) - 1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(2, 6*(d*
x + c)*f/d)/((d*x + c)*a^3*d) - 3/8*e^(-4*e + 4*c*f/d)*exp_integral_e(2, 4*
(d*x + c)*f/d)/((d*x + c)*a^3*d) - 3/8*e^(-2*e + 2*c*f/d)*exp_integral_e(2,
 2*(d*x + c)*f/d)/((d*x + c)*a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+a\tanh\left(e+f\,x\right)\right)^3\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*tanh(e + f*x))^3*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*tanh(e + f*x))^3*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c^2\tanh^3\left(e+fx\right)+3c^2\tanh^2\left(e+fx\right)+3c^2\tanh\left(e+fx\right)+c^2+2cdx\tanh^3\left(e+fx\right)+6cdx\tanh^2\left(e+fx\right)+6cdx\tanh\left(e+fx\right)+2cdx+d^2x^2\tanh^3\left(e+fx\right)+3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*tanh(f*x+e))**3,x)
```

```
[Out] Integral(1/(c**2*tanh(e + f*x)**3 + 3*c**2*tanh(e + f*x)**2 + 3*c**2*tanh(e
 + f*x) + c**2 + 2*c*d*x*tanh(e + f*x)**3 + 6*c*d*x*tanh(e + f*x)**2 + 6*c*
d*x*tanh(e + f*x) + 2*c*d*x + d**2*x**2*tanh(e + f*x)**3 + 3*d**2*x**2*tanh
(e + f*x)**2 + 3*d**2*x**2*tanh(e + f*x) + d**2*x**2), x)/a**3
```

3.48 $\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx$

Optimal. Leaf size=23

$$\text{Int}\left((c+dx)^m (a\tanh(e+fx)+a)^2, x\right)$$

```
[Out] Unintegrable((d*x+c)^m*(a+a*tanh(f*x+e))^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + a*Tanh[e + f*x])^2, x]
```

Rubi steps

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \int (c+dx)^m (a+a\tanh(e+fx))^2\,dx$$

Mathematica [A] time = 34.65, size = 0, normalized size = 0.00

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x])^2, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\tanh\left(fx+e\right)^2 + 2\,a^2\tanh\left(fx+e\right) + a^2\right)(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*tanh(f*x + e)^2 + 2*a^2*tanh(f*x + e) + a^2)*(d*x + c)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\tanh\left(fx+e\right)+a\right)^2 (dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*tanh(f*x + e) + a)^2*(d*x + c)^m, x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(a+a\tanh\left(fx+e\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*tanh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+a*tanh(f*x+e))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(dx+c)^{m+1}a^2}{d(m+1)} + \int \frac{2\,(dx+c)^m a^2\left(e^{(fx+e)} - e^{(-fx-e)}\right)}{e^{(fx+e)} + e^{(-fx-e)}} + \frac{(dx+c)^m a^2\left(e^{(fx+e)} - e^{(-fx-e)}\right)^2}{\left(e^{(fx+e)} + e^{(-fx-e)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] (d*x + c)^(m + 1)*a^2/(d*(m + 1)) + integrate(2*(d*x + c)^m*a^2*(e^(f*x + e
) - e^(-f*x - e))/(e^(f*x + e) + e^(-f*x - e)) + (d*x + c)^m*a^2*(e^(f*x +
e) - e^(-f*x - e))^2/(e^(f*x + e) + e^(-f*x - e))^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \left(a + a\tanh\left(e + f\,x\right)\right)^2 (c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tanh(e + f*x))^2*(c + d*x)^m,x)
```

```
[Out] int((a + a*tanh(e + f*x))^2*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\,(c+dx)^m \tanh\left(e+fx\right)dx + \int (c+dx)^m \tanh^2\left(e+fx\right)dx + \int (c+dx)^m\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*tanh(f*x+e))**2,x)
```

```
[Out] a**2*(Integral(2*(c + d*x)**m*tanh(e + f*x), x) + Integral((c + d*x)**m*tan
h(e + f*x)**2, x) + Integral((c + d*x)**m, x))
```

3.49 $\int (c+dx)^m (a+a\tanh(e+fx))\,dx$

Optimal. Leaf size=21

$$\text{Int}\left((c+dx)^m (a\tanh(e+fx)+a), x\right)$$

```
[Out] Unintegrable((d*x+c)^m*(a+a*tanh(f*x+e)),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m (a+a\tanh(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + a*Tanh[e + f*x]),x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + a*Tanh[e + f*x]), x]
```

Rubi steps

$$\int (c+dx)^m (a+a\tanh(e+fx))\,dx = \int (c+dx)^m (a+a\tanh(e+fx))\,dx$$

Mathematica [A] time = 13.98, size = 0, normalized size = 0.00

$$\int (c+dx)^m (a+a\tanh(e+fx))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x]),x]
```

```
[Out] Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x]), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a\tanh\left(fx+e\right)+a\right)(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((a*tanh(f*x + e) + a)*(d*x + c)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\tanh\left(fx+e\right)+a\right)(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*tanh(f*x + e) + a)*(d*x + c)^m, x)
```

maple [A] time = 0.34, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(a+a\tanh\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*tanh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+a*tanh(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$a \int \frac{(dx+c)^m \left(e^{(fx+e)} - e^{(-fx-e)}\right)}{e^{(fx+e)} + e^{(-fx-e)}}\, dx + \frac{(dx+c)^{m+1} a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] a*integrate((d*x + c)^m*(e^(f*x + e) - e^(-f*x - e))/(e^(f*x + e) + e^(-f*x
 - e)), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \left(a + a \tanh\left(e + f\,x\right)\right)\,(c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*tanh(e + f*x))*(c + d*x)^m,x)
```

```
[Out] int((a + a*tanh(e + f*x))*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (c+dx)^m \tanh\left(e+fx\right) dx + \int (c+dx)^m\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*tanh(f*x+e)),x)
```

```
[Out] a*(Integral((c + d*x)**m*tanh(e + f*x), x) + Integral((c + d*x)**m, x))
```

3.50 $\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx$

Optimal. Leaf size=89

$$\frac{(c+dx)^{m+1}}{2ad(m+1)} - \frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{af}$$

```
[Out] 1/2*(d*x+c)^(1+m)/a/d/(1+m)-2^(-2-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,
2*f*(d*x+c)/d)/a/f/((f*(d*x+c)/d)^m)
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3727, 2181}

$$\frac{(c+dx)^{m+1}}{2ad(m+1)} - \frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + a*Tanh[e + f*x]),x]
```

```
[Out] (c + d*x)^(1 + m)/(2*a*d*(1 + m)) - (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c + d
*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a*f*((f*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3727

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + Dist[1/(2*a), Int[(c +
 d*x)^m*E^((2*a*(e + f*x))/b), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[a^2 + b^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx &= \frac{(c+dx)^{1+m}}{2ad(1+m)} + \frac{\int e^{2i(ie+ifx)}(c+dx)^m\,dx}{2a}\\ &= \frac{(c+dx)^{1+m}}{2ad(1+m)} - \frac{2^{-2-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{af}\end{aligned}$$

Mathematica [B] time = 1.20, size = 186, normalized size = 2.09

$$\frac{2^{-m-2}(c+dx)^m\text{sech}(e+fx)\left(-\frac{f(c+dx)}{d}\right)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(\sinh\left(f\left(\frac{c}{d}+x\right)\right)+\cosh\left(f\left(\frac{c}{d}+x\right)\right)\right)\left(f2^{m+1}(c+dx)\left(f\right.\right.}{adf(m+1)(\tanh(e$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + a*Tanh[e + f*x]),x]
```

```
[Out] (2^(-2 - m)*(c + d*x)^m*(-((f*(c + d*x))/d))^m*Sech[e + f*x]*(d*(1 + m)*Gam
ma[1 + m, (2*f*(c + d*x))/d]*(-Cosh[e - (c*f)/d] + Sinh[e - (c*f)/d]) + 2^(
1 + m)*f*(f*(c/d + x))^m*(c + d*x)*(Cosh[e - (c*f)/d] + Sinh[e - (c*f)/d]))
*(Cosh[f*(c/d + x)] + Sinh[f*(c/d + x)]))/(a*d*f*(1 + m)*(-((f^2*(c + d*x)^
2)/d^2))^m*(1 + Tanh[e + f*x]))
```

fricas [A] time = 0.47, size = 148, normalized size = 1.66

$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2\left(dfx+cf\right)}{d}\right)-(dm+d)\Gamma\left(m+1,\frac{2\left(dfx+cf\right)}{d}\right)\sinh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2}{d}\right.}{4\left(adfm+adf\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/4*((d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1, 2*(d*
f*x + c*f)/d) - (d*m + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f
/d) + 2*d*e - 2*c*f)/d) - 2*(d*f*x + c*f)*cosh(m*log(d*x + c)) - 2*(d*f*x +
 c*f)*sinh(m*log(d*x + c)))/(a*d*f*m + a*d*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a\tanh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*tanh(f*x + e) + a), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+a\tanh\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*tanh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+a*tanh(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a\tanh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(a*tanh(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^m}{a+a\tanh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*tanh(e + f*x)),x)
```

```
[Out] int((c + d*x)^m/(a + a*tanh(e + f*x)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c+dx)^m}{\tanh\left(e+fx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*tanh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**m/(tanh(e + f*x) + 1), x)/a
```

3.51 $\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=153

$$-\frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^2f}-\frac{4^{-m-2}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4f(c+dx)}{d}\right)}{a^2f}+$$

```
[Out] 1/4*(d*x+c)^(1+m)/a^2/d/(1+m)-2^(-2-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+
m,2*f*(d*x+c)/d)/a^2/f/((f*(d*x+c)/d)^m)-4^(-2-m)*exp(-4*e+4*c*f/d)*(d*x+c)
^m*GAMMA(1+m,4*f*(d*x+c)/d)/a^2/f/((f*(d*x+c)/d)^m)
```

Rubi [A] time = 0.18, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3729, 2181}

$$-\frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^2f}-\frac{4^{-m-2}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) - (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c +
 d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^2*f*((f*(c + d*x))/d)^m) - (4^(
-2 - m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*x))/d])/(
a^2*f*((f*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx &= \int\left(\frac{(c+dx)^m}{4a^2}+\frac{e^{-4e-4fx}(c+dx)^m}{4a^2}+\frac{e^{-2e-2fx}(c+dx)^m}{2a^2}\right)dx\\ &= \frac{(c+dx)^{1+m}}{4a^2d(1+m)}+\frac{\int e^{-4e-4fx}(c+dx)^m\,dx}{4a^2}+\frac{\int e^{-2e-2fx}(c+dx)^m\,dx}{2a^2}\\ &= \frac{(c+dx)^{1+m}}{4a^2d(1+m)}-\frac{2^{-2-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{a^2f}-\frac{4^{-2-m}e^{}}{}\end{aligned}$$

Mathematica [A] time = 11.17, size = 195, normalized size = 1.27

$$\frac{4^{-m-2}(c+dx)^m\text{sech}^2(e+fx)(\sinh(2fx)+\cosh(2fx))\left(-\frac{f(c+dx)}{d}\right)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(d4^{m+1}(\sinh(e)+\cosh(e))^2\right.}{a^2df(m+1)(\tanh$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m/(a + a*Tanh[e + f*x])^2,x]
```

```
[Out] (4^(-2 - m)*(c + d*x)^m*(-((f*(c + d*x))/d))^m*Sech[e + f*x]^2*(-(2^(2 + m)
*d*E^((2*c*f)/d)*(1 + m)*Gamma[1 + m, (2*f*(c + d*x))/d]) + 4^(1 + m)*d*((f
*(c + d*x))/d)^(1 + m)*(Cosh[e] + Sinh[e])^2 - d*E^((4*c*f)/d)*(1 + m)*Gamm
a[1 + m, (4*f*(c + d*x))/d]*(Cosh[2*e] - Sinh[2*e]))*(Cosh[2*f*x] + Sinh[2*
f*x]))/(a^2*d*f*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m*(1 + Tanh[e + f*x])^2)
```

fricas [A] time = 0.48, size = 248, normalized size = 1.62

$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{4f}{d}\right)+4de-4cf}{d}\right)\Gamma\left(m+1,\frac{4(dfx+cf)}{d}\right)+4(dm+d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2(dfx+c}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/16*((d*m + d)*cosh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d)*gamma(m + 1, 4*(d
*f*x + c*f)/d) + 4*(d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma
(m + 1, 2*(d*f*x + c*f)/d) - (d*m + d)*gamma(m + 1, 4*(d*f*x + c*f)/d)*sinh
((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d) - 4*(d*m + d)*gamma(m + 1, 2*(d*f*x +
c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) - 4*(d*f*x + c*f)*cosh(m*l
og(d*x + c)) - 4*(d*f*x + c*f)*sinh(m*log(d*x + c)))/(a^2*d*f*m + a^2*d*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a\tanh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^2, x)
```

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a+a\tanh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*tanh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m/(a+a*tanh(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a\tanh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^m}{\left(a + a\tanh\left(e + f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*tanh(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^m/(a + a*tanh(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c+dx)^m}{\tanh^2\left(e+fx\right)+2\tanh\left(e+fx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*tanh(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**m/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x)/a**2
```

3.52 $\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx$

Optimal. Leaf size=224

$$-\frac{3\,2^{-m-4}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^3f}-\frac{3\,2^{-2m-5}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4f(c+dx)}{d}\right)}{a^3f}$$

```
[Out] 1/8*(d*x+c)^(1+m)/a^3/d/(1+m)-3*2^(-4-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(
1+m,2*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)/d)^m)-3*2^(-5-2*m)*exp(-4*e+4*c*f/d)*(
d*x+c)^m*GAMMA(1+m,4*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)/d)^m)-2^(-4-m)*3^(-1-m)
*exp(-6*e+6*c*f/d)*(d*x+c)^m*GAMMA(1+m,6*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)/d)^
m)
```

Rubi [A] time = 0.24, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3729, 2181}

$$-\frac{3\,2^{-m-4}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^3f}-\frac{3\,2^{-2m-5}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{4f(c+dx)}{d}\right)}{a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m/(a + a*Tanh[e + f*x])^3,x]
```

```
[Out] (c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) - (3*2^(-4 - m)*E^(-2*e + (2*c*f)/d)*(c
 + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m) - (3
*2^(-5 - 2*m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*x))
/d])/(a^3*f*((f*(c + d*x))/d)^m) - (2^(-4 - m)*3^(-1 - m)*E^(-6*e + (6*c*f)
/d)*(c + d*x)^m*Gamma[1 + m, (6*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m
)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3729

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^((2*a*(e + f*x))
/b)/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int\frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx &= \int\left(\frac{(c+dx)^m}{8a^3}+\frac{e^{-6e-6fx}(c+dx)^m}{8a^3}+\frac{3e^{-4e-4fx}(c+dx)^m}{8a^3}+\frac{3e^{-2e-2fx}(c+dx)^m}{8a^3}\right)dx\\ &= \frac{(c+dx)^{1+m}}{8a^3d(1+m)}+\frac{\int e^{-6e-6fx}(c+dx)^m\,dx}{8a^3}+\frac{3\int e^{-4e-4fx}(c+dx)^m\,dx}{8a^3}+\frac{3\int e^{-2e-2fx}(c\\ &= \frac{(c+dx)^{1+m}}{8a^3d(1+m)}-\frac{3\,2^{-4-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{a^3f}-\frac{3\,2^{-5-2m}}{}\end{aligned}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + a*Tanh[e + f*x])^3,x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.69, size = 345, normalized size = 1.54

$$-\frac{2\,(dm+d)\cosh\left(\frac{dm\log\left(\frac{6f}{d}\right)+6de-6cf}{d}\right)\Gamma\left(m+1,\frac{6\left(dfx+cf\right)}{d}\right)+9\,(dm+d)\cosh\left(\frac{dm\log\left(\frac{4f}{d}\right)+4de-4cf}{d}\right)\Gamma\left(m+1,\frac{4(d}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/96*(2*(d*m + d)*cosh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d)*gamma(m + 1, 6*
(d*f*x + c*f)/d) + 9*(d*m + d)*cosh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d)*gam
ma(m + 1, 4*(d*f*x + c*f)/d) + 18*(d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e -
2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 2*(d*m + d)*gamma(m + 1, 6*(d*f
*x + c*f)/d)*sinh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d) - 9*(d*m + d)*gamma(m
 + 1, 4*(d*f*x + c*f)/d)*sinh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d) - 18*(d*m
 + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)
/d) - 12*(d*f*x + c*f)*cosh(m*log(d*x + c)) - 12*(d*f*x + c*f)*sinh(m*log(d
*x + c)))/(a^3*d*f*m + a^3*d*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a\tanh\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^3, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a+a\tanh\left(fx+e\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*tanh(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m/(a+a*tanh(f*x+e))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a\tanh\left(fx+e\right)+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,x)^m}{\left(a + a\tanh\left(e + f\,x\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*tanh(e + f*x))^3,x)
```

```
[Out] int((c + d*x)^m/(a + a*tanh(e + f*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c+dx)^m}{\tanh^3\left(e+fx\right)+3\tanh^2\left(e+fx\right)+3\tanh\left(e+fx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*tanh(f*x+e))**3,x)
```

```
[Out] Integral((c + d*x)**m/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f
*x) + 1), x)/a**3
```

3.53 $\int (c+dx)^3(a+b\tanh(e+fx))\,dx$

Optimal. Leaf size=137

$$\frac{a(c+dx)^4}{4d}-\frac{3bd^2(c+dx)\mathrm{Li}_3\left(-e^{2(e+fx)}\right)}{2f^3}+\frac{3bd(c+dx)^2\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}+\frac{b(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b(c+dx)^4}{4d}$$

```
[Out] 1/4*a*(d*x+c)^4/d-1/4*b*(d*x+c)^4/d+b*(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+3/2*
b*d*(d*x+c)^2*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*b*d^2*(d*x+c)*polylog(3,-e
xp(2*f*x+2*e))/f^3+3/4*b*d^3*polylog(4,-exp(2*f*x+2*e))/f^4
```

Rubi [A] time = 0.27, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3722, 3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3bd^2(c+dx)\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{3bd(c+dx)^2\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{3bd^3\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}+\frac{a(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Tanh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - (b*(c + d*x)^4)/(4*d) + (b*(c + d*x)^3*Log[1 + E^(2
*(e + f*x))])/f + (3*b*d*(c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2)
- (3*b*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3) + (3*b*d^3*PolyL
og[4, -E^(2*(e + f*x))])/(4*f^4)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3(a+b\tanh(e+fx))\,dx &= \int \left(a(c+dx)^3 + b(c+dx)^3\tanh(e+fx)\right)dx \\
&= \frac{a(c+dx)^4}{4d} + b\int (c+dx)^3\tanh(e+fx)\,dx \\
&= \frac{a(c+dx)^4}{4d} - \frac{b(c+dx)^4}{4d} + (2b)\int \frac{e^{2(e+fx)}(c+dx)^3}{1+e^{2(e+fx)}}\,dx \\
&= \frac{a(c+dx)^4}{4d} - \frac{b(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} - \frac{(3bd)\int (c+dx)^2 \l}{} \\
&= \frac{a(c+dx)^4}{4d} - \frac{b(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-}{2f^2} \\
&= \frac{a(c+dx)^4}{4d} - \frac{b(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-}{2f^2} \\
&= \frac{a(c+dx)^4}{4d} - \frac{b(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-}{2f^2} \\
&= \frac{a(c+dx)^4}{4d} - \frac{b(c+dx)^4}{4d} + \frac{b(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-}{2f^2}
\end{aligned}$$

Mathematica [A] time = 2.17, size = 161, normalized size = 1.18

$$\frac{1}{4}\left(x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)(a+b\tanh(e)) + b\left(-\frac{3d\left(2f^2(c+dx)^2\mathrm{Li}_2\left(-e^{-2(e+fx)}\right)+d\left(2f(c+dx)\mathrm{Li}_3\left(-e\right.\right.\right.\right.}{f^4}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Tanh[e + f*x]),x]
```

```
[Out] (b*((2*(c + d*x)^4)/(d*(1 + E^(2*e))) + (4*(c + d*x)^3*Log[1 + E^(-2*(e + f
*x))])/f - (3*d*(2*f^2*(c + d*x)^2*PolyLog[2, -E^(-2*(e + f*x))] + d*(2*f*(
c + d*x)*PolyLog[3, -E^(-2*(e + f*x))] + d*PolyLog[4, -E^(-2*(e + f*x))])))
/f^4) + x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(a + b*Tanh[e]))/4
```

fricas [C] time = 0.56, size = 583, normalized size = 4.26

$$(a-b)d^3f^4x^4 + 4(a-b)cd^2f^4x^3 + 6(a-b)c^2df^4x^2 + 4(a-b)c^3f^4x + 24bd^3\mathrm{polylog}\left(4, i\cosh\left(fx+e\right)+i\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*((a - b)*d^3*f^4*x^4 + 4*(a - b)*c*d^2*f^4*x^3 + 6*(a - b)*c^2*d*f^4*x^
2 + 4*(a - b)*c^3*f^4*x + 24*b*d^3*polylog(4, I*cosh(f*x + e) + I*sinh(f*x
+ e)) + 24*b*d^3*polylog(4, -I*cosh(f*x + e) - I*sinh(f*x + e)) + 12*(b*d^3
*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(I*cosh(f*x + e) + I*sinh(f*
x + e)) + 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-I*cosh(
f*x + e) - I*sinh(f*x + e)) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*
f^2 - b*c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 4*(b*d^3*e^3 - 3*
b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(cosh(f*x + e) + sinh(f*x +
 e) - I) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e
^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(I*cosh(f*x + e) + I*sinh(f*x +
e) + 1) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^
3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-I*cosh(f*x + e) - I*sinh(f*x +
e) + 1) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, I*cosh(f*x + e) + I*sinh(f*
x + e)) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, -I*cosh(f*x + e) - I*sinh(f
*x + e)))/f^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \left(b \tanh\left(fx + e\right) + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*tanh(f*x + e) + a), x)
```

maple [B] time = 0.30, size = 452, normalized size = 3.30

$$ac\,d^2x^3 - bc\,d^2x^3 + \frac{3a\,c^2d\,x^2}{2} - \frac{3b\,c^2d\,x^2}{2} - \frac{3b\,e^4d^3}{2f^4} + \frac{b\,c^3\ln\left(\mathrm{e}^{2fx+2e}+1\right)}{f} - \frac{2b\,c^3\ln\left(\mathrm{e}^{fx+e}\right)}{f} + \frac{3b\,d^3\,\mathrm{polylog}\left(4,-\mathrm{e}^{2fx}\right.}{4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*tanh(f*x+e)),x)
```

```
[Out] a*c*d^2*x^3-b*c*d^2*x^3+3/2*a*c^2*d*x^2-3/2*b*c^2*d*x^2-3/2/f^4*b*e^4*d^3+1
/f*b*c^3*ln(exp(2*f*x+2*e)+1)-2/f*b*c^3*ln(exp(f*x+e))+3/4*b*d^3*polylog(4,
-exp(2*f*x+2*e))/f^4+1/4*a*d^3*x^4-1/4*b*d^3*x^4+c^3*a*x+b*c^3*x-6/f*b*c^2*
d*e*x+6/f^2*b*c*d^2*e^2*x+3/f^2*b*c*d^2*polylog(2,-exp(2*f*x+2*e))*x+4/f^3*
b*c*d^2*e^3-2/f^3*b*e^3*d^3*x-3/f^2*b*c^2*d*e^2-3/2/f^3*b*d^3*polylog(3,-ex
p(2*f*x+2*e))*x+1/f*b*d^3*ln(exp(2*f*x+2*e)+1)*x^3+3/2/f^2*b*c^2*d*polylog(
2,-exp(2*f*x+2*e))+2/f^4*b*d^3*e^3*ln(exp(f*x+e))-3/2/f^3*b*c*d^2*polylog(3
,-exp(2*f*x+2*e))+3/2/f^2*b*d^3*polylog(2,-exp(2*f*x+2*e))*x^2+3/f*b*c^2*d*
ln(exp(2*f*x+2*e)+1)*x-6/f^3*b*c*d^2*e^2*ln(exp(f*x+e))+3/f*b*c*d^2*ln(exp(
2*f*x+2*e)+1)*x^2+6/f^2*b*c^2*d*e*ln(exp(f*x+e))
```

maxima [B] time = 0.39, size = 304, normalized size = 2.22

$$\frac{1}{4}\,ad^3x^4 + \frac{1}{4}\,bd^3x^4 + acd^2x^3 + bcd^2x^3 + \frac{3}{2}\,ac^2dx^2 + \frac{3}{2}\,bc^2dx^2 + ac^3x + \frac{bc^3\log\left(\cosh\left(fx+e\right)\right)}{f} + \frac{3\left(2\,fx\log\left(e^{\left(2\,fx+2\,e\right.}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + 1/4*b*d^3*x^4 + a*c*d^2*x^3 + b*c*d^2*x^3 + 3/2*a*c^2*d*x^2
 + 3/2*b*c^2*d*x^2 + a*c^3*x + b*c^3*log(cosh(f*x + e))/f + 3/2*(2*f*x*log(
```

```
e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))*b*c^2*d/f^2 + 3/2*(2*f^2*x^
2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^
(2*f*x + 2*e)))*b*c*d^2/f^3 + 1/3*(4*f^3*x^3*log(e^(2*f*x + 2*e) + 1) + 6*f
^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*polylog(3, -e^(2*f*x + 2*e)) + 3*pol
ylog(4, -e^(2*f*x + 2*e)))*b*d^3/f^4 - 1/2*(b*d^3*f^4*x^4 + 4*b*c*d^2*f^4*x
^3 + 6*b*c^2*d*f^4*x^2)/f^4
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b \tanh\left(e + f\,x\right)\right) (c + d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))*(c + d*x)^3,x)
```

```
[Out] int((a + b*tanh(e + f*x))*(c + d*x)^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh\left(e + f\,x\right)\right) (c + dx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*tanh(f*x+e)),x)
```

```
[Out] Integral((a + b*tanh(e + f*x))*(c + d*x)**3, x)
```

3.54 $\int (c+dx)^2(a+b\tanh(e+fx))\,dx$

Optimal. Leaf size=103

$$\frac{a(c+dx)^3}{3d}+\frac{bd(c+dx)\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2}+\frac{b(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b(c+dx)^3}{3d}-\frac{bd^2\mathrm{Li}_3\left(-e^{2(e+fx)}\right)}{2f^3}$$

```
[Out] 1/3*a*(d*x+c)^3/d-1/3*b*(d*x+c)^3/d+b*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+b*d*
(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2-1/2*b*d^2*polylog(3,-exp(2*f*x+2*e))
/f^3
```

Rubi [A] time = 0.21, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3722, 3718, 2190, 2531, 2282, 6589}

$$\frac{bd(c+dx)\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{bd^2\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{a(c+dx)^3}{3d}+\frac{b(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Tanh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) - (b*(c + d*x)^3)/(3*d) + (b*(c + d*x)^2*Log[1 + E^(2
*(e + f*x))])/f + (b*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (b*d^2
*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2(a+b\tanh(e+fx))\,dx &= \int \left(a(c+dx)^2 + b(c+dx)^2\tanh(e+fx)\right)dx \\
&= \frac{a(c+dx)^3}{3d} + b\int (c+dx)^2\tanh(e+fx)\,dx \\
&= \frac{a(c+dx)^3}{3d} - \frac{b(c+dx)^3}{3d} + (2b)\int \frac{e^{2(e+fx)}(c+dx)^2}{1+e^{2(e+fx)}}\,dx \\
&= \frac{a(c+dx)^3}{3d} - \frac{b(c+dx)^3}{3d} + \frac{b(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} - \frac{(2bd)\int (c+dx)\log}{} \\
&= \frac{a(c+dx)^3}{3d} - \frac{b(c+dx)^3}{3d} + \frac{b(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{bd(c+dx)\mathrm{Li}_2\left(-e^{} \right.}{f^2} \\
&= \frac{a(c+dx)^3}{3d} - \frac{b(c+dx)^3}{3d} + \frac{b(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{bd(c+dx)\mathrm{Li}_2\left(-e^{} \right.}{f^2} \\
&= \frac{a(c+dx)^3}{3d} - \frac{b(c+dx)^3}{3d} + \frac{b(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{bd(c+dx)\mathrm{Li}_2\left(-e^{} \right.}{f^2}
\end{aligned}$$

Mathematica [A] time = 1.77, size = 147, normalized size = 1.43

$$\frac{1}{6}\left(2x\left(3c^2+3cdx+d^2x^2\right)(a+b\tanh(e)) + \frac{be^{2e}\left(-\frac{3d\left(e^{-2e}+1\right)\left(2f(c+dx)\mathrm{Li}_2\left(-e^{-2(e+fx)}\right)+d\mathrm{Li}_3\left(-e^{-2(e+fx)}\right)\right)}{f^3} + \frac{6\left(e^{-2e}+1\right)(c+dx)^2\log}{f}\right.}{e^{2e}+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Tanh[e + f*x]),x]
```

```
[Out] ((b*E^(2*e)*((4*(c + d*x)^3)/(d*E^(2*e)) + (6*(1 + E^(-2*e))*(c + d*x)^2*Lo
g[1 + E^(-2*(e + f*x))])/f - (3*d*(1 + E^(-2*e))*(2*f*(c + d*x)*PolyLog[2,
-E^(-2*(e + f*x))] + d*PolyLog[3, -E^(-2*(e + f*x))]))/f^3))/(1 + E^(2*e))
+ 2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*(a + b*Tanh[e]))/6
```

fricas [C] time = 0.77, size = 364, normalized size = 3.53

$$\frac{(a-b)d^2f^3x^3 + 3(a-b)cdf^3x^2 + 3(a-b)c^2f^3x - 6bd^2\mathrm{polylog}\left(3, i\cosh\left(fx+e\right) + i\sinh\left(fx+e\right)\right) - 6bd^2\mathrm{po}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*((a - b)*d^2*f^3*x^3 + 3*(a - b)*c*d*f^3*x^2 + 3*(a - b)*c^2*f^3*x - 6*
b*d^2*polylog(3, I*cosh(f*x + e) + I*sinh(f*x + e)) - 6*b*d^2*polylog(3, -I
*cosh(f*x + e) - I*sinh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(I*cosh(f*
x + e) + I*sinh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(-I*cosh(f*x + e)
```

```
- I*sinh(f*x + e)) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(cosh(f*x +
 e) + sinh(f*x + e) + I) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(cosh
(f*x + e) + sinh(f*x + e) - I) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e
^2 + 2*b*c*d*e*f)*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) + 3*(b*d^2*f^2
*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-I*cosh(f*x + e) - I*si
nh(f*x + e) + 1))/f^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\left(b\tanh\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*tanh(f*x + e) + a), x)
```

maple [B] time = 0.28, size = 272, normalized size = 2.64

$$\frac{a\,d^2x^3}{3}-\frac{b\,d^2x^3}{3}+acd\,x^2-bcd\,x^2+c^2ax+b\,c^2x+\frac{b\,c^2\ln\left(\mathrm{e}^{2fx+2e}+1\right)}{f}-\frac{2b\,c^2\ln\left(\mathrm{e}^{fx+e}\right)}{f}-\frac{2b\,d^2e^2\ln\left(\mathrm{e}^{fx+e}\right)}{f^3}+\frac{2b\,d^2e^2}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*tanh(f*x+e)),x)
```

```
[Out] 1/3*a*d^2*x^3-1/3*b*d^2*x^3+a*c*d*x^2-b*c*d*x^2+c^2*a*x+b*c^2*x+1/f*b*c^2*l
n(exp(2*f*x+2*e)+1)-2/f*b*c^2*ln(exp(f*x+e))-2/f^3*b*d^2*e^2*ln(exp(f*x+e))
+2/f^2*b*d^2*e^2*x+4/3/f^3*b*d^2*e^3+1/f*b*d^2*ln(exp(2*f*x+2*e)+1)*x^2+1/f
^2*b*d^2*polylog(2,-exp(2*f*x+2*e))*x-1/2*b*d^2*polylog(3,-exp(2*f*x+2*e))/
f^3+4/f^2*b*c*d*e*ln(exp(f*x+e))-4/f*b*d*c*e*x-2/f^2*b*d*c*e^2+2/f*b*c*d*ln
(exp(2*f*x+2*e)+1)*x+1/f^2*b*c*d*polylog(2,-exp(2*f*x+2*e))
```

maxima [A] time = 0.40, size = 179, normalized size = 1.74

$$\frac{1}{3}\,ad^2x^3+\frac{1}{3}\,bd^2x^3+acdx^2+bcdx^2+ac^2x+\frac{bc^2\log\left(\cosh\left(fx+e\right)\right)}{f}+\frac{\left(2\,fx\log\left(e^{\left(2\,fx+2\,e\right)}+1\right)+\mathrm{Li}_2\left(-e^{\left(2\,fx+2\,e\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3 + 1/3*b*d^2*x^3 + a*c*d*x^2 + b*c*d*x^2 + a*c^2*x + b*c^2*log
(cosh(f*x + e))/f + (2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e
)))*b*c*d/f^2 + 1/2*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2
*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*b*d^2/f^3 - 2/3*(b*d^2*f^3*x^3
 + 3*b*c*d*f^3*x^2)/f^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\tanh\left(e+f\,x\right)\right)\,\left(c+d\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))*(c + d*x)^2,x)
```

```
[Out] int((a + b*tanh(e + f*x))*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh\left(e+fx\right)\right)\,(c+dx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*tanh(f*x+e)),x)
[Out] Integral((a + b*tanh(e + f*x))*(c + d*x)**2, x)
```

3.55 $\int (c+dx)(a+b\tanh(e+fx))\,dx$

Optimal. Leaf size=75

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b(c+dx)^2}{2d}+\frac{bd\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}$$

```
[Out] 1/2*a*(d*x+c)^2/d-1/2*b*(d*x+c)^2/d+b*(d*x+c)*ln(1+exp(2*f*x+2*e))/f+1/2*b*
d*polylog(2,-exp(2*f*x+2*e))/f^2
```

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3722, 3718, 2190, 2279, 2391}

$$\frac{bd\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Tanh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (b*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Log[1 + E^(2*(
e + f*x))])/f + (b*d*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+b\tanh(e+fx))\,dx &= \int (a(c+dx)+b(c+dx)\tanh(e+fx))\,dx \\ &= \frac{a(c+dx)^2}{2d} + b\int (c+dx)\tanh(e+fx)\,dx \\ &= \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)^2}{2d} + (2b)\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}\,dx \\ &= \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)^2}{2d} + \frac{b(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} - \frac{(bd)\int \log\left(1+e^{2(e+\right.}{f} \\ &= \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)^2}{2d} + \frac{b(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} - \frac{(bd)\,\mathrm{Subst}\left(\int \frac{\log(1+}{x}\right.}{2f^2} \\ &= \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)^2}{2d} + \frac{b(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{bd\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}\end{aligned}$$

Mathematica [C] time = 4.26, size = 226, normalized size = 3.01

$$acx+\frac{1}{2}adx^2+\frac{bc\log(\cosh(e+fx))}{f}+bd\mathrm{csch}(e)\mathrm{sech}(e)\left(f^2x^2e^{-\tanh^{-1}(\coth(e))} - \frac{i\coth(e)\left(i\mathrm{Li}_2\left(e^{2i\left(ifx+i\tanh^{-1}(\coth(e))\right)}\right)-fx\left(-\pi+\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*(a + b*Tanh[e + f*x]),x]
```

```
[Out] a*c*x + (a*d*x^2)/2 + (b*c*Log[Cosh[e + f*x]])/f + (b*d*Csch[e]*((f^2*x^2)/
E^ArcTanh[Coth[e]] - (I*Coth[e]*(-(f*x*(-Pi + (2*I)*ArcTanh[Coth[e]])) - Pi
*Log[1 + E^(2*f*x)] - 2*(I*f*x + I*ArcTanh[Coth[e]])*Log[1 - E^((2*I)*(I*f*
x + I*ArcTanh[Coth[e]]))] + Pi*Log[Cosh[f*x]] + (2*I)*ArcTanh[Coth[e]]*Log[
I*Sinh[f*x + ArcTanh[Coth[e]]]] + I*PolyLog[2, E^((2*I)*(I*f*x + I*ArcTanh[
Coth[e]]))]))/Sqrt[1 - Coth[e]^2])*Sech[e])/(2*f^2*Sqrt[Csch[e]^2*(-Cosh[e]
^2 + Sinh[e]^2)]) + (b*d*x^2*Tanh[e])/2
```

fricas [C] time = 0.53, size = 191, normalized size = 2.55

$$\frac{(a-b)df^2x^2+2\,(a-b)cf^2x+2\,bd\mathrm{Li}_2\left(i\cosh\left(fx+e\right)+i\sinh\left(fx+e\right)\right)+2\,bd\mathrm{Li}_2\left(-i\cosh\left(fx+e\right)-i\sinh\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((a - b)*d*f^2*x^2 + 2*(a - b)*c*f^2*x + 2*b*d*dilog(I*cosh(f*x + e) +
I*sinh(f*x + e)) + 2*b*d*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 2*(b*d
*e - b*c*f)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 2*(b*d*e - b*c*f)*log(
cosh(f*x + e) + sinh(f*x + e) - I) + 2*(b*d*f*x + b*d*e)*log(I*cosh(f*x + e
) + I*sinh(f*x + e) + 1) + 2*(b*d*f*x + b*d*e)*log(-I*cosh(f*x + e) - I*sin
h(f*x + e) + 1))/f^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\tanh\left(fx+e\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*tanh(f*x + e) + a), x)
```

maple [A] time = 0.29, size = 129, normalized size = 1.72

$$\frac{ad\,x^2}{2}-\frac{bd\,x^2}{2}+acx+bcx+\frac{bc\ln\left(\mathrm{e}^{2fx+2e}+1\right)}{f}-\frac{2bc\ln\left(\mathrm{e}^{fx+e}\right)}{f}-\frac{2bdex}{f}-\frac{bd\,e^2}{f^2}+\frac{bd\ln\left(\mathrm{e}^{2fx+2e}+1\right)x}{f}+\frac{bd\,\mathrm{polylog}\left(}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*tanh(f*x+e)),x)
```

```
[Out] 1/2*a*d*x^2-1/2*b*d*x^2+a*c*x+b*c*x+1/f*b*c*ln(exp(2*f*x+2*e)+1)-2/f*b*c*ln
(exp(f*x+e))-2/f*b*d*e*x-1/f^2*b*d*e^2+1/f*b*d*ln(exp(2*f*x+2*e)+1)*x+1/2*b
*d*polylog(2,-exp(2*f*x+2*e))/f^2+2/f^2*b*d*e*ln(exp(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2-4\int\frac{x}{e^{\left(2\,fx+2\,e\right)}+1}\,dx\right)bd+acx+\frac{bc\log\left(\cosh\left(fx+e\right)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + 1/2*(x^2 - 4*integrate(x/(e^(2*f*x + 2*e) + 1), x))*b*d + a*c
*x + b*c*log(cosh(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a+b\tanh\left(e+f\,x\right)\right)\,\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))*(c + d*x),x)
```

```
[Out] int((a + b*tanh(e + f*x))*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\tanh\left(e+fx\right)\right)\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e)),x)
```

```
[Out] Integral((a + b*tanh(e + f*x))*(c + d*x), x)
```

3.56 $\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{a+b\tanh(e+fx)}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*tanh(f*x+e))/(d*x+c),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Tanh[e + f*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*Tanh[e + f*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx = \int \frac{a+b\tanh(e+fx)}{c+dx}\,dx$$

Mathematica [A] time = 4.55, size = 0, normalized size = 0.00

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Tanh[e + f*x])/(c + d*x),x]
```

```
[Out] Integrate[(a + b*Tanh[e + f*x])/(c + d*x), x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\tanh\left(fx+e\right)+a}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*tanh(f*x + e) + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\tanh\left(fx+e\right)+a}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*tanh(f*x + e) + a)/(d*x + c), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{a + b \tanh\left(fx + e\right)}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(f*x+e))/(d*x+c),x)
```

```
[Out] int((a+b*tanh(f*x+e))/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{\log\left(dx + c\right)}{d} - 2\int \frac{1}{dx + \left(dxe^{(2e)} + ce^{(2e)}\right)e^{\left(2fx\right)} + c}\, dx\right) + \frac{a\log\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] b*(log(d*x + c)/d - 2*integrate(1/(d*x + (d*x*e^(2*e) + c*e^(2*e))*e^(2*f*x
) + c), x)) + a*log(d*x + c)/d
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a + b \tanh\left(e + f\,x\right)}{c + d\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + b*tanh(e + f*x))/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \tanh\left(e + fx\right)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*tanh(e + f*x))/(c + d*x), x)
```

3.57 $\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{a+b\tanh(e+fx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Tanh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Tanh[e + f*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 11.40, size = 0, normalized size = 0.00

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Tanh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Tanh[e + f*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\tanh\left(fx+e\right)+a}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*tanh(f*x + e) + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\tanh\left(fx+e\right)+a}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*tanh(f*x + e) + a)/(d*x + c)^2, x)
```

maple [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{a + b \tanh\left(fx + e\right)}{(dx + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

```
[Out] int((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-b\left(\frac{1}{d^2x + cd} + 2\int \frac{1}{d^2x^2 + 2\,cdx + c^2 + \left(d^2x^2e^{(2\,e)} + 2\,cdxe^{(2\,e)} + c^2e^{(2\,e)}\right)e^{\left(2\,fx\right)}}\, dx\right) - \frac{a}{d^2x + cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -b*(1/(d^2*x + c*d) + 2*integrate(1/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2*e^(
2*e) + 2*c*d*x*e^(2*e) + c^2*e^(2*e))*e^(2*f*x)), x)) - a/(d^2*x + c*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{a + b \tanh\left(e + f\,x\right)}{(c + d\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + b*tanh(e + f*x))/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \tanh\left(e + fx\right)}{(c + dx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*tanh(e + f*x))/(c + d*x)**2, x)
```

3.58 $\int (c+dx)^3 (a+b\tanh(e+fx))^2\,dx$

Optimal. Leaf size=277

$$\frac{a^2(c+dx)^4}{4d}-\frac{3abd^2(c+dx)\mathrm{Li}_3\left(-e^{2(e+fx)}\right)}{f^3}+\frac{3abd(c+dx)^2\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{ab(c+$$

```
[Out] -b^2*(d*x+c)^3/f+1/4*a^2*(d*x+c)^4/d-1/2*a*b*(d*x+c)^4/d+1/4*b^2*(d*x+c)^4/
d+3*b^2*d*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f^2+2*a*b*(d*x+c)^3*ln(1+exp(2*f*x
+2*e))/f+3*b^2*d^2*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^3+3*a*b*d*(d*x+c)^2
*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*b^2*d^3*polylog(3,-exp(2*f*x+2*e))/f^4-
3*a*b*d^2*(d*x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+3/2*a*b*d^3*polylog(4,-exp
(2*f*x+2*e))/f^4-b^2*(d*x+c)^3*tanh(f*x+e)/f
```

Rubi [A] time = 0.55, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3722, 3718, 2190, 2531, 6609, 2282, 6589, 3720, 32}

$$-\frac{3abd^2(c+dx)\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3}+\frac{3abd(c+dx)^2\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{3abd^3\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{2f^4}+\frac{3b^2d$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] -((b^2*(c + d*x)^3)/f) + (a^2*(c + d*x)^4)/(4*d) - (a*b*(c + d*x)^4)/(2*d)
+ (b^2*(c + d*x)^4)/(4*d) + (3*b^2*d*(c + d*x)^2*Log[1 + E^(2*(e + f*x))])/
f^2 + (2*a*b*(c + d*x)^3*Log[1 + E^(2*(e + f*x))])/f + (3*b^2*d^2*(c + d*x)
*PolyLog[2, -E^(2*(e + f*x))])/f^3 + (3*a*b*d*(c + d*x)^2*PolyLog[2, -E^(2*
(e + f*x))])/f^2 - (3*b^2*d^3*PolyLog[3, -E^(2*(e + f*x))])/(2*f^4) - (3*a*
b*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/f^3 + (3*a*b*d^3*PolyLog[4, -
E^(2*(e + f*x))])/(2*f^4) - (b^2*(c + d*x)^3*Tanh[e + f*x])/f
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \int \left(a^2(c+dx)^3 + 2ab(c+dx)^3\tanh(e+fx) + b^2(c+dx)^3\tanh^2(e+fx)\right)$$

$$= \frac{a^2(c+dx)^4}{4d} + (2ab)\int (c+dx)^3\tanh(e+fx)\,dx + b^2\int (c+dx)^3\tanh^2(e+$$

$$= \frac{a^2(c+dx)^4}{4d} - \frac{ab(c+dx)^4}{2d} - \frac{b^2(c+dx)^3\tanh(e+fx)}{f} + (4ab)\int \frac{e^{2(e+fx)}(c$$

$$= -\frac{b^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{ab(c+dx)^4}{2d} + \frac{b^2(c+dx)^4}{4d} + \frac{2ab(c+dx)^3\,\mathrm{lo}}{f}$$

$$= -\frac{b^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{ab(c+dx)^4}{2d} + \frac{b^2(c+dx)^4}{4d} + \frac{3b^2d(c+dx)^2\,\mathrm{l}}{f}$$

$$= -\frac{b^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{ab(c+dx)^4}{2d} + \frac{b^2(c+dx)^4}{4d} + \frac{3b^2d(c+dx)^2\,\mathrm{l}}{f}$$

$$= -\frac{b^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{ab(c+dx)^4}{2d} + \frac{b^2(c+dx)^4}{4d} + \frac{3b^2d(c+dx)^2\,\mathrm{l}}{f}$$

$$= -\frac{b^2(c+dx)^3}{f} + \frac{a^2(c+dx)^4}{4d} - \frac{ab(c+dx)^4}{2d} + \frac{b^2(c+dx)^4}{4d} + \frac{3b^2d(c+dx)^2\,\mathrm{l}}{f}$$

Mathematica [A] time = 7.90, size = 508, normalized size = 1.83

$$\frac{1}{8}\left(\frac{\operatorname{sech}(e)\operatorname{sech}(e+fx)\left(fx\left(a^2+b^2\right)\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\cosh(2e+fx) + fx\left(a^2+b^2\right)\left(4c^3+6c^2dx+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] (4*b*((4*b*(c + d*x)^3)/((1 + E^(2*e))*f) + (2*a*(c + d*x)^4)/(d*(1 + E^(2*
e))) + (12*c*d*(b*d + a*c*f)*x*Log[1 + E^(-2*(e + f*x))])/f^2 + (6*d^2*(b*d
 + 2*a*c*f)*x^2*Log[1 + E^(-2*(e + f*x))])/f^2 + (4*a*d^3*x^3*Log[1 + E^(-2
*(e + f*x))])/f - (2*c^2*(3*b*d + 2*a*c*f)*(2*f*x - Log[1 + E^(2*(e + f*x))
]))/f^2 - (6*c*d*(b*d + a*c*f)*PolyLog[2, -E^(-2*(e + f*x))])/f^3 - (3*d^2*
(b*d + 2*a*c*f)*(2*f*x*PolyLog[2, -E^(-2*(e + f*x))] + PolyLog[3, -E^(-2*(e
 + f*x))]))/f^4 - (3*a*d^3*(2*f^2*x^2*PolyLog[2, -E^(-2*(e + f*x))] + 2*f*x
*PolyLog[3, -E^(-2*(e + f*x))] + PolyLog[4, -E^(-2*(e + f*x))]))/f^4) + (Se
ch[e]*Sech[e + f*x]*((a^2 + b^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3
*x^3)*Cosh[f*x] + (a^2 + b^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^
3)*Cosh[2*e + f*x] - 2*b*(4*b*(c + d*x)^3 + a*f*x*(4*c^3 + 6*c^2*d*x + 4*c*
d^2*x^2 + d^3*x^3))*Sinh[f*x] + 2*a*b*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2
 + d^3*x^3)*Sinh[2*e + f*x]))/f)/8
```

fricas [C] time = 1.02, size = 3744, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^2 - 2*a*b + b^2)*d^3*f^4*x^4 + 4*(a^2 - 2*a*b + b^2)*c*d^2*f^4*x^3
 + 6*(a^2 - 2*a*b + b^2)*c^2*d*f^4*x^2 + 4*a*b*d^3*e^4 + 4*(a^2 - 2*a*b + b^
2)*c^3*f^4*x - 8*b^2*d^3*e^3 - 8*(2*a*b*c^3*e - b^2*c^3)*f^3 + 24*(a*b*c^2*
d*e^2 - b^2*c^2*d*e)*f^2 + ((a^2 - 2*a*b + b^2)*d^3*f^4*x^4 + 4*a*b*d^3*e^4
 - 16*a*b*c^3*e*f^3 - 8*b^2*d^3*e^3 - 4*(2*b^2*d^3*f^3 - (a^2 - 2*a*b + b^2
```

```
)*c*d^2*f^4)*x^3 + 24*(a*b*c^2*d*e^2 - b^2*c^2*d*e)*f^2 - 6*(4*b^2*c*d^2*f^
3 - (a^2 - 2*a*b + b^2)*c^2*d*f^4)*x^2 - 8*(2*a*b*c*d^2*e^3 - 3*b^2*c*d^2*e
^2)*f - 4*(6*b^2*c^2*d*f^3 - (a^2 - 2*a*b + b^2)*c^3*f^4)*x)*cosh(f*x + e)^
2 + 2*((a^2 - 2*a*b + b^2)*d^3*f^4*x^4 + 4*a*b*d^3*e^4 - 16*a*b*c^3*e*f^3 -
 8*b^2*d^3*e^3 - 4*(2*b^2*d^3*f^3 - (a^2 - 2*a*b + b^2)*c*d^2*f^4)*x^3 + 24
*(a*b*c^2*d*e^2 - b^2*c^2*d*e)*f^2 - 6*(4*b^2*c*d^2*f^3 - (a^2 - 2*a*b + b^
2)*c^2*d*f^4)*x^2 - 8*(2*a*b*c*d^2*e^3 - 3*b^2*c*d^2*e^2)*f - 4*(6*b^2*c^2*
d*f^3 - (a^2 - 2*a*b + b^2)*c^3*f^4)*x)*cosh(f*x + e)*sinh(f*x + e) + ((a^2
 - 2*a*b + b^2)*d^3*f^4*x^4 + 4*a*b*d^3*e^4 - 16*a*b*c^3*e*f^3 - 8*b^2*d^3*
e^3 - 4*(2*b^2*d^3*f^3 - (a^2 - 2*a*b + b^2)*c*d^2*f^4)*x^3 + 24*(a*b*c^2*d
*e^2 - b^2*c^2*d*e)*f^2 - 6*(4*b^2*c*d^2*f^3 - (a^2 - 2*a*b + b^2)*c^2*d*f^
4)*x^2 - 8*(2*a*b*c*d^2*e^3 - 3*b^2*c*d^2*e^2)*f - 4*(6*b^2*c^2*d*f^3 - (a^
2 - 2*a*b + b^2)*c^3*f^4)*x)*sinh(f*x + e)^2 - 8*(2*a*b*c*d^2*e^3 - 3*b^2*c
*d^2*e^2)*f + 24*(a*b*d^3*f^2*x^2 + a*b*c^2*d*f^2 + b^2*c*d^2*f + (a*b*d^3*
f^2*x^2 + a*b*c^2*d*f^2 + b^2*c*d^2*f + (2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*co
sh(f*x + e)^2 + 2*(a*b*d^3*f^2*x^2 + a*b*c^2*d*f^2 + b^2*c*d^2*f + (2*a*b*c
*d^2*f^2 + b^2*d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) + (a*b*d^3*f^2*x^2 + a
*b*c^2*d*f^2 + b^2*c*d^2*f + (2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*sinh(f*x + e)
^2 + (2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*dilog(I*cosh(f*x + e) + I*sinh(f*x +
e)) + 24*(a*b*d^3*f^2*x^2 + a*b*c^2*d*f^2 + b^2*c*d^2*f + (a*b*d^3*f^2*x^2
+ a*b*c^2*d*f^2 + b^2*c*d^2*f + (2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*cosh(f*x +
 e)^2 + 2*(a*b*d^3*f^2*x^2 + a*b*c^2*d*f^2 + b^2*c*d^2*f + (2*a*b*c*d^2*f^2
 + b^2*d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) + (a*b*d^3*f^2*x^2 + a*b*c^2*d
*f^2 + b^2*c*d^2*f + (2*a*b*c*d^2*f^2 + b^2*d^3*f)*x)*sinh(f*x + e)^2 + (2*
a*b*c*d^2*f^2 + b^2*d^3*f)*x)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 4
*(2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c^
2*d)*f^2 + (2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*
e - b^2*c^2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f)*cosh(f*x + e)^2 + 2
*(2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c^
2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f)*cosh(f*x + e)*sinh(f*x + e) +
 (2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c^
2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f)*sinh(f*x + e)^2 - 6*(a*b*c*d^
2*e^2 - b^2*c*d^2*e)*f)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 4*(2*a*b*d
^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c^2*d)*f^2
+ (2*a*b*d^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c
^2*d)*f^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f)*cosh(f*x + e)^2 + 2*(2*a*b*d
^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c^2*d)*f^2
- 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f)*cosh(f*x + e)*sinh(f*x + e) + (2*a*b*d
^3*e^3 - 2*a*b*c^3*f^3 - 3*b^2*d^3*e^2 + 3*(2*a*b*c^2*d*e - b^2*c^2*d)*f^2
- 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f)*sinh(f*x + e)^2 - 6*(a*b*c*d^2*e^2 - b
^2*c*d^2*e)*f)*log(cosh(f*x + e) + sinh(f*x + e) - I) + 4*(2*a*b*d^3*f^3*x^
3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3
+ b^2*d^3*f^2)*x^2 + (2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2
 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 - 6*(a*b*c*d^2*e^2
 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*cosh(f*x + e)^2 +
2*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 +
3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f +
 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*cosh(f*x + e)*sinh(f*x + e) + (2*a*b*
d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*
c*d^2*f^3 + b^2*d^3*f^2)*x^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c
^2*d*f^3 + b^2*c*d^2*f^2)*x)*sinh(f*x + e)^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2
*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*log(I*cosh(f*x + e) + I*sinh(f
*x + e) + 1) + 4*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2 - 3
*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 + (2*a*b*d^3*f^3*x^3 +
 2*a*b*d^3*e^3 + 6*a*b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b
^2*d^3*f^2)*x^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^
2*c*d^2*f^2)*x)*cosh(f*x + e)^2 + 2*(2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*
a*b*c^2*d*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 - 6
*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*cos
```

```
h(f*x + e)*sinh(f*x + e) + (2*a*b*d^3*f^3*x^3 + 2*a*b*d^3*e^3 + 6*a*b*c^2*d
*e*f^2 - 3*b^2*d^3*e^2 + 3*(2*a*b*c*d^2*f^3 + b^2*d^3*f^2)*x^2 - 6*(a*b*c*d
^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)*x)*sinh(f*x + e
)^2 - 6*(a*b*c*d^2*e^2 - b^2*c*d^2*e)*f + 6*(a*b*c^2*d*f^3 + b^2*c*d^2*f^2)
*x)*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1) + 48*(a*b*d^3*cosh(f*x + e)
^2 + 2*a*b*d^3*cosh(f*x + e)*sinh(f*x + e) + a*b*d^3*sinh(f*x + e)^2 + a*b*
d^3)*polylog(4, I*cosh(f*x + e) + I*sinh(f*x + e)) + 48*(a*b*d^3*cosh(f*x +
 e)^2 + 2*a*b*d^3*cosh(f*x + e)*sinh(f*x + e) + a*b*d^3*sinh(f*x + e)^2 + a
*b*d^3)*polylog(4, -I*cosh(f*x + e) - I*sinh(f*x + e)) - 24*(2*a*b*d^3*f*x
+ 2*a*b*c*d^2*f + b^2*d^3 + (2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*cosh(
f*x + e)^2 + 2*(2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*cosh(f*x + e)*sinh
(f*x + e) + (2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*sinh(f*x + e)^2)*poly
log(3, I*cosh(f*x + e) + I*sinh(f*x + e)) - 24*(2*a*b*d^3*f*x + 2*a*b*c*d^2
*f + b^2*d^3 + (2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*cosh(f*x + e)^2 +
2*(2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*cosh(f*x + e)*sinh(f*x + e) + (
2*a*b*d^3*f*x + 2*a*b*c*d^2*f + b^2*d^3)*sinh(f*x + e)^2)*polylog(3, -I*cos
h(f*x + e) - I*sinh(f*x + e)))/(f^4*cosh(f*x + e)^2 + 2*f^4*cosh(f*x + e)*s
inh(f*x + e) + f^4*sinh(f*x + e)^2 + f^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \left(b \tanh\left(fx+e\right)+a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*tanh(f*x + e) + a)^2, x)
```

maple [B] time = 0.53, size = 873, normalized size = 3.15

$$\frac{6b^2d^3e^2x}{f^3}-\frac{6b^2d^2c\,x^2}{f}-\frac{6b^2d^2c\,e^2}{f^3}+b^2c^3x+\frac{12bac\,d^2e^2x}{f^2}-\frac{12ba\,c^2dex}{f}+\frac{12ba\,c^2de\ln\left(e^{fx+e}\right)}{f^2}+\frac{6bac\,d^2\ln\left(e^{2fx+2e}+1\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*tanh(f*x+e))^2,x)
```

```
[Out] 6/f^3*b^2*d^3*e^2*x-6/f*b^2*d^2*c*x^2-6/f^3*b^2*d^2*c*e^2+b^2*c^3*x+3/2*a*b
*d^3*polylog(4,-exp(2*f*x+2*e))/f^4-3/2*b^2*d^3*polylog(3,-exp(2*f*x+2*e))/
f^4+12/f^2*b*a*c*d^2*e^2*x-12/f*b*a*c^2*d*e*x+12/f^2*b*a*c^2*d*e*ln(exp(f*x
+e))+6/f*b*a*c*d^2*ln(exp(2*f*x+2*e)+1)*x^2+6/f^2*b*a*c*d^2*polylog(2,-exp(
2*f*x+2*e))*x+1/4*a^2*d^3*x^4+b^2*c*d^2*x^3+a^2*c*d^2*x^3-2/f*b^2*d^3*x^3+4
/f^4*b^2*d^3*e^3+6/f*b*ln(exp(2*f*x+2*e)+1)*a*c^2*d*x-12/f^3*b*a*c*d^2*e^2*
ln(exp(f*x+e))-3/f^4*b*e^4*a*d^3+3/f^2*b^2*d^3*ln(exp(2*f*x+2*e)+1)*x^2+3/f
^3*b^2*d^3*polylog(2,-exp(2*f*x+2*e))*x-6/f^4*b^2*d^3*e^2*ln(exp(f*x+e))+3/
f^3*b^2*c*d^2*polylog(2,-exp(2*f*x+2*e))+2/f*b*a*c^3*ln(exp(2*f*x+2*e)+1)+3
/f^2*b^2*c^2*d*ln(exp(2*f*x+2*e)+1)-4/f*b*a*c^3*ln(exp(f*x+e))-6/f^2*b^2*c^
2*d*ln(exp(f*x+e))+2/f*b^2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(exp(2*f*x+2
*e)+1)-1/2*a*b*d^3*x^4+3/2*a^2*c^2*d*x^2+3/2*b^2*c^2*d*x^2+2*c^3*a*b*x-2*a*
b*c*d^2*x^3-3*a*b*c^2*d*x^2+1/4*b^2*d^3*x^4+c^3*a^2*x-6/f^2*b*a*c^2*d*e^2+8
/f^3*b*a*c*d^2*e^3-4/f^3*b*e^3*a*d^3*x-12/f^2*b^2*d^2*c*e*x+2/f*b*a*d^3*ln(
exp(2*f*x+2*e)+1)*x^3+3/f^2*b*a*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3/f^3*b*
a*d^3*polylog(3,-exp(2*f*x+2*e))*x+4/f^4*b*a*d^3*e^3*ln(exp(f*x+e))+3/f^2*b
*a*c^2*d*polylog(2,-exp(2*f*x+2*e))-3/f^3*b*a*c*d^2*polylog(3,-exp(2*f*x+2*
e))+6/f^2*b^2*c*d^2*ln(exp(2*f*x+2*e)+1)*x+12/f^3*b^2*c*d^2*e*ln(exp(f*x+e)
)
```

maxima [B] time = 0.43, size = 625, normalized size = 2.26

$$\frac{1}{4}a^2d^3x^4+a^2cd^2x^3+\frac{3}{2}a^2c^2dx^2+b^2c^3\left(x+\frac{e}{f}-\frac{2}{f\left(e^{(-2fx-2e)}+1\right)}\right)+a^2c^3x+\frac{3}{2}b^2c^2d\left(\frac{fx^2+\left(fx^2e^{(2e)}-4xe^{(2e)}\right)e^{(2}}{fe^{(2fx+2e)}+f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + b^2*c^3*(x + e/f - 2/
(f*(e^(-2*f*x - 2*e) + 1))) + a^2*c^3*x + 3/2*b^2*c^2*d*((f*x^2 + (f*x^2*e^
(2*e) - 4*x*e^(2*e))*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 2*log((e^(2*f*x +
 2*e) + 1)*e^(-2*e))/f^2) + 2*a*b*c^3*log(cosh(f*x + e))/f + 2/3*(4*f^3*x^3
*log(e^(2*f*x + 2*e) + 1) + 6*f^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*polyl
og(3, -e^(2*f*x + 2*e)) + 3*polylog(4, -e^(2*f*x + 2*e)))*a*b*d^3/f^4 + 1/4
*((2*a*b*d^3*f + b^2*d^3*f)*x^4 + 4*(2*a*b*c*d^2*f + (c*d^2*f + 2*d^3)*b^2)
*x^3 + 12*(a*b*c^2*d*f + 2*b^2*c*d^2)*x^2 + (12*a*b*c^2*d*f*x^2*e^(2*e) + (
2*a*b*d^3*f*e^(2*e) + b^2*d^3*f*e^(2*e))*x^4 + 4*(2*a*b*c*d^2*f*e^(2*e) + b
^2*c*d^2*f*e^(2*e))*x^3)*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 3*(a*b*c^2*d*
f + b^2*c*d^2)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f
^3 + 3/2*(2*a*b*c*d^2*f + b^2*d^3)*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*
f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))/f^4 - (a*b*d^3*
f^4*x^4 + 2*(2*a*b*c*d^2*f + b^2*d^3)*f^3*x^3 + 6*(a*b*c^2*d*f^2 + b^2*c*d^
2*f)*f^2*x^2)/f^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\tanh\left(e+fx\right)\right)^2(c+dx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^2*(c + d*x)^3,x)
```

```
[Out] int((a + b*tanh(e + f*x))^2*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh\left(e+fx\right)\right)^2(c+dx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*tanh(f*x+e))**2,x)
```

```
[Out] Integral((a + b*tanh(e + f*x))**2*(c + d*x)**3, x)
```

3.59 $\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx$

Optimal. Leaf size=211

$$\frac{a^2(c+dx)^3}{3d}+\frac{2abd(c+dx)\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{2ab(c+dx)^3}{3d}-\frac{abd^2\mathrm{Li}_3\left(-e^{2(e+fx)}\right)}{f^3}+\frac{2b^2}{}$$

```
[Out] -b^2*(d*x+c)^2/f+1/3*a^2*(d*x+c)^3/d-2/3*a*b*(d*x+c)^3/d+1/3*b^2*(d*x+c)^3/
d+2*b^2*d*(d*x+c)*ln(1+exp(2*f*x+2*e))/f^2+2*a*b*(d*x+c)^2*ln(1+exp(2*f*x+2
*e))/f+b^2*d^2*polylog(2,-exp(2*f*x+2*e))/f^3+2*a*b*d*(d*x+c)*polylog(2,-ex
p(2*f*x+2*e))/f^2-a*b*d^2*polylog(3,-exp(2*f*x+2*e))/f^3-b^2*(d*x+c)^2*tanh
(f*x+e)/f
```

Rubi [A] time = 0.40, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3722, 3718, 2190, 2531, 2282, 6589, 3720, 2279, 2391, 32}

$$\frac{2abd(c+dx)\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{abd^2\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3}+\frac{b^2d^2\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}+\frac{a^2(c+dx)^3}{3d}+\frac{2ab(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] -((b^2*(c + d*x)^2)/f) + (a^2*(c + d*x)^3)/(3*d) - (2*a*b*(c + d*x)^3)/(3*d
) + (b^2*(c + d*x)^3)/(3*d) + (2*b^2*d*(c + d*x)*Log[1 + E^(2*(e + f*x))])/
f^2 + (2*a*b*(c + d*x)^2*Log[1 + E^(2*(e + f*x))])/f + (b^2*d^2*PolyLog[2,
-E^(2*(e + f*x))])/f^3 + (2*a*b*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f
^2 - (a*b*d^2*PolyLog[3, -E^(2*(e + f*x))])/f^3 - (b^2*(c + d*x)^2*Tanh[e +
 f*x])/f
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \int \left(a^2(c+dx)^2 + 2ab(c+dx)^2\tanh(e+fx) + b^2(c+dx)^2\tanh^2(e+fx)\right)$$

$$= \frac{a^2(c+dx)^3}{3d} + (2ab)\int (c+dx)^2\tanh(e+fx)\,dx + b^2\int (c+dx)^2\tanh^2(e+$$

$$= \frac{a^2(c+dx)^3}{3d} - \frac{2ab(c+dx)^3}{3d} - \frac{b^2(c+dx)^2\tanh(e+fx)}{f} + (4ab)\int \frac{e^{2(e+fx)}(}{1+e}$$

$$= -\frac{b^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2ab(c+dx)^3}{3d} + \frac{b^2(c+dx)^3}{3d} + \frac{2ab(c+dx)^2}{}$$

$$= -\frac{b^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2ab(c+dx)^3}{3d} + \frac{b^2(c+dx)^3}{3d} + \frac{2b^2d(c+dx)}{}$$

$$= -\frac{b^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2ab(c+dx)^3}{3d} + \frac{b^2(c+dx)^3}{3d} + \frac{2b^2d(c+dx)}{}$$

$$= -\frac{b^2(c+dx)^2}{f} + \frac{a^2(c+dx)^3}{3d} - \frac{2ab(c+dx)^3}{3d} + \frac{b^2(c+dx)^3}{3d} + \frac{2b^2d(c+dx)}{}$$

Mathematica [A] time = 5.76, size = 232, normalized size = 1.10

$$\frac{1}{3}\left(x\left(3c^2+3cdx+d^2x^2\right)\left(a^2+2ab\tanh(e)+b^2\right) + \frac{b\left(2f\left(\frac{fx\left(2af\left(-3c^2e^{2e}+3cdx+d^2x^2\right)+3bd\left(dx-2ce^{2e}\right)\right)}{e^{2e}+1} + 3dx\log\left(e^{-2(e+fx}\right.\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] ((b*(2*f*((f*x*(3*b*d*(-2*c*E^(2*e) + d*x) + 2*a*f*(-3*c^2*E^(2*e) + 3*c*d*
x + d^2*x^2)))/(1 + E^(2*e)) + 3*d*x*(b*d + a*f*(2*c + d*x))*Log[1 + E^(-2*
(e + f*x))] + 3*c*(b*d + a*c*f)*Log[1 + E^(2*(e + f*x))]) - 3*d*(b*d + 2*a*
f*(c + d*x))*PolyLog[2, -E^(-2*(e + f*x))] - 3*a*d^2*PolyLog[3, -E^(-2*(e +
 f*x))]))/f^3 - (3*b^2*(c + d*x)^2*Sech[e]*Sech[e + f*x]*Sinh[f*x])/f + x*(
3*c^2 + 3*c*d*x + d^2*x^2)*(a^2 + b^2 + 2*a*b*Tanh[e]))/3
```

fricas [C] time = 0.59, size = 2123, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 + 3*(a^2 - 2*a*b + b^2)*c*d*f^3*x^2 -
4*a*b*d^2*e^3 + 3*(a^2 - 2*a*b + b^2)*c^2*f^3*x + 6*b^2*d^2*e^2 - 6*(2*a*b*
c^2*e - b^2*c^2)*f^2 + ((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3 - 1
2*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2)*c*
d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2 - 2*a
*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)^2 + 2*((a^2 - 2*a*b + b^2)*d^2*f^3*x^3
- 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^
2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c
*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)*sinh(f*x + e) + ((a^
2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 + 6*b^2*d^2
*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^
2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f^3)*x)*sinh(
f*x + e)^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f + 6*(2*a*b*d^2*f*x + 2*a*b*c*d*
```

```
f + b^2*d^2 + (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)^2 + 2*(
2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)*sinh(f*x + e) + (2*a*b
*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*sinh(f*x + e)^2)*dilog(I*cosh(f*x + e) +
I*sinh(f*x + e)) + 6*(2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2 + (2*a*b*d^2*f*
x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)^2 + 2*(2*a*b*d^2*f*x + 2*a*b*c*d*f
 + b^2*d^2)*cosh(f*x + e)*sinh(f*x + e) + (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^
2*d^2)*sinh(f*x + e)^2)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) + 6*(a*b*
d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e + (a*b*d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e
- (2*a*b*c*d*e - b^2*c*d)*f)*cosh(f*x + e)^2 + 2*(a*b*d^2*e^2 + a*b*c^2*f^2
 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*cosh(f*x + e)*sinh(f*x + e) + (a*
b*d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*sinh(f*x +
 e)^2 - (2*a*b*c*d*e - b^2*c*d)*f)*log(cosh(f*x + e) + sinh(f*x + e) + I) +
 6*(a*b*d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e + (a*b*d^2*e^2 + a*b*c^2*f^2 - b^
2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*cosh(f*x + e)^2 + 2*(a*b*d^2*e^2 + a*b
*c^2*f^2 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*cosh(f*x + e)*sinh(f*x +
e) + (a*b*d^2*e^2 + a*b*c^2*f^2 - b^2*d^2*e - (2*a*b*c*d*e - b^2*c*d)*f)*si
nh(f*x + e)^2 - (2*a*b*c*d*e - b^2*c*d)*f)*log(cosh(f*x + e) + sinh(f*x + e
) - I) + 6*(a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*d^2*e + (a*
b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*d^2*e + (2*a*b*c*d*f^2 +
b^2*d^2*f)*x)*cosh(f*x + e)^2 + 2*(a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*
d*e*f + b^2*d^2*e + (2*a*b*c*d*f^2 + b^2*d^2*f)*x)*cosh(f*x + e)*sinh(f*x +
 e) + (a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*d^2*e + (2*a*b*c
*d*f^2 + b^2*d^2*f)*x)*sinh(f*x + e)^2 + (2*a*b*c*d*f^2 + b^2*d^2*f)*x)*log
(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) + 6*(a*b*d^2*f^2*x^2 - a*b*d^2*e^2
+ 2*a*b*c*d*e*f + b^2*d^2*e + (a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*
f + b^2*d^2*e + (2*a*b*c*d*f^2 + b^2*d^2*f)*x)*cosh(f*x + e)^2 + 2*(a*b*d^2
*f^2*x^2 - a*b*d^2*e^2 + 2*a*b*c*d*e*f + b^2*d^2*e + (2*a*b*c*d*f^2 + b^2*d
^2*f)*x)*cosh(f*x + e)*sinh(f*x + e) + (a*b*d^2*f^2*x^2 - a*b*d^2*e^2 + 2*a
*b*c*d*e*f + b^2*d^2*e + (2*a*b*c*d*f^2 + b^2*d^2*f)*x)*sinh(f*x + e)^2 + (
2*a*b*c*d*f^2 + b^2*d^2*f)*x)*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1) -
 12*(a*b*d^2*cosh(f*x + e)^2 + 2*a*b*d^2*cosh(f*x + e)*sinh(f*x + e) + a*b*
d^2*sinh(f*x + e)^2 + a*b*d^2)*polylog(3, I*cosh(f*x + e) + I*sinh(f*x + e)
) - 12*(a*b*d^2*cosh(f*x + e)^2 + 2*a*b*d^2*cosh(f*x + e)*sinh(f*x + e) + a
*b*d^2*sinh(f*x + e)^2 + a*b*d^2)*polylog(3, -I*cosh(f*x + e) - I*sinh(f*x
+ e)))/(f^3*cosh(f*x + e)^2 + 2*f^3*cosh(f*x + e)*sinh(f*x + e) + f^3*sinh(
f*x + e)^2 + f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \left(b \tanh\left(fx + e\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*tanh(f*x + e) + a)^2, x)
```

maple [B] time = 0.47, size = 510, normalized size = 2.42

$$-\frac{4bacd\,e^2}{f^2}+\frac{4ba\,d^2e^2x}{f^2}+\frac{2ba\,d^2\ln\left(\mathrm{e}^{2fx+2e}+1\right)x^2}{f}-\frac{4ba\,d^2e^2\ln\left(\mathrm{e}^{fx+e}\right)}{f^3}+\frac{2bacd\,\mathrm{polylog}\left(2,-\mathrm{e}^{2fx+2e}\right)}{f^2}+\frac{2ba\,d^2\,\mathrm{p}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*tanh(f*x+e))^2,x)
```

```
[Out] b^2*d^2*polylog(2,-exp(2*f*x+2*e))/f^3-4/f^2*b*a*c*d*e^2+4/f^2*b*a*d^2*e^2*
x+2/f^2*b*a*c*d*polylog(2,-exp(2*f*x+2*e))+2/f*b*a*d^2*ln(exp(2*f*x+2*e)+1)
*x^2+2/f^2*b*a*d^2*polylog(2,-exp(2*f*x+2*e))*x-4/f^3*b*a*d^2*e^2*ln(exp(f*
x+e))+b^2*c*d*x^2+a^2*c*d*x^2-8/f*b*a*c*d*e*x+4/f*b*ln(exp(2*f*x+2*e)+1)*a*
```

```
c*d*x+8/f^2*b*a*c*d*e*ln(exp(f*x+e))+1/3*a^2*d^2*x^3+1/3*b^2*d^2*x^3+c^2*a^
2*x+c^2*b^2*x-2/f*b^2*d^2*x^2-2/f^3*b^2*d^2*e^2-4/f^2*b^2*d^2*e*x+2/f*b^2*(
d^2*x^2+2*c*d*x+c^2)/(exp(2*f*x+2*e)+1)-2/3*a*b*d^2*x^3+2*c^2*a*b*x-a*b*d^2
*polylog(3,-exp(2*f*x+2*e))/f^3+8/3/f^3*b*a*d^2*e^3-4/f*b*a*c^2*ln(exp(f*x+
e))+2/f*b*a*c^2*ln(exp(2*f*x+2*e)+1)-4/f^2*b^2*c*d*ln(exp(f*x+e))+2/f^2*b^2
*c*d*ln(exp(2*f*x+2*e)+1)+4/f^3*b^2*d^2*e*ln(exp(f*x+e))+2/f^2*b^2*d^2*ln(e
xp(2*f*x+2*e)+1)*x-2*a*b*c*d*x^2
```

maxima [B] time = 0.43, size = 411, normalized size = 1.95

$$\frac{1}{3}a^2d^2x^3+a^2cdx^2+b^2c^2\left(x+\frac{e}{f}-\frac{2}{f\left(e^{\left(-2fx-2e\right)}+1\right)}\right)+a^2c^2x+b^2cd\left(\frac{fx^2+\left(fx^2e^{(2e)}-4xe^{(2e)}\right)e^{(2fx)}}{fe^{\left(2fx+2e\right)}+f}+\frac{2\log\left(\left(e^{(2f}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + b^2*c^2*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) +
 1))) + a^2*c^2*x + b^2*c*d*((f*x^2 + (f*x^2*e^(2*e) - 4*x*e^(2*e))*e^(2*f*
x))/(f*e^(2*f*x + 2*e) + f) + 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2) +
2*a*b*c^2*log(cosh(f*x + e))/f + (2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*
x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*a*b*d^2/f^3 + 1/3
*((2*a*b*d^2*f + b^2*d^2*f)*x^3 + 6*(a*b*c*d*f + b^2*d^2)*x^2 + (6*a*b*c*d*
f*x^2*e^(2*e) + (2*a*b*d^2*f*e^(2*e) + b^2*d^2*f*e^(2*e))*x^3)*e^(2*f*x))/(
f*e^(2*f*x + 2*e) + f) + (2*a*b*c*d*f + b^2*d^2)*(2*f*x*log(e^(2*f*x + 2*e)
 + 1) + dilog(-e^(2*f*x + 2*e)))/f^3 - 2/3*(2*a*b*d^2*f^3*x^3 + 3*(2*a*b*c*
d*f + b^2*d^2)*f^2*x^2)/f^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\tanh\left(e+fx\right)\right)^2(c+dx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^2*(c + d*x)^2,x)
```

```
[Out] int((a + b*tanh(e + f*x))^2*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh\left(e+fx\right)\right)^2(c+dx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*tanh(f*x+e))**2,x)
```

```
[Out] Integral((a + b*tanh(e + f*x))**2*(c + d*x)**2, x)
```

3.60 $\int (c+dx)(a+b\tanh(e+fx))^2\,dx$

Optimal. Leaf size=127

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{ab(c+dx)^2}{d}+\frac{abd\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)\tanh(e+fx)}{f}+b^2cx+\frac{b}{}$$

```
[Out] b^2*c*x+1/2*b^2*d*x^2+1/2*a^2*(d*x+c)^2/d-a*b*(d*x+c)^2/d+2*a*b*(d*x+c)*ln(
1+exp(2*f*x+2*e))/f+b^2*d*ln(cosh(f*x+e))/f^2+a*b*d*polylog(2,-exp(2*f*x+2*
e))/f^2-b^2*(d*x+c)*tanh(f*x+e)/f
```

Rubi [A] time = 0.18, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3722, 3718, 2190, 2279, 2391, 3720, 3475}

$$\frac{abd\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{ab(c+dx)^2}{d}-\frac{b^2(c+dx)\tanh(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] b^2*c*x + (b^2*d*x^2)/2 + (a^2*(c + d*x)^2)/(2*d) - (a*b*(c + d*x)^2)/d + (
2*a*b*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f + (b^2*d*Log[Cosh[e + f*x]])/f^
2 + (a*b*d*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (b^2*(c + d*x)*Tanh[e + f*x]
)/f
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+b\tanh(e+fx))^2\,dx &= \int \left(a^2(c+dx)+2ab(c+dx)\tanh(e+fx)+b^2(c+dx)\tanh^2(e+fx)\right)dx \\
&= \frac{a^2(c+dx)^2}{2d}+(2ab)\int (c+dx)\tanh(e+fx)\,dx+b^2\int (c+dx)\tanh^2(e+f \\
&= \frac{a^2(c+dx)^2}{2d}-\frac{ab(c+dx)^2}{d}-\frac{b^2(c+dx)\tanh(e+fx)}{f}+(4ab)\int \frac{e^{2(e+fx)}(c+}{1+e^{2(e+} \\
&= b^2cx+\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{ab(c+dx)^2}{d}+\frac{2ab(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} \\
&= b^2cx+\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{ab(c+dx)^2}{d}+\frac{2ab(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} \\
&= b^2cx+\frac{1}{2}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{ab(c+dx)^2}{d}+\frac{2ab(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f}
\end{aligned}$$

Mathematica [A] time = 2.45, size = 192, normalized size = 1.51

$$\frac{\cosh(e+fx)(a+b\tanh(e+fx))^2\left(\cosh(e+fx)\left(-\left((e+fx)\left(a^2(-2cf+de-dfx)-2abd(e+fx)+b^2(-2cf+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] (Cosh[e + f*x]*(Cosh[e + f*x]*(-((e + f*x)*(-2*a*b*d*(e + f*x) + a^2*(d*e -
 2*c*f - d*f*x) + b^2*(d*e - 2*c*f - d*f*x))) + 4*a*b*d*(e + f*x)*Log[1 + E
^(-2*(e + f*x))] + 2*b*(b*d - 2*a*d*e + 2*a*c*f)*Log[Cosh[e + f*x]]) - 2*a*
b*d*Cosh[e + f*x]*PolyLog[2, -E^(-2*(e + f*x))] - 2*b^2*f*(c + d*x)*Sinh[e
+ f*x])*(a + b*Tanh[e + f*x])^2)/(2*f^2*(a*Cosh[e + f*x] + b*Sinh[e + f*x])
^2)
```

fricas [C] time = 1.00, size = 944, normalized size = 7.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 + 2*(a^2 - 2*a*b + b^2)*c*
f^2*x - 4*b^2*d*e + ((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*
e*f - 4*b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*cosh(f*x + e
)^2 + 2*((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*e*f - 4*b^2*
```

```
d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*cosh(f*x + e)*sinh(f*x +
 e) + ((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*e*f - 4*b^2*d*
e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*sinh(f*x + e)^2 - 4*(2*a*b
*c*e - b^2*c)*f + 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)*sinh(f*x
 + e) + a*b*d*sinh(f*x + e)^2 + a*b*d)*dilog(I*cosh(f*x + e) + I*sinh(f*x +
 e)) + 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)*sinh(f*x + e) + a*b
*d*sinh(f*x + e)^2 + a*b*d)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 2*(
2*a*b*d*e - 2*a*b*c*f - b^2*d + (2*a*b*d*e - 2*a*b*c*f - b^2*d)*cosh(f*x +
e)^2 + 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d)*cosh(f*x + e)*sinh(f*x + e) + (2*a
*b*d*e - 2*a*b*c*f - b^2*d)*sinh(f*x + e)^2)*log(cosh(f*x + e) + sinh(f*x +
 e) + I) - 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d + (2*a*b*d*e - 2*a*b*c*f - b^2*
d)*cosh(f*x + e)^2 + 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d)*cosh(f*x + e)*sinh(f
*x + e) + (2*a*b*d*e - 2*a*b*c*f - b^2*d)*sinh(f*x + e)^2)*log(cosh(f*x + e
) + sinh(f*x + e) - I) + 4*(a*b*d*f*x + a*b*d*e + (a*b*d*f*x + a*b*d*e)*cos
h(f*x + e)^2 + 2*(a*b*d*f*x + a*b*d*e)*cosh(f*x + e)*sinh(f*x + e) + (a*b*d
*f*x + a*b*d*e)*sinh(f*x + e)^2)*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1)
 + 4*(a*b*d*f*x + a*b*d*e + (a*b*d*f*x + a*b*d*e)*cosh(f*x + e)^2 + 2*(a*b*
d*f*x + a*b*d*e)*cosh(f*x + e)*sinh(f*x + e) + (a*b*d*f*x + a*b*d*e)*sinh(f
*x + e)^2)*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1))/(f^2*cosh(f*x + e)^
2 + 2*f^2*cosh(f*x + e)*sinh(f*x + e) + f^2*sinh(f*x + e)^2 + f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\left(b \tanh\left(fx + e\right) + a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*tanh(f*x + e) + a)^2, x)
```

maple [A] time = 0.41, size = 221, normalized size = 1.74

$$\frac{a^2 d\, x^2}{2} - abd\, x^2 + \frac{b^2 d\, x^2}{2} + a^2 cx + 2cabx + b^2 cx + \frac{2b^2 (dx + c)}{f\left(\mathrm{e}^{2fx+2e} + 1\right)} + \frac{b^2 d \ln\left(\mathrm{e}^{2fx+2e} + 1\right)}{f^2} - \frac{2b^2 d \ln\left(\mathrm{e}^{fx+e}\right)}{f^2} - \frac{4bac \ln\left(\mathrm{e}^{f}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*tanh(f*x+e))^2,x)
```

```
[Out] 1/2*a^2*d*x^2-a*b*d*x^2+1/2*b^2*d*x^2+a^2*c*x+2*c*a*b*x+b^2*c*x+2/f*b^2*(d*
x+c)/(exp(2*f*x+2*e)+1)+1/f^2*b^2*d*ln(exp(2*f*x+2*e)+1)-2/f^2*b^2*d*ln(exp
(f*x+e))-4/f*b*a*c*ln(exp(f*x+e))+2/f*b*a*c*ln(exp(2*f*x+2*e)+1)+4/f^2*b*d*
a*e*ln(exp(f*x+e))-4/f*b*a*d*e*x-2/f^2*b*a*d*e^2+2/f*b*ln(exp(2*f*x+2*e)+1)
*a*d*x+a*b*d*polylog(2,-exp(2*f*x+2*e))/f^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} a^2 dx^2 + \left(x^2 - 4 \int \frac{x}{e^{\left(2fx+2e\right)} + 1}\, dx\right) abd + b^2 c \left(x + \frac{e}{f} - \frac{2}{f\left(e^{\left(-2fx-2e\right)} + 1\right)}\right) + a^2 cx + \frac{1}{2} b^2 d \left(\frac{fx^2 + \left(fx^2 e^{(2e)} - 4x\right.}{fe^{\left(2fx+2e\right)} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 + (x^2 - 4*integrate(x/(e^(2*f*x + 2*e) + 1), x))*a*b*d + b^2
*c*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) + a^2*c*x + 1/2*b^2*d*((f*x^2 +
 (f*x^2*e^(2*e) - 4*x*e^(2*e))*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 2*log((
e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2) + 2*a*b*c*log(cosh(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b \tanh\left(e + f\,x\right)\right)^2 \, (c + d\,x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^2*(c + d*x),x)
```

```
[Out] int((a + b*tanh(e + f*x))^2*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh\left(e + f\,x\right)\right)^2 (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e))**2,x)
```

```
[Out] Integral((a + b*tanh(e + f*x))**2*(c + d*x), x)
```

3.61 $\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\tanh(e+fx))^2}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Tanh[e + f*x])^2/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*Tanh[e + f*x])^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$$

Mathematica [A] time = 41.59, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x),x]
```

```
[Out] Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\tanh\left(fx+e\right)^2+2\,ab\tanh\left(fx+e\right)+a^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*tanh(f*x + e)^2 + 2*a*b*tanh(f*x + e) + a^2)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\tanh\left(fx+e\right)+a\right)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*tanh(f*x + e) + a)^2/(d*x + c), x)
```

maple [A] time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \tanh\left(fx + e\right)\right)^2}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

```
[Out] int((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \log\left(dx + c\right)}{d} + \frac{2\, b^2}{dfx + cf + \left(dfxe^{(2\,e)} + cfe^{(2\,e)}\right)e^{\left(2\,fx\right)}} + \frac{\left(2\, ab + b^2\right)\log\left(dx + c\right)}{d} - \int \frac{2\left(2\, abdf$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] a^2*log(d*x + c)/d + 2*b^2/(d*f*x + c*f + (d*f*x*e^(2*e) + c*f*e^(2*e))*e^(
2*f*x)) + (2*a*b + b^2)*log(d*x + c)/d - integrate(2*(2*a*b*d*f*x + 2*a*b*c
*f - b^2*d)/(d^2*f*x^2 + 2*c*d*f*x + c^2*f + (d^2*f*x^2*e^(2*e) + 2*c*d*f*x
*e^(2*e) + c^2*f*e^(2*e))*e^(2*f*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b \tanh\left(e + f\, x\right)\right)^2}{c + d\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^2/(c + d*x),x)
```

```
[Out] int((a + b*tanh(e + f*x))^2/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \tanh\left(e + fx\right)\right)^2}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))**2/(d*x+c),x)
```

```
[Out] Integral((a + b*tanh(e + f*x))**2/(c + d*x), x)
```

3.62 $\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\tanh(e+fx))^2}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Tanh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Tanh[e + f*x])^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$

Mathematica [A] time = 27.17, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x)^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\tanh\left(fx+e\right)^2 + 2\,ab\tanh\left(fx+e\right) + a^2}{d^2x^2 + 2\,cdx + c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*tanh(f*x + e)^2 + 2*a*b*tanh(f*x + e) + a^2)/(d^2*x^2 + 2*c*d
*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\tanh\left(fx+e\right)+a\right)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*tanh(f*x + e) + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\tanh\left(fx + e\right)\right)^2}{\left(dx + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

```
[Out] int((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2}{d^2x + cd} - \frac{2\,abcf + \left(cf - 2\,d\right)b^2 + \left(2\,abdf + b^2df\right)x + \left(2\,abcfe^{(2\,e)} + b^2cfe^{(2\,e)} + \left(2\,abdfe^{(2\,e)} + b^2dfe^{(2\,e)}\right)x\right)e^{(2\,}}{d^3fx^2 + 2\,cd^2fx + c^2df + \left(d^3fx^2e^{(2\,e)} + 2\,cd^2fxe^{(2\,e)} + c^2dfe^{(2\,e)}\right)e^{(2\,fx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -a^2/(d^2*x + c*d) - (2*a*b*c*f + (c*f - 2*d)*b^2 + (2*a*b*d*f + b^2*d*f)*x
 + (2*a*b*c*f*e^(2*e) + b^2*c*f*e^(2*e) + (2*a*b*d*f*e^(2*e) + b^2*d*f*e^(2
*e))*x)*e^(2*f*x))/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2*e^(2*e)
+ 2*c*d^2*f*x*e^(2*e) + c^2*d*f*e^(2*e))*e^(2*f*x)) - integrate(4*(a*b*d*f*
x + a*b*c*f - b^2*d)/(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f + (d^
3*f*x^3*e^(2*e) + 3*c*d^2*f*x^2*e^(2*e) + 3*c^2*d*f*x*e^(2*e) + c^3*f*e^(2*
e))*e^(2*f*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\tanh\left(e + f\,x\right)\right)^2}{\left(c + d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + b*tanh(e + f*x))^2/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\tanh\left(e + fx\right)\right)^2}{\left(c + dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*tanh(e + f*x))**2/(c + d*x)**2, x)
```

3.63 $\int (c+dx)^3 (a+b\tanh(e+fx))^3\,dx$

Optimal. Leaf size=566

$$\frac{a^3(c+dx)^4}{4d}-\frac{9a^2bd^2(c+dx)\mathrm{Li}_3\left(-e^{2(e+fx)}\right)}{2f^3}+\frac{9a^2bd(c+dx)^2\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}+\frac{3a^2b(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{3}{}$$

```
[Out] -3/2*b^3*d*(d*x+c)^2/f^2-3*a*b^2*(d*x+c)^3/f+1/2*b^3*(d*x+c)^3/f+1/4*a^3*(d
*x+c)^4/d-3/4*a^2*b*(d*x+c)^4/d+3/4*a*b^2*(d*x+c)^4/d-1/4*b^3*(d*x+c)^4/d+3
*b^3*d^2*(d*x+c)*ln(1+exp(2*f*x+2*e))/f^3+9*a*b^2*d*(d*x+c)^2*ln(1+exp(2*f*
x+2*e))/f^2+3*a^2*b*(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+b^3*(d*x+c)^3*ln(1+exp
(2*f*x+2*e))/f+3/2*b^3*d^3*polylog(2,-exp(2*f*x+2*e))/f^4+9*a*b^2*d^2*(d*x+
c)*polylog(2,-exp(2*f*x+2*e))/f^3+9/2*a^2*b*d*(d*x+c)^2*polylog(2,-exp(2*f*
x+2*e))/f^2+3/2*b^3*d*(d*x+c)^2*polylog(2,-exp(2*f*x+2*e))/f^2-9/2*a*b^2*d^
3*polylog(3,-exp(2*f*x+2*e))/f^4-9/2*a^2*b*d^2*(d*x+c)*polylog(3,-exp(2*f*x
+2*e))/f^3-3/2*b^3*d^2*(d*x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+9/4*a^2*b*d^3
*polylog(4,-exp(2*f*x+2*e))/f^4+3/4*b^3*d^3*polylog(4,-exp(2*f*x+2*e))/f^4-
3/2*b^3*d*(d*x+c)^2*tanh(f*x+e)/f^2-3*a*b^2*(d*x+c)^3*tanh(f*x+e)/f-1/2*b^3
*(d*x+c)^3*tanh(f*x+e)^2/f
```

Rubi [A] time = 1.02, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3722, 3718, 2190, 2531, 6609, 2282, 6589, 3720, 32, 2279, 2391}

$$-\frac{9a^2bd^2(c+dx)\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{9a^2bd(c+dx)^2\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{9a^2bd^3\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Tanh[e + f*x])^3,x]
```

```
[Out] (-3*b^3*d*(c + d*x)^2)/(2*f^2) - (3*a*b^2*(c + d*x)^3)/f + (b^3*(c + d*x)^3
)/(2*f) + (a^3*(c + d*x)^4)/(4*d) - (3*a^2*b*(c + d*x)^4)/(4*d) + (3*a*b^2*
(c + d*x)^4)/(4*d) - (b^3*(c + d*x)^4)/(4*d) + (3*b^3*d^2*(c + d*x)*Log[1 +
 E^(2*(e + f*x))])/f^3 + (9*a*b^2*d*(c + d*x)^2*Log[1 + E^(2*(e + f*x))])/f
^2 + (3*a^2*b*(c + d*x)^3*Log[1 + E^(2*(e + f*x))])/f + (b^3*(c + d*x)^3*Lo
g[1 + E^(2*(e + f*x))])/f + (3*b^3*d^3*PolyLog[2, -E^(2*(e + f*x))])/(2*f^4
) + (9*a*b^2*d^2*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^3 + (9*a^2*b*d*(
c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) + (3*b^3*d*(c + d*x)^2*Pol
yLog[2, -E^(2*(e + f*x))])/(2*f^2) - (9*a*b^2*d^3*PolyLog[3, -E^(2*(e + f*x
))])/(2*f^4) - (9*a^2*b*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3)
 - (3*b^3*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3) + (9*a^2*b*d^
3*PolyLog[4, -E^(2*(e + f*x))])/(4*f^4) + (3*b^3*d^3*PolyLog[4, -E^(2*(e +
f*x))])/(4*f^4) - (3*b^3*d*(c + d*x)^2*Tanh[e + f*x])/(2*f^2) - (3*a*b^2*(c
 + d*x)^3*Tanh[e + f*x])/f - (b^3*(c + d*x)^3*Tanh[e + f*x]^2)/(2*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \int \left(a^3(c+dx)^3 + 3a^2b(c+dx)^3\tanh(e+fx) + 3ab^2(c+dx)^3\tanh^2(e+\right.$$

$$= \frac{a^3(c+dx)^4}{4d} + \left(3a^2b\right)\int (c+dx)^3\tanh(e+fx)\,dx + \left(3ab^2\right)\int (c+dx)^3$$

$$= \frac{a^3(c+dx)^4}{4d} - \frac{3a^2b(c+dx)^4}{4d} - \frac{3ab^2(c+dx)^3\tanh(e+fx)}{f} - \frac{b^3(c+dx)}{}$$

$$= -\frac{3ab^2(c+dx)^3}{f} + \frac{a^3(c+dx)^4}{4d} - \frac{3a^2b(c+dx)^4}{4d} + \frac{3ab^2(c+dx)^4}{4d} - \frac{b^3(c}{}$$

$$= -\frac{3b^3d(c+dx)^2}{2f^2} - \frac{3ab^2(c+dx)^3}{f} + \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3a^2b(c}{}$$

$$= -\frac{3b^3d(c+dx)^2}{2f^2} - \frac{3ab^2(c+dx)^3}{f} + \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3a^2b(c}{}$$

$$= -\frac{3b^3d(c+dx)^2}{2f^2} - \frac{3ab^2(c+dx)^3}{f} + \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3a^2b(c}{}$$

$$= -\frac{3b^3d(c+dx)^2}{2f^2} - \frac{3ab^2(c+dx)^3}{f} + \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3a^2b(c}{}$$

$$= -\frac{3b^3d(c+dx)^2}{2f^2} - \frac{3ab^2(c+dx)^3}{f} + \frac{b^3(c+dx)^3}{2f} + \frac{a^3(c+dx)^4}{4d} - \frac{3a^2b(c}{}$$

Mathematica [B] time = 14.61, size = 2010, normalized size = 3.55

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Tanh[e + f*x])^3,x]
```

```
[Out] (b*E^(2*e)*(-24*b^2*c*d^2*x - 72*a*b*c^2*d*f*x - 24*a^2*c^3*f^2*x - 8*b^2*c
^3*f^2*x - 12*b^2*d^3*x^2 - 72*a*b*c*d^2*f*x^2 - 36*a^2*c^2*d*f^2*x^2 - 12*
b^2*c^2*d*f^2*x^2 - 24*a*b*d^3*f*x^3 - 24*a^2*c*d^2*f^2*x^3 - 8*b^2*c*d^2*f
^2*x^3 - 6*a^2*d^3*f^2*x^4 - 2*b^2*d^3*f^2*x^4 + 36*a*b*c^2*d*Log[1 + E^(2*
(e + f*x))] + (36*a*b*c^2*d*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (12*b^2*c*d
^2*Log[1 + E^(2*(e + f*x))])/f + (12*b^2*c*d^2*Log[1 + E^(2*(e + f*x))])/(E
^(2*e)*f) + 12*a^2*c^3*f*Log[1 + E^(2*(e + f*x))] + 4*b^2*c^3*f*Log[1 + E^(
2*(e + f*x))] + (12*a^2*c^3*f*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (4*b^2*c^
3*f*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 72*a*b*c*d^2*x*Log[1 + E^(2*(e + f*
x))] + (72*a*b*c*d^2*x*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (12*b^2*d^3*x*Lo
g[1 + E^(2*(e + f*x))])/f + (12*b^2*d^3*x*Log[1 + E^(2*(e + f*x))])/(E^(2*e
)*f) + 36*a^2*c^2*d*f*x*Log[1 + E^(2*(e + f*x))] + 12*b^2*c^2*d*f*x*Log[1 +
 E^(2*(e + f*x))] + (36*a^2*c^2*d*f*x*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (
12*b^2*c^2*d*f*x*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 36*a*b*d^3*x^2*Log[1 +
 E^(2*(e + f*x))] + (36*a*b*d^3*x^2*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 36*
a^2*c*d^2*f*x^2*Log[1 + E^(2*(e + f*x))] + 12*b^2*c*d^2*f*x^2*Log[1 + E^(2*
(e + f*x))] + (36*a^2*c*d^2*f*x^2*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (12*b
^2*c*d^2*f*x^2*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 12*a^2*d^3*f*x^3*Log[1 +
 E^(2*(e + f*x))] + 4*b^2*d^3*f*x^3*Log[1 + E^(2*(e + f*x))] + (12*a^2*d^3*
f*x^3*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (4*b^2*d^3*f*x^3*Log[1 + E^(2*(e
```

```
+ f*x))])/E^(2*e) + (6*d*(1 + E^(2*e))*(6*a*b*d*f*(c + d*x) + 3*a^2*f^2*(c
+ d*x)^2 + b^2*(d^2 + c^2*f^2 + 2*c*d*f^2*x + d^2*f^2*x^2))*PolyLog[2, -E^(
2*(e + f*x))])/(E^(2*e)*f^2) - (6*d^2*(1 + E^(2*e))*(3*a*b*d + 3*a^2*f*(c +
 d*x) + b^2*f*(c + d*x))*PolyLog[3, -E^(2*(e + f*x))])/(E^(2*e)*f^2) + (9*a
^2*d^3*PolyLog[4, -E^(2*(e + f*x))])/f^2 + (3*b^2*d^3*PolyLog[4, -E^(2*(e +
 f*x))])/f^2 + (9*a^2*d^3*PolyLog[4, -E^(2*(e + f*x))])/(E^(2*e)*f^2) + (3*
b^2*d^3*PolyLog[4, -E^(2*(e + f*x))])/(E^(2*e)*f^2)))/(4*(1 + E^(2*e))*f^2)
 + ((b^3*c^3 + 3*b^3*c^2*d*x + 3*b^3*c*d^2*x^2 + b^3*d^3*x^3)*Sech[e + f*x]
^2)/(2*f) + (3*x^2*(a^3*c^2*d - 3*a^2*b*c^2*d + 3*a*b^2*c^2*d - b^3*c^2*d +
 a^3*c^2*d*Cosh[2*e] + 3*a^2*b*c^2*d*Cosh[2*e] + 3*a*b^2*c^2*d*Cosh[2*e] +
b^3*c^2*d*Cosh[2*e] + a^3*c^2*d*Sinh[2*e] + 3*a^2*b*c^2*d*Sinh[2*e] + 3*a*b
^2*c^2*d*Sinh[2*e] + b^3*c^2*d*Sinh[2*e]))/(2*(1 + Cosh[2*e] + Sinh[2*e]))
+ (x^3*(a^3*c*d^2 - 3*a^2*b*c*d^2 + 3*a*b^2*c*d^2 - b^3*c*d^2 + a^3*c*d^2*C
osh[2*e] + 3*a^2*b*c*d^2*Cosh[2*e] + 3*a*b^2*c*d^2*Cosh[2*e] + b^3*c*d^2*Co
sh[2*e] + a^3*c*d^2*Sinh[2*e] + 3*a^2*b*c*d^2*Sinh[2*e] + 3*a*b^2*c*d^2*Sin
h[2*e] + b^3*c*d^2*Sinh[2*e]))/(1 + Cosh[2*e] + Sinh[2*e]) + (x^4*(a^3*d^3
- 3*a^2*b*d^3 + 3*a*b^2*d^3 - b^3*d^3 + a^3*d^3*Cosh[2*e] + 3*a^2*b*d^3*Cos
h[2*e] + 3*a*b^2*d^3*Cosh[2*e] + b^3*d^3*Cosh[2*e] + a^3*d^3*Sinh[2*e] + 3*
a^2*b*d^3*Sinh[2*e] + 3*a*b^2*d^3*Sinh[2*e] + b^3*d^3*Sinh[2*e]))/(4*(1 + C
osh[2*e] + Sinh[2*e])) + x*(a^3*c^3 + 3*a*b^2*c^3 - (3*a^2*b*c^3)/(1 + Cosh
[2*e] + Sinh[2*e]) + (3*a^2*b*c^3*Cosh[2*e] + 3*a^2*b*c^3*Sinh[2*e])/(1 + C
osh[2*e] + Sinh[2*e]) + (2*b^3*c^3*Cosh[2*e] + 2*b^3*c^3*Sinh[2*e])/((1 + C
osh[2*e] + Sinh[2*e])*(1 - Cosh[2*e] + Cosh[4*e] - Sinh[2*e] + Sinh[4*e]))
+ (-2*b^3*c^3*Cosh[4*e] - 2*b^3*c^3*Sinh[4*e])/((1 + Cosh[2*e] + Sinh[2*e])
*(1 - Cosh[2*e] + Cosh[4*e] - Sinh[2*e] + Sinh[4*e])) - (b^3*c^3)/(1 + Cosh
[6*e] + Sinh[6*e]) + (b^3*c^3*Cosh[6*e] + b^3*c^3*Sinh[6*e])/(1 + Cosh[6*e]
 + Sinh[6*e])) - (3*Sech[e]*Sech[e + f*x]*(b^3*c^2*d*Sinh[f*x] + 2*a*b^2*c^
3*f*Sinh[f*x] + 2*b^3*c*d^2*x*Sinh[f*x] + 6*a*b^2*c^2*d*f*x*Sinh[f*x] + b^3
*d^3*x^2*Sinh[f*x] + 6*a*b^2*c*d^2*f*x^2*Sinh[f*x] + 2*a*b^2*d^3*f*x^3*Sinh
[f*x]))/(2*f^2)
```

fricas [C] time = 1.20, size = 12909, normalized size = 22.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tanh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^3*f^4*x^4 + 4*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*c*d^2*f^4*x^3 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*d*f^4*x^2 -
 24*a*b^2*d^3*e^3 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^3*f^4*x + 12*b^3*d^
3*e^2 + 2*(3*a^2*b + b^3)*d^3*e^4 + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^3*f^
4*x^4 - 24*a*b^2*d^3*e^3 + 12*b^3*d^3*e^2 + 2*(3*a^2*b + b^3)*d^3*e^4 - 8*(
3*a^2*b + b^3)*c^3*e*f^3 - 4*(6*a*b^2*d^3*f^3 - (a^3 - 3*a^2*b + 3*a*b^2 -
b^3)*c*d^2*f^4)*x^3 - 12*(6*a*b^2*c^2*d*e - (3*a^2*b + b^3)*c^2*d*e^2)*f^2
- 6*(12*a*b^2*c*d^2*f^3 + 2*b^3*d^3*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c
^2*d*f^4)*x^2 + 8*(9*a*b^2*c*d^2*e^2 - 3*b^3*c*d^2*e - (3*a^2*b + b^3)*c*d^
2*e^3)*f - 4*(18*a*b^2*c^2*d*f^3 + 6*b^3*c*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*c^3*f^4)*x)*cosh(f*x + e)^4 + 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
d^3*f^4*x^4 - 24*a*b^2*d^3*e^3 + 12*b^3*d^3*e^2 + 2*(3*a^2*b + b^3)*d^3*e^4
 - 8*(3*a^2*b + b^3)*c^3*e*f^3 - 4*(6*a*b^2*d^3*f^3 - (a^3 - 3*a^2*b + 3*a*
b^2 - b^3)*c*d^2*f^4)*x^3 - 12*(6*a*b^2*c^2*d*e - (3*a^2*b + b^3)*c^2*d*e^2
)*f^2 - 6*(12*a*b^2*c*d^2*f^3 + 2*b^3*d^3*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 -
b^3)*c^2*d*f^4)*x^2 + 8*(9*a*b^2*c*d^2*e^2 - 3*b^3*c*d^2*e - (3*a^2*b + b^3
)*c*d^2*e^3)*f - 4*(18*a*b^2*c^2*d*f^3 + 6*b^3*c*d^2*f^2 - (a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*c^3*f^4)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d^3*f^4*x^4 - 24*a*b^2*d^3*e^3 + 12*b^3*d^3*e^2 + 2*(3*a^
2*b + b^3)*d^3*e^4 - 8*(3*a^2*b + b^3)*c^3*e*f^3 - 4*(6*a*b^2*d^3*f^3 - (a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d^2*f^4)*x^3 - 12*(6*a*b^2*c^2*d*e - (3*a^2*
b + b^3)*c^2*d*e^2)*f^2 - 6*(12*a*b^2*c*d^2*f^3 + 2*b^3*d^3*f^2 - (a^3 - 3*
```

```
a^2*b + 3*a*b^2 - b^3)*c^2*d*f^4)*x^2 + 8*(9*a*b^2*c*d^2*e^2 - 3*b^3*c*d^2*
e - (3*a^2*b + b^3)*c*d^2*e^3)*f - 4*(18*a*b^2*c^2*d*f^3 + 6*b^3*c*d^2*f^2
 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^3*f^4)*x)*sinh(f*x + e)^4 + 8*(3*a*b^2*
c^3 - (3*a^2*b + b^3)*c^3*e)*f^3 - 12*(6*a*b^2*c^2*d*e - b^3*c^2*d - (3*a^2
*b + b^3)*c^2*d*e^2)*f^2 + 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^3*f^4*x^4 -
 24*a*b^2*d^3*e^3 + 12*b^3*d^3*e^2 + 2*(3*a^2*b + b^3)*d^3*e^4 - 4*(2*(3*a^
2*b + b^3)*c^3*e - (3*a*b^2 + b^3)*c^3)*f^3 + 4*((a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*c*d^2*f^4 - (3*a*b^2 - b^3)*d^3*f^3)*x^3 - 6*(12*a*b^2*c^2*d*e - b^3*
c^2*d - 2*(3*a^2*b + b^3)*c^2*d*e^2)*f^2 - 6*(b^3*d^3*f^2 - (a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*c^2*d*f^4 + 2*(3*a*b^2 - b^3)*c*d^2*f^3)*x^2 + 8*(9*a*b^2*
c*d^2*e^2 - 3*b^3*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^3)*f - 4*(3*b^3*c*d^2*f
^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^3*f^4 + 3*(3*a*b^2 - b^3)*c^2*d*f^3)
*x)*cosh(f*x + e)^2 + 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^3*f^4*x^4 - 24*a
*b^2*d^3*e^3 + 12*b^3*d^3*e^2 + 2*(3*a^2*b + b^3)*d^3*e^4 - 4*(2*(3*a^2*b +
 b^3)*c^3*e - (3*a*b^2 + b^3)*c^3)*f^3 + 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*c*d^2*f^4 - (3*a*b^2 - b^3)*d^3*f^3)*x^3 - 6*(12*a*b^2*c^2*d*e - b^3*c^2*d
 - 2*(3*a^2*b + b^3)*c^2*d*e^2)*f^2 - 6*(b^3*d^3*f^2 - (a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*c^2*d*f^4 + 2*(3*a*b^2 - b^3)*c*d^2*f^3)*x^2 + 3*((a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*d^3*f^4*x^4 - 24*a*b^2*d^3*e^3 + 12*b^3*d^3*e^2 + 2*(3*a
^2*b + b^3)*d^3*e^4 - 8*(3*a^2*b + b^3)*c^3*e*f^3 - 4*(6*a*b^2*d^3*f^3 - (a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d^2*f^4)*x^3 - 12*(6*a*b^2*c^2*d*e - (3*a^2
*b + b^3)*c^2*d*e^2)*f^2 - 6*(12*a*b^2*c*d^2*f^3 + 2*b^3*d^3*f^2 - (a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*c^2*d*f^4)*x^2 + 8*(9*a*b^2*c*d^2*e^2 - 3*b^3*c*d^2
*e - (3*a^2*b + b^3)*c*d^2*e^3)*f - 4*(18*a*b^2*c^2*d*f^3 + 6*b^3*c*d^2*f^2
 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^3*f^4)*x)*cosh(f*x + e)^2 + 8*(9*a*b^2
*c*d^2*e^2 - 3*b^3*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^3)*f - 4*(3*b^3*c*d^2*
f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^3*f^4 + 3*(3*a*b^2 - b^3)*c^2*d*f^3
)*x)*sinh(f*x + e)^2 + 8*(9*a*b^2*c*d^2*e^2 - 3*b^3*c*d^2*e - (3*a^2*b + b^
3)*c*d^2*e^3)*f + 12*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d
^3 + (3*a^2*b + b^3)*c^2*d*f^2 + ((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d
^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b +
b^3)*c*d^2*f^2)*x)*cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b
^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^
2*b + b^3)*c*d^2*f^2)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d
^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a
*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*sinh(f*x + e)^4 + 2*((3*a^2*b +
b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 +
2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*cosh(f*x + e)^2 + 2*((3*a^
2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*
f^2 + 3*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b
 + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*cosh(f
*x + e)^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*sinh(f*x + e)^
2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x + 4*(((3*a^2*b + b^3)*d
^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a
*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*cosh(f*x + e)^3 + ((3*a^2*b + b^
3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*
(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*cosh(f*x + e))*sinh(f*x + e)
)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) + 12*((3*a^2*b + b^3)*d^3*f^2*x^
2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + ((3*a^2*b + b^3
)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(
3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*cosh(f*x + e)^4 + 4*((3*a^2*b
 + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2
 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*cosh(f*x + e)*sinh(f*x
+ e)^3 + ((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*
b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*sinh(
f*x + e)^4 + 2*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (
3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)
*cosh(f*x + e)^2 + 2*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d
^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 3*((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c
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*d^2*f + b^3*d^3 + (3*a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b
+ b^3)*c*d^2*f^2)*x)*cosh(f*x + e)^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c
*d^2*f^2)*x)*sinh(f*x + e)^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2
)*x + 4*(((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*a^2*
b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*cosh(
f*x + e)^3 + ((3*a^2*b + b^3)*d^3*f^2*x^2 + 6*a*b^2*c*d^2*f + b^3*d^3 + (3*
a^2*b + b^3)*c^2*d*f^2 + 2*(3*a*b^2*d^3*f + (3*a^2*b + b^3)*c*d^2*f^2)*x)*c
osh(f*x + e))*sinh(f*x + e))*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) + 4*
(9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^3)*
c^3*f^3 + (9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2
*b + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6*
a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*cosh(f*x + e)^4 +
 4*(9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^
3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c
*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*cosh(f*x + e)*sinh(f*x +
 e)^3 + (9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b
 + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6*a*
b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*sinh(f*x + e)^4 + 3
*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 + 2*(9*a*b^2*d^3*e^2 - 3*b^3
*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2
*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2
*b + b^3)*c*d^2*e^2)*f)*cosh(f*x + e)^2 + 2*(9*a*b^2*d^3*e^2 - 3*b^3*d^3*e
- (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3
*a^2*b + b^3)*c^2*d*e)*f^2 + 3*(9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b +
b^3)*d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)
*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)
*f)*cosh(f*x + e)^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^
2*e^2)*f)*sinh(f*x + e)^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3
)*c*d^2*e^2)*f + 4*((9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^
3 + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f
^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*cosh(f*
x + e)^3 + (9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^
2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6
*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*cosh(f*x + e))*s
inh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) + I) + 4*(9*a*b^2*d^3*e^2 -
 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3 + (9*a*b^2
*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3
+ 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^
3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*cosh(f*x + e)^4 + 4*(9*a*b^2*d^3*e^
2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*
a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2
 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (9*a*b^2*d
^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3 +
3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*
c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*sinh(f*x + e)^4 + 3*(3*a*b^2*c^2*d -
(3*a^2*b + b^3)*c^2*d*e)*f^2 + 2*(9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b
+ b^3)*d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^
3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^
2)*f)*cosh(f*x + e)^2 + 2*(9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*
d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*
d*e)*f^2 + 3*(9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*
a^2*b + b^3)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*
(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*cosh(f*x + e)^
2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*sinh(f*x
 + e)^2 - 3*(6*a*b^2*c*d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f + 4
*((9*a*b^2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^3
)*c^3*f^3 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*
d^2*e - b^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*cosh(f*x + e)^3 + (9*a*b^
2*d^3*e^2 - 3*b^3*d^3*e - (3*a^2*b + b^3)*d^3*e^3 + (3*a^2*b + b^3)*c^3*f^3
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 + 3*(3*a*b^2*c^2*d - (3*a^2*b + b^3)*c^2*d*e)*f^2 - 3*(6*a*b^2*c*d^2*e - b
^3*c*d^2 - (3*a^2*b + b^3)*c*d^2*e^2)*f)*cosh(f*x + e))*sinh(f*x + e))*log(
cosh(f*x + e) + sinh(f*x + e) - I) + 4*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b
^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*
d*e*f^2 + ((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3
*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2
 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*
d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*
x)*cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b
^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*
b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*
b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*
c^2*d*f^3)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^3*f^3*x^3
 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^
3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6
*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*
d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*sinh(f*x + e)^4 + 3*(3*a*b^2*d^3*f^2
 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 2*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2
*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*
e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*
d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (
3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)^2 + 2*((3*a^2*b + b^3)*d^3*f^3*x
^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b +
 b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3
*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b +
b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*
b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f
 + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*
x + e)^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c
*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*sinh(f*x + e)^2 + 3*(6
*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*
d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x + 4*(((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*
a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c
^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b
^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*
f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)^3 + ((3*a^2*b + b^3)*d^3*f^
3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*
b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2
 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2
 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e))*sinh(f*x + e))*l
og(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) + 4*((3*a^2*b + b^3)*d^3*f^3*x^3
 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^
3)*c^2*d*e*f^2 + ((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3
*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d
^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b
^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d
*f^3)*x)*cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2
 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 +
3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e -
(3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b
+ b^3)*c^2*d*f^3)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^3*f
^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2
*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2
 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2
 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*sinh(f*x + e)^4 + 3*(3*a*b^2*d
^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 2*((3*a^2*b + b^3)*d^3*f^3*x^3 -
9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)
*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a
*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^
```

```
3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)^2 + 2*((3*a^2*b + b^3)*d^
3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*
a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*
x^2 + 3*((3*a^2*b + b^3)*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a
^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 +
(3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2
*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*
cosh(f*x + e)^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*
a*b^2*c*d^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*sinh(f*x + e)^2
 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2
 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x + 4*(((3*a^2*b + b^3)*d^3*f^3*x
^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*(3*a^2*b +
 b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^3)*x^2 + 3
*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d^2*f^2 + b
^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e)^3 + ((3*a^2*b + b^3)
*d^3*f^3*x^3 - 9*a*b^2*d^3*e^2 + 3*b^3*d^3*e + (3*a^2*b + b^3)*d^3*e^3 + 3*
(3*a^2*b + b^3)*c^2*d*e*f^2 + 3*(3*a*b^2*d^3*f^2 + (3*a^2*b + b^3)*c*d^2*f^
3)*x^2 + 3*(6*a*b^2*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^2)*f + 3*(6*a*b^2*c*d
^2*f^2 + b^3*d^3*f + (3*a^2*b + b^3)*c^2*d*f^3)*x)*cosh(f*x + e))*sinh(f*x
+ e))*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1) + 24*((3*a^2*b + b^3)*d^3
*cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d^3*cosh(f*x + e)*sinh(f*x + e)^3 + (3
*a^2*b + b^3)*d^3*sinh(f*x + e)^4 + 2*(3*a^2*b + b^3)*d^3*cosh(f*x + e)^2 +
 (3*a^2*b + b^3)*d^3 + 2*(3*(3*a^2*b + b^3)*d^3*cosh(f*x + e)^2 + (3*a^2*b
+ b^3)*d^3)*sinh(f*x + e)^2 + 4*((3*a^2*b + b^3)*d^3*cosh(f*x + e)^3 + (3*a
^2*b + b^3)*d^3*cosh(f*x + e))*sinh(f*x + e))*polylog(4, I*cosh(f*x + e) +
I*sinh(f*x + e)) + 24*((3*a^2*b + b^3)*d^3*cosh(f*x + e)^4 + 4*(3*a^2*b + b
^3)*d^3*cosh(f*x + e)*sinh(f*x + e)^3 + (3*a^2*b + b^3)*d^3*sinh(f*x + e)^4
 + 2*(3*a^2*b + b^3)*d^3*cosh(f*x + e)^2 + (3*a^2*b + b^3)*d^3 + 2*(3*(3*a^
2*b + b^3)*d^3*cosh(f*x + e)^2 + (3*a^2*b + b^3)*d^3)*sinh(f*x + e)^2 + 4*(
(3*a^2*b + b^3)*d^3*cosh(f*x + e)^3 + (3*a^2*b + b^3)*d^3*cosh(f*x + e))*si
nh(f*x + e))*polylog(4, -I*cosh(f*x + e) - I*sinh(f*x + e)) - 24*(3*a*b^2*d
^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f + (3*a*b^2*d^3 + (3*
a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e)^4 + 4*(3*a*b^
2*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e)*si
nh(f*x + e)^3 + (3*a*b^2*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*
d^2*f)*sinh(f*x + e)^4 + 2*(3*a*b^2*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*
b + b^3)*c*d^2*f)*cosh(f*x + e)^2 + 2*(3*a*b^2*d^3 + (3*a^2*b + b^3)*d^3*f*
x + (3*a^2*b + b^3)*c*d^2*f + 3*(3*a*b^2*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3
*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((3*a*b^2*d^3 +
 (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e)^3 + (3*a*
b^2*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e))
*sinh(f*x + e))*polylog(3, I*cosh(f*x + e) + I*sinh(f*x + e)) - 24*(3*a*b^2
*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f + (3*a*b^2*d^3 + (
3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e)^4 + 4*(3*a*
b^2*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e)*
sinh(f*x + e)^3 + (3*a*b^2*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*
c*d^2*f)*sinh(f*x + e)^4 + 2*(3*a*b^2*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^
2*b + b^3)*c*d^2*f)*cosh(f*x + e)^2 + 2*(3*a*b^2*d^3 + (3*a^2*b + b^3)*d^3*
f*x + (3*a^2*b + b^3)*c*d^2*f + 3*(3*a*b^2*d^3 + (3*a^2*b + b^3)*d^3*f*x +
(3*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((3*a*b^2*d^3
 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e)^3 + (3*
a*b^2*d^3 + (3*a^2*b + b^3)*d^3*f*x + (3*a^2*b + b^3)*c*d^2*f)*cosh(f*x + e
))*sinh(f*x + e))*polylog(3, -I*cosh(f*x + e) - I*sinh(f*x + e)) + 4*(((a^3
 - 3*a^2*b + 3*a*b^2 - b^3)*d^3*f^4*x^4 - 24*a*b^2*d^3*e^3 + 12*b^3*d^3*e^2
 + 2*(3*a^2*b + b^3)*d^3*e^4 - 8*(3*a^2*b + b^3)*c^3*e*f^3 - 4*(6*a*b^2*d^3
*f^3 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d^2*f^4)*x^3 - 12*(6*a*b^2*c^2*d*e
 - (3*a^2*b + b^3)*c^2*d*e^2)*f^2 - 6*(12*a*b^2*c*d^2*f^3 + 2*b^3*d^3*f^2 -
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*d*f^4)*x^2 + 8*(9*a*b^2*c*d^2*e^2 - 3*
b^3*c*d^2*e - (3*a^2*b + b^3)*c*d^2*e^3)*f - 4*(18*a*b^2*c^2*d*f^3 + 6*b^3*
```

```
c*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^3*f^4)*x)*cosh(f*x + e)^3 + (
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^3*f^4*x^4 - 24*a*b^2*d^3*e^3 + 12*b^3*d^3
*e^2 + 2*(3*a^2*b + b^3)*d^3*e^4 - 4*(2*(3*a^2*b + b^3)*c^3*e - (3*a*b^2 +
b^3)*c^3)*f^3 + 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d^2*f^4 - (3*a*b^2 - b
^3)*d^3*f^3)*x^3 - 6*(12*a*b^2*c^2*d*e - b^3*c^2*d - 2*(3*a^2*b + b^3)*c^2*
d*e^2)*f^2 - 6*(b^3*d^3*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*d*f^4 + 2
*(3*a*b^2 - b^3)*c*d^2*f^3)*x^2 + 8*(9*a*b^2*c*d^2*e^2 - 3*b^3*c*d^2*e - (3
*a^2*b + b^3)*c*d^2*e^3)*f - 4*(3*b^3*c*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*c^3*f^4 + 3*(3*a*b^2 - b^3)*c^2*d*f^3)*x)*cosh(f*x + e))*sinh(f*x +
e))/(f^4*cosh(f*x + e)^4 + 4*f^4*cosh(f*x + e)*sinh(f*x + e)^3 + f^4*sinh(f
*x + e)^4 + 2*f^4*cosh(f*x + e)^2 + f^4 + 2*(3*f^4*cosh(f*x + e)^2 + f^4)*s
inh(f*x + e)^2 + 4*(f^4*cosh(f*x + e)^3 + f^4*cosh(f*x + e))*sinh(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \left(b \tanh\left(fx + e\right) + a\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tanh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*(b*tanh(f*x + e) + a)^3, x)
```

maple [B] time = 0.67, size = 1788, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*tanh(f*x+e))^3,x)
```

```
[Out] 1/4*a^3*d^3*x^4-1/4*b^3*d^3*x^4+c^3*a^3*x+c^3*b^3*x+a^3*c*d^2*x^3-b^3*c*d^2
*x^3-36/f^2*b^2*a*c*d^2*e*x+18/f^2*b*a^2*c*d^2*e^2*x+4/f^3*b^3*c*d^2*e^3-2/
f^3*b^3*e^3*d^3*x-9/2/f^4*b*e^4*a^2*d^3-6/f*b^2*a*d^3*x^3+12/f^4*b^2*a*d^3*
e^3-6/f^3*b^3*d^3*e*x-3/f^2*b^3*c^2*d*e^2+3/2/f^2*b^3*c^2*d*polylog(2,-exp(
2*f*x+2*e))+2/f^4*b^3*d^3*e^3*ln(exp(f*x+e))+6/f^4*b^3*d^3*e*ln(exp(f*x+e))
+3/f^3*b^3*d^3*ln(exp(2*f*x+2*e)+1)*x+1/f*b^3*d^3*ln(exp(2*f*x+2*e)+1)*x^3+
3/2/f^2*b^3*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3/2/f^3*b^3*d^3*polylog(3,-e
xp(2*f*x+2*e))*x+3/f^3*b^3*c*d^2*ln(exp(2*f*x+2*e)+1)+3/f*b*a^2*c^3*ln(exp(
2*f*x+2*e)+1)-6/f*b*a^2*c^3*ln(exp(f*x+e))-6/f^3*b^3*c*d^2*ln(exp(f*x+e))-3
/2/f^3*b^3*c*d^2*polylog(3,-exp(2*f*x+2*e))-3/f^4*b^3*e^2*d^3-3/f^2*b^3*d^3
*x^2-3/2/f^4*b^3*e^4*d^3-9/2*a*b^2*d^3*polylog(3,-exp(2*f*x+2*e))/f^4+3/2*b
^3*d^3*polylog(2,-exp(2*f*x+2*e))/f^4+3/4*b^3*d^3*polylog(4,-exp(2*f*x+2*e)
)/f^4-3*a^2*b*c*d^2*x^3+3*a*b^2*c*d^2*x^3-9/2*a^2*b*c^2*d*x^2+9/2*a*b^2*c^2
*d*x^2-18/f*b*a^2*c^2*d*e*x+1/f*b^3*c^3*ln(exp(2*f*x+2*e)+1)-2/f*b^3*c^3*ln
(exp(f*x+e))+9/4*a^2*b*d^3*polylog(4,-exp(2*f*x+2*e))/f^4+b^2*(6*a*d^3*f*x^
3*exp(2*f*x+2*e)+2*b*d^3*f*x^3*exp(2*f*x+2*e)+18*a*c*d^2*f*x^2*exp(2*f*x+2*
e)+6*b*c*d^2*f*x^2*exp(2*f*x+2*e)+18*a*c^2*d*f*x*exp(2*f*x+2*e)+6*a*d^3*f*x
^3+6*b*c^2*d*f*x*exp(2*f*x+2*e)+3*b*d^3*x^2*exp(2*f*x+2*e)+6*a*c^3*f*exp(2*
f*x+2*e)+18*a*c*d^2*f*x^2+2*b*c^3*f*exp(2*f*x+2*e)+6*b*c*d^2*x*exp(2*f*x+2*
e)+18*a*c^2*d*f*x+3*b*c^2*d*exp(2*f*x+2*e)+3*b*d^3*x^2+6*a*c^3*f+6*b*c*d^2*
x+3*b*c^2*d)/f^2/(exp(2*f*x+2*e)+1)^2-3/4*a^2*b*d^3*x^4+3/4*a*b^2*d^3*x^4+3
/2*a^3*c^2*d*x^2-3/2*b^3*c^2*d*x^2+3*c^3*a^2*b*x+3*c^3*a*b^2*x+12/f^3*b*a^2
*c*d^2*e^3-9/f^2*b*a^2*c^2*d*e^2+18/f^3*b^2*a*d^3*e^2*x-18/f^3*b^2*a*c*d^2*
e^2-6/f^3*b*e^3*a^2*d^3*x+6/f^2*b^3*c*d^2*e^2*x-18/f*b^2*a*c*d^2*x^2-6/f*b^
3*c^2*d*e*x+9/f^2*b^2*a*d^3*ln(exp(2*f*x+2*e)+1)*x^2+9/f^3*b^2*a*c*d^2*poly
log(2,-exp(2*f*x+2*e))+6/f^4*b*a^2*d^3*e^3*ln(exp(f*x+e))+9/2/f^2*b*a^2*c^2
*d*polylog(2,-exp(2*f*x+2*e))-6/f^3*b^3*c*d^2*e^2*ln(exp(f*x+e))-18/f^2*b^2
*a*c^2*d*ln(exp(f*x+e))+6/f^2*b^3*c^2*d*e*ln(exp(f*x+e))-18/f^4*b^2*a*d^3*e
^2*ln(exp(f*x+e))+9/f^2*b^2*a*c^2*d*ln(exp(2*f*x+2*e)+1)+3/f*b*a^2*d^3*ln(e
xp(2*f*x+2*e)+1)*x^3+9/2/f^2*b*a^2*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-9/2/f
```

```
^3*b*a^2*d^3*polylog(3,-exp(2*f*x+2*e))*x-9/2/f^3*b*a^2*c*d^2*polylog(3,-ex
p(2*f*x+2*e))+9/f^3*b^2*a*d^3*polylog(2,-exp(2*f*x+2*e))*x+3/f*b^3*ln(exp(2
*f*x+2*e)+1)*c*d^2*x^2+3/f^2*b^3*polylog(2,-exp(2*f*x+2*e))*c*d^2*x+3/f*b^3
*ln(exp(2*f*x+2*e)+1)*c^2*d*x+18/f^2*b*a^2*c^2*d*e*ln(exp(f*x+e))+36/f^3*b^
2*a*c*d^2*e*ln(exp(f*x+e))+9/f*b*ln(exp(2*f*x+2*e)+1)*a^2*c*d^2*x^2+9/f^2*b
*polylog(2,-exp(2*f*x+2*e))*a^2*c*d^2*x-18/f^3*b*a^2*c*d^2*e^2*ln(exp(f*x+e
))+9/f*b*ln(exp(2*f*x+2*e)+1)*a^2*c^2*d*x+18/f^2*b^2*ln(exp(2*f*x+2*e)+1)*a
*c*d^2*x
```

maxima [B] time = 1.36, size = 1297, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*tanh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*c^2*d*x^2 + b^3*c^3*(x + e/f + lo
g(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2*f*x - 2*e) + e^(
-4*f*x - 4*e) + 1))) + a^3*c^3*x + 3*a^2*b*c^3*log(cosh(f*x + e))/f + 1/4*(
24*a*b^2*c^3*f + 12*b^3*c^2*d + (3*a^2*b*d^3*f^2 + 3*a*b^2*d^3*f^2 + b^3*d^
3*f^2)*x^4 + 4*(3*a^2*b*c*d^2*f^2 + b^3*c*d^2*f^2 + 3*(c*d^2*f^2 + 2*d^3*f)
*a*b^2)*x^3 + 6*(3*a^2*b*c^2*d*f^2 + 3*(c^2*d*f^2 + 4*c*d^2*f)*a*b^2 + (c^2
*d*f^2 + 2*d^3)*b^3)*x^2 + 12*(2*b^3*c*d^2 + (c^3*f^2 + 6*c^2*d*f)*a*b^2)*x
 + (12*a*b^2*c^3*f^2*x*e^(4*e) + (3*a^2*b*d^3*f^2*e^(4*e) + 3*a*b^2*d^3*f^2
*e^(4*e) + b^3*d^3*f^2*e^(4*e))*x^4 + 4*(3*a^2*b*c*d^2*f^2*e^(4*e) + 3*a*b^
2*c*d^2*f^2*e^(4*e) + b^3*c*d^2*f^2*e^(4*e))*x^3 + 6*(3*a^2*b*c^2*d*f^2*e^(
4*e) + 3*a*b^2*c^2*d*f^2*e^(4*e) + b^3*c^2*d*f^2*e^(4*e))*x^2)*e^(4*f*x) +
2*(12*a*b^2*c^3*f*e^(2*e) + 6*b^3*c^2*d*e^(2*e) + (3*a^2*b*d^3*f^2*e^(2*e)
+ 3*a*b^2*d^3*f^2*e^(2*e) + b^3*d^3*f^2*e^(2*e))*x^4 + 4*(3*a^2*b*c*d^2*f^2
*e^(2*e) + 3*(c*d^2*f^2*e^(2*e) + d^3*f*e^(2*e))*a*b^2 + (c*d^2*f^2*e^(2*e)
 + d^3*f*e^(2*e))*b^3)*x^3 + 6*(3*a^2*b*c^2*d*f^2*e^(2*e) + 3*(c^2*d*f^2*e^
(2*e) + 2*c*d^2*f*e^(2*e))*a*b^2 + (c^2*d*f^2*e^(2*e) + 2*c*d^2*f*e^(2*e) +
 d^3*e^(2*e))*b^3)*x^2 + 12*((c^3*f^2*e^(2*e) + 3*c^2*d*f*e^(2*e))*a*b^2 +
(c^2*d*f*e^(2*e) + c*d^2*e^(2*e))*b^3)*x)*e^(2*f*x))/(f^2*e^(4*f*x + 4*e) +
 2*f^2*e^(2*f*x + 2*e) + f^2) - 6*(3*a*b^2*c^2*d*f + b^3*c*d^2)*x/f^2 + 3*(
3*a*b^2*c^2*d*f + b^3*c*d^2)*log(e^(2*f*x + 2*e) + 1)/f^3 + 1/3*(4*f^3*x^3*
log(e^(2*f*x + 2*e) + 1) + 6*f^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*polylo
g(3, -e^(2*f*x + 2*e)) + 3*polylog(4, -e^(2*f*x + 2*e)))*(3*a^2*b*d^3 + b^3
*d^3)/f^4 + 3/2*(3*a^2*b*c*d^2*f + b^3*c*d^2*f + 3*a*b^2*d^3)*(2*f^2*x^2*lo
g(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f
*x + 2*e)))/f^4 + 3/2*(3*a^2*b*c^2*d*f^2 + 6*a*b^2*c*d^2*f + (c^2*d*f^2 + d
^3)*b^3)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f^4 - 1
/2*((3*a^2*b*d^3 + b^3*d^3)*f^4*x^4 + 4*(3*a^2*b*c*d^2*f + b^3*c*d^2*f + 3*
a*b^2*d^3)*f^3*x^3 + 6*(3*a^2*b*c^2*d*f^2 + 6*a*b^2*c*d^2*f + (c^2*d*f^2 +
d^3)*b^3)*f^2*x^2)/f^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \tanh\left(e + f\,x\right)\right)^3 (c + d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^3*(c + d*x)^3,x)
```

```
[Out] int((a + b*tanh(e + f*x))^3*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh\left(e + f\,x\right)\right)^3 (c + d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*tanh(f*x+e))**3,x)
[Out] Integral((a + b*tanh(e + f*x))**3*(c + d*x)**3, x)
```

3.64 $\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx$

Optimal. Leaf size=405

$$\frac{a^3(c+dx)^3}{3d}+\frac{3a^2bd(c+dx)\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{f^2}+\frac{3a^2b(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{a^2b(c+dx)^3}{d}-\frac{3a^2bd^2\mathrm{Li}_3\left(-e^{2(e+fx)}\right)}{2f^3}$$

```
[Out] b^3*c*d*x/f+1/2*b^3*d^2*x^2/f-3*a*b^2*(d*x+c)^2/f+1/3*a^3*(d*x+c)^3/d-a^2*b
*(d*x+c)^3/d+a*b^2*(d*x+c)^3/d-1/3*b^3*(d*x+c)^3/d+6*a*b^2*d*(d*x+c)*ln(1+e
xp(2*f*x+2*e))/f^2+3*a^2*b*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+b^3*(d*x+c)^2*l
n(1+exp(2*f*x+2*e))/f+b^3*d^2*ln(cosh(f*x+e))/f^3+3*a*b^2*d^2*polylog(2,-ex
p(2*f*x+2*e))/f^3+3*a^2*b*d*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2+b^3*d*(d
*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*a^2*b*d^2*polylog(3,-exp(2*f*x+2*e
))/f^3-1/2*b^3*d^2*polylog(3,-exp(2*f*x+2*e))/f^3-b^3*d*(d*x+c)*tanh(f*x+e)
/f^2-3*a*b^2*(d*x+c)^2*tanh(f*x+e)/f-1/2*b^3*(d*x+c)^2*tanh(f*x+e)^2/f
```

Rubi [A] time = 0.70, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3722, 3718, 2190, 2531, 2282, 6589, 3720, 2279, 2391, 32, 3475}

$$\frac{3a^2bd(c+dx)\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{3a^2bd^2\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{3ab^2d^2\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}+\frac{b^3d(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Tanh[e + f*x])^3,x]
```

```
[Out] (b^3*c*d*x)/f + (b^3*d^2*x^2)/(2*f) - (3*a*b^2*(c + d*x)^2)/f + (a^3*(c + d
*x)^3)/(3*d) - (a^2*b*(c + d*x)^3)/d + (a*b^2*(c + d*x)^3)/d - (b^3*(c + d*
x)^3)/(3*d) + (6*a*b^2*d*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f^2 + (3*a^2*b
*(c + d*x)^2*Log[1 + E^(2*(e + f*x))])/f + (b^3*(c + d*x)^2*Log[1 + E^(2*(e
 + f*x))])/f + (b^3*d^2*Log[Cosh[e + f*x]])/f^3 + (3*a*b^2*d^2*PolyLog[2, -
E^(2*(e + f*x))])/f^3 + (3*a^2*b*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/
f^2 + (b^3*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (3*a^2*b*d^2*Pol
yLog[3, -E^(2*(e + f*x))])/(2*f^3) - (b^3*d^2*PolyLog[3, -E^(2*(e + f*x))])
/(2*f^3) - (b^3*d*(c + d*x)*Tanh[e + f*x])/f^2 - (3*a*b^2*(c + d*x)^2*Tanh[
e + f*x])/f - (b^3*(c + d*x)^2*Tanh[e + f*x]^2)/(2*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx = \int \left(a^3(c+dx)^2 + 3a^2b(c+dx)^2\tanh(e+fx) + 3ab^2(c+dx)^2\tanh^2(e+fx)\right.$$

$$= \frac{a^3(c+dx)^3}{3d} + \left(3a^2b\right)\int (c+dx)^2\tanh(e+fx)\,dx + \left(3ab^2\right)\int (c+dx)^2\tan$$

$$= \frac{a^3(c+dx)^3}{3d} - \frac{a^2b(c+dx)^3}{d} - \frac{3ab^2(c+dx)^2\tanh(e+fx)}{f} - \frac{b^3(c+dx)^2\tan}{2f}$$

$$= -\frac{3ab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{a^2b(c+dx)^3}{d} + \frac{ab^2(c+dx)^3}{d} - \frac{b^3(c+dx)^3}{3d}$$

$$= \frac{b^3cdx}{f} + \frac{b^3d^2x^2}{2f} - \frac{3ab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{a^2b(c+dx)^3}{d} + \frac{ab^2(c+d}{d}$$

$$= \frac{b^3cdx}{f} + \frac{b^3d^2x^2}{2f} - \frac{3ab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{a^2b(c+dx)^3}{d} + \frac{ab^2(c+d}{d}$$

$$= \frac{b^3cdx}{f} + \frac{b^3d^2x^2}{2f} - \frac{3ab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{a^2b(c+dx)^3}{d} + \frac{ab^2(c+d}{d}$$

$$= \frac{b^3cdx}{f} + \frac{b^3d^2x^2}{2f} - \frac{3ab^2(c+dx)^2}{f} + \frac{a^3(c+dx)^3}{3d} - \frac{a^2b(c+dx)^3}{d} + \frac{ab^2(c+d}{d}$$

Mathematica [B] time = 11.85, size = 1142, normalized size = 2.82

$$\frac{\operatorname{sech}(e)\left(2d^2f^2x^3\cosh(e)a^3 + 6cdf^2x^2\cosh(e)a^3 + 6c^2f^2x\cosh(e)a^3 + d^2f^2x^3\cosh(e+2fx)a^3 + 3cdf^2x^2\cosh(e\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Tanh[e + f*x])^3,x]
```

```
[Out] (b*((-4*E^(2*e)*f*x*(9*a*b*d*f*(2*c + d*x) + 3*a^2*f^2*(3*c^2 + 3*c*d*x + d
^2*x^2) + b^2*(3*c^2*f^2 + 3*c*d*f^2*x + d^2*(3 + f^2*x^2))))/(1 + E^(2*e))
 + 6*(6*a*b*d*f*(c + d*x) + 3*a^2*f^2*(c + d*x)^2 + b^2*(c^2*f^2 + 2*c*d*f^
2*x + d^2*(1 + f^2*x^2)))*Log[1 + E^(2*(e + f*x))] + 6*d*(3*a*b*d + 3*a^2*f
*(c + d*x) + b^2*f*(c + d*x))*PolyLog[2, -E^(2*(e + f*x))] - 3*(3*a^2 + b^2
)*d^2*PolyLog[3, -E^(2*(e + f*x))]))/(6*f^3) + (Sech[e]*Sech[e + f*x]^2*(6*
b^3*c^2*f*Cosh[e] + 12*b^3*c*d*f*x*Cosh[e] + 6*a^3*c^2*f^2*x*Cosh[e] + 18*a
*b^2*c^2*f^2*x*Cosh[e] + 6*b^3*d^2*f*x^2*Cosh[e] + 6*a^3*c*d*f^2*x^2*Cosh[e
] + 18*a*b^2*c*d*f^2*x^2*Cosh[e] + 2*a^3*d^2*f^2*x^3*Cosh[e] + 6*a*b^2*d^2*
f^2*x^3*Cosh[e] + 3*a^3*c^2*f^2*x*Cosh[e + 2*f*x] + 9*a*b^2*c^2*f^2*x*Cosh[
e + 2*f*x] + 3*a^3*c*d*f^2*x^2*Cosh[e + 2*f*x] + 9*a*b^2*c*d*f^2*x^2*Cosh[e
 + 2*f*x] + a^3*d^2*f^2*x^3*Cosh[e + 2*f*x] + 3*a*b^2*d^2*f^2*x^3*Cosh[e +
2*f*x] + 3*a^3*c^2*f^2*x*Cosh[3*e + 2*f*x] + 9*a*b^2*c^2*f^2*x*Cosh[3*e + 2
*f*x] + 3*a^3*c*d*f^2*x^2*Cosh[3*e + 2*f*x] + 9*a*b^2*c*d*f^2*x^2*Cosh[3*e
+ 2*f*x] + a^3*d^2*f^2*x^3*Cosh[3*e + 2*f*x] + 3*a*b^2*d^2*f^2*x^3*Cosh[3*e
 + 2*f*x] + 6*b^3*c*d*Sinh[e] + 18*a*b^2*c^2*f*Sinh[e] + 6*b^3*d^2*x*Sinh[e
] + 36*a*b^2*c*d*f*x*Sinh[e] + 18*a^2*b*c^2*f^2*x*Sinh[e] + 6*b^3*c^2*f^2*x
*Sinh[e] + 18*a*b^2*d^2*f*x^2*Sinh[e] + 18*a^2*b*c*d*f^2*x^2*Sinh[e] + 6*b^
3*c*d*f^2*x^2*Sinh[e] + 6*a^2*b*d^2*f^2*x^3*Sinh[e] + 2*b^3*d^2*f^2*x^3*Sin
h[e] - 6*b^3*c*d*Sinh[e + 2*f*x] - 18*a*b^2*c^2*f*Sinh[e + 2*f*x] - 6*b^3*d
^2*x*Sinh[e + 2*f*x] - 36*a*b^2*c*d*f*x*Sinh[e + 2*f*x] - 9*a^2*b*c^2*f^2*x
*Sinh[e + 2*f*x] - 3*b^3*c^2*f^2*x*Sinh[e + 2*f*x] - 18*a*b^2*d^2*f*x^2*Sin
h[e + 2*f*x] - 9*a^2*b*c*d*f^2*x^2*Sinh[e + 2*f*x] - 3*b^3*c*d*f^2*x^2*Sinh
[e + 2*f*x] - 3*a^2*b*d^2*f^2*x^3*Sinh[e + 2*f*x] - b^3*d^2*f^2*x^3*Sinh[e
+ 2*f*x] + 9*a^2*b*c^2*f^2*x*Sinh[3*e + 2*f*x] + 3*b^3*c^2*f^2*x*Sinh[3*e +
```

```
 2*f*x] + 9*a^2*b*c*d*f^2*x^2*Sinh[3*e + 2*f*x] + 3*b^3*c*d*f^2*x^2*Sinh[3*
e + 2*f*x] + 3*a^2*b*d^2*f^2*x^3*Sinh[3*e + 2*f*x] + b^3*d^2*f^2*x^3*Sinh[3
*e + 2*f*x]))/(12*f^2)
```

fricas [C] time = 0.84, size = 7298, normalized size = 18.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tanh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*f^3*x^3 + 3*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*c*d*f^3*x^2 + 18*a*b^2*d^2*e^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*c^2*f^3*x - 6*b^3*d^2*e - 2*(3*a^2*b + b^3)*d^2*e^3 + ((a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d^2*f^3*x^3 + 18*a*b^2*d^2*e^2 - 6*b^3*d^2*e - 2*(3*a^2*b + b
^3)*d^2*e^3 - 6*(3*a^2*b + b^3)*c^2*e*f^2 - 3*(6*a*b^2*d^2*f^2 - (a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*c*d*f^3)*x^2 - 6*(6*a*b^2*c*d*e - (3*a^2*b + b^3)*c*d
*e^2)*f - 3*(12*a*b^2*c*d*f^2 + 2*b^3*d^2*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*c^2*f^3)*x)*cosh(f*x + e)^4 + 4*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*f^3
*x^3 + 18*a*b^2*d^2*e^2 - 6*b^3*d^2*e - 2*(3*a^2*b + b^3)*d^2*e^3 - 6*(3*a^
2*b + b^3)*c^2*e*f^2 - 3*(6*a*b^2*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*c*d*f^3)*x^2 - 6*(6*a*b^2*c*d*e - (3*a^2*b + b^3)*c*d*e^2)*f - 3*(12*a*b^2
*c*d*f^2 + 2*b^3*d^2*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*f^3)*x)*cosh(f
*x + e)*sinh(f*x + e)^3 + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*f^3*x^3 + 18
*a*b^2*d^2*e^2 - 6*b^3*d^2*e - 2*(3*a^2*b + b^3)*d^2*e^3 - 6*(3*a^2*b + b^3
)*c^2*e*f^2 - 3*(6*a*b^2*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d*f^3)
*x^2 - 6*(6*a*b^2*c*d*e - (3*a^2*b + b^3)*c*d*e^2)*f - 3*(12*a*b^2*c*d*f^2
+ 2*b^3*d^2*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*f^3)*x)*sinh(f*x + e)^4
 + 6*(3*a*b^2*c^2 - (3*a^2*b + b^3)*c^2*e)*f^2 + 2*((a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*d^2*f^3*x^3 + 18*a*b^2*d^2*e^2 - 6*b^3*d^2*e - 2*(3*a^2*b + b^3)*d
^2*e^3 - 3*(2*(3*a^2*b + b^3)*c^2*e - (3*a*b^2 + b^3)*c^2)*f^2 + 3*((a^3 -
3*a^2*b + 3*a*b^2 - b^3)*c*d*f^3 - (3*a*b^2 - b^3)*d^2*f^2)*x^2 - 3*(12*a*b
^2*c*d*e - b^3*c*d - 2*(3*a^2*b + b^3)*c*d*e^2)*f - 3*(b^3*d^2*f - (a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*c^2*f^3 + 2*(3*a*b^2 - b^3)*c*d*f^2)*x)*cosh(f*x +
e)^2 + 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*f^3*x^3 + 18*a*b^2*d^2*e^2 -
6*b^3*d^2*e - 2*(3*a^2*b + b^3)*d^2*e^3 - 3*(2*(3*a^2*b + b^3)*c^2*e - (3*a
*b^2 + b^3)*c^2)*f^2 + 3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d*f^3 - (3*a*b^
2 - b^3)*d^2*f^2)*x^2 + 3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*f^3*x^3 + 18
*a*b^2*d^2*e^2 - 6*b^3*d^2*e - 2*(3*a^2*b + b^3)*d^2*e^3 - 6*(3*a^2*b + b^3
)*c^2*e*f^2 - 3*(6*a*b^2*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d*f^3)
*x^2 - 6*(6*a*b^2*c*d*e - (3*a^2*b + b^3)*c*d*e^2)*f - 3*(12*a*b^2*c*d*f^2
+ 2*b^3*d^2*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*f^3)*x)*cosh(f*x + e)^2
 - 3*(12*a*b^2*c*d*e - b^3*c*d - 2*(3*a^2*b + b^3)*c*d*e^2)*f - 3*(b^3*d^2*
f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*f^3 + 2*(3*a*b^2 - b^3)*c*d*f^2)*x)
*sinh(f*x + e)^2 - 6*(6*a*b^2*c*d*e - b^3*c*d - (3*a^2*b + b^3)*c*d*e^2)*f
+ 6*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a*b^2*d^2 + (3*a^2*b + b^3)
*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*cosh(f*x + e)^4 + 4*(3*a*b^2*d^2 + (3*a^2
*b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*cosh(f*x + e)*sinh(f*x + e)^3 +
(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*sinh(f*x +
e)^4 + (3*a^2*b + b^3)*c*d*f + 2*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (
3*a^2*b + b^3)*c*d*f)*cosh(f*x + e)^2 + 2*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^
2*f*x + (3*a^2*b + b^3)*c*d*f + 3*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x +
(3*a^2*b + b^3)*c*d*f)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((3*a*b^2*d^2 +
 (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*cosh(f*x + e)^3 + (3*a*b^
2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*cosh(f*x + e))*sin
h(f*x + e))*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) + 6*(3*a*b^2*d^2 + (3*
a^2*b + b^3)*d^2*f*x + (3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b +
b^3)*c*d*f)*cosh(f*x + e)^4 + 4*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3
*a^2*b + b^3)*c*d*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (3*a*b^2*d^2 + (3*a^2*
b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*sinh(f*x + e)^4 + (3*a^2*b + b^3)
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*c*d*f + 2*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*
cosh(f*x + e)^2 + 2*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b + b^3
)*c*d*f + 3*(3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*((3*a*b^2*d^2 + (3*a^2*b + b^3)*d^2*f
*x + (3*a^2*b + b^3)*c*d*f)*cosh(f*x + e)^3 + (3*a*b^2*d^2 + (3*a^2*b + b^3
)*d^2*f*x + (3*a^2*b + b^3)*c*d*f)*cosh(f*x + e))*sinh(f*x + e))*dilog(-I*c
osh(f*x + e) - I*sinh(f*x + e)) - 3*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b
^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 + (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b
 + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3
)*c*d*e)*f)*cosh(f*x + e)^4 + 4*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*
d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)
*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b
^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*
d*e)*f)*sinh(f*x + e)^4 + 2*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*
e^2 - (3*a^2*b + b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*
cosh(f*x + e)^2 + 2*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3
*a^2*b + b^3)*c^2*f^2 + 3*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^
2 - (3*a^2*b + b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*co
sh(f*x + e)^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*sinh(f*x + e)^2
 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f + 4*((6*a*b^2*d^2*e - b^3*d^2 -
 (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^
2*b + b^3)*c*d*e)*f)*cosh(f*x + e)^3 + (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b
 + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)
*c*d*e)*f)*cosh(f*x + e))*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e)
 + I) - 3*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^
3)*c^2*f^2 + (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b
 + b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*cosh(f*x + e)^4
 + 4*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c
^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*cosh(f*x + e)*sinh(f*x
 + e)^3 + (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^
3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*sinh(f*x + e)^4 + 2
*(6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f
^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*cosh(f*x + e)^2 + 2*(6*a*b^
2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2 + 3*(
6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b + b^3)*c^2*f^2
 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*cosh(f*x + e)^2 - 2*(3*a*b^2*
c*d - (3*a^2*b + b^3)*c*d*e)*f)*sinh(f*x + e)^2 - 2*(3*a*b^2*c*d - (3*a^2*b
 + b^3)*c*d*e)*f + 4*((6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 -
(3*a^2*b + b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*cosh(f
*x + e)^3 + (6*a*b^2*d^2*e - b^3*d^2 - (3*a^2*b + b^3)*d^2*e^2 - (3*a^2*b +
 b^3)*c^2*f^2 - 2*(3*a*b^2*c*d - (3*a^2*b + b^3)*c*d*e)*f)*cosh(f*x + e))*s
inh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) - I) + 3*((3*a^2*b + b^3)*d
^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*
d*e*f + ((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*
e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^
2)*x)*cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3
*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a
^2*b + b^3)*c*d*f^2)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^
2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d
*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*sinh(f*x + e)^4 + 2*(
(3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(
3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*cos
h(f*x + e)^2 + 2*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b +
b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 3*((3*a^2*b + b^3)*d^2*f^2*x^2 +
 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3
*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*cosh(f*x + e)^2 + 2*(3*a*b^2*d^2
*f + (3*a^2*b + b^3)*c*d*f^2)*x)*sinh(f*x + e)^2 + 2*(3*a*b^2*d^2*f + (3*a^
2*b + b^3)*c*d*f^2)*x + 4*(((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (
3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*
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a^2*b + b^3)*c*d*f^2)*x)*cosh(f*x + e)^3 + ((3*a^2*b + b^3)*d^2*f^2*x^2 + 6
*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a
*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*cosh(f*x + e))*sinh(f*x + e))*log(
I*cosh(f*x + e) + I*sinh(f*x + e) + 1) + 3*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6
*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + ((3*a^
2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2
*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*cosh(f*x
 + e)^4 + 4*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*
d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*
d*f^2)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*
a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*
b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*sinh(f*x + e)^4 + 2*((3*a^2*b + b^3
)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)
*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d*f^2)*x)*cosh(f*x + e)^2 +
 2*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)*d^2*e^2 +
 2*(3*a^2*b + b^3)*c*d*e*f + 3*((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e
 - (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f +
 (3*a^2*b + b^3)*c*d*f^2)*x)*cosh(f*x + e)^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b
+ b^3)*c*d*f^2)*x)*sinh(f*x + e)^2 + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c*d
*f^2)*x + 4*(((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e - (3*a^2*b + b^3)
*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (3*a^2*b + b^3)*c
*d*f^2)*x)*cosh(f*x + e)^3 + ((3*a^2*b + b^3)*d^2*f^2*x^2 + 6*a*b^2*d^2*e -
 (3*a^2*b + b^3)*d^2*e^2 + 2*(3*a^2*b + b^3)*c*d*e*f + 2*(3*a*b^2*d^2*f + (
3*a^2*b + b^3)*c*d*f^2)*x)*cosh(f*x + e))*sinh(f*x + e))*log(-I*cosh(f*x +
e) - I*sinh(f*x + e) + 1) - 6*((3*a^2*b + b^3)*d^2*cosh(f*x + e)^4 + 4*(3*a
^2*b + b^3)*d^2*cosh(f*x + e)*sinh(f*x + e)^3 + (3*a^2*b + b^3)*d^2*sinh(f*
x + e)^4 + 2*(3*a^2*b + b^3)*d^2*cosh(f*x + e)^2 + (3*a^2*b + b^3)*d^2 + 2*
(3*(3*a^2*b + b^3)*d^2*cosh(f*x + e)^2 + (3*a^2*b + b^3)*d^2)*sinh(f*x + e)
^2 + 4*((3*a^2*b + b^3)*d^2*cosh(f*x + e)^3 + (3*a^2*b + b^3)*d^2*cosh(f*x
+ e))*sinh(f*x + e))*polylog(3, I*cosh(f*x + e) + I*sinh(f*x + e)) - 6*((3*
a^2*b + b^3)*d^2*cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d^2*cosh(f*x + e)*sinh
(f*x + e)^3 + (3*a^2*b + b^3)*d^2*sinh(f*x + e)^4 + 2*(3*a^2*b + b^3)*d^2*c
osh(f*x + e)^2 + (3*a^2*b + b^3)*d^2 + 2*(3*(3*a^2*b + b^3)*d^2*cosh(f*x +
e)^2 + (3*a^2*b + b^3)*d^2)*sinh(f*x + e)^2 + 4*((3*a^2*b + b^3)*d^2*cosh(f
*x + e)^3 + (3*a^2*b + b^3)*d^2*cosh(f*x + e))*sinh(f*x + e))*polylog(3, -I
*cosh(f*x + e) - I*sinh(f*x + e)) + 4*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2
*f^3*x^3 + 18*a*b^2*d^2*e^2 - 6*b^3*d^2*e - 2*(3*a^2*b + b^3)*d^2*e^3 - 6*(
3*a^2*b + b^3)*c^2*e*f^2 - 3*(6*a*b^2*d^2*f^2 - (a^3 - 3*a^2*b + 3*a*b^2 -
b^3)*c*d*f^3)*x^2 - 6*(6*a*b^2*c*d*e - (3*a^2*b + b^3)*c*d*e^2)*f - 3*(12*a
*b^2*c*d*f^2 + 2*b^3*d^2*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*f^3)*x)*co
sh(f*x + e)^3 + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d^2*f^3*x^3 + 18*a*b^2*d^2
*e^2 - 6*b^3*d^2*e - 2*(3*a^2*b + b^3)*d^2*e^3 - 3*(2*(3*a^2*b + b^3)*c^2*e
 - (3*a*b^2 + b^3)*c^2)*f^2 + 3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*d*f^3 -
(3*a*b^2 - b^3)*d^2*f^2)*x^2 - 3*(12*a*b^2*c*d*e - b^3*c*d - 2*(3*a^2*b + b
^3)*c*d*e^2)*f - 3*(b^3*d^2*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c^2*f^3 + 2
*(3*a*b^2 - b^3)*c*d*f^2)*x)*cosh(f*x + e))*sinh(f*x + e))/(f^3*cosh(f*x +
e)^4 + 4*f^3*cosh(f*x + e)*sinh(f*x + e)^3 + f^3*sinh(f*x + e)^4 + 2*f^3*co
sh(f*x + e)^2 + f^3 + 2*(3*f^3*cosh(f*x + e)^2 + f^3)*sinh(f*x + e)^2 + 4*(
f^3*cosh(f*x + e)^3 + f^3*cosh(f*x + e))*sinh(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\left(b\tanh\left(fx+e\right)+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tanh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*(b*tanh(f*x + e) + a)^3, x)
```

maple [B] time = 0.59, size = 1022, normalized size = 2.52

$$-3a^2bcd\,x^2+3a\,b^2cd\,x^2+\frac{4b\,a^2d^2e^3}{f^3}+\frac{2b^3d^2e^2x}{f^2}-\frac{6b^2a\,d^2e^2}{f^3}-\frac{2b^3cd\,e^2}{f^2}-\frac{6b^2a\,d^2x^2}{f}+\frac{3b\,a^2c^2\ln\left(e^{2fx+2e}+1\right)}{f}-\frac{2b^3d^2e^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*tanh(f*x+e))^3,x)
```

```
[Out] -3*a^2*b*c*d*x^2+3*a*b^2*c*d*x^2+4/f^3*b*a^2*d^2*e^3+2/f^2*b^3*d^2*e^2*x-6/
f^3*b^2*a*d^2*e^2-2/f^2*b^3*c*d*e^2-6/f*b^2*a*d^2*x^2+1/f^2*b^3*c*d*polylog
(2,-exp(2*f*x+2*e))+3/f*b*a^2*c^2*ln(exp(2*f*x+2*e)+1)-2/f^3*b^3*d^2*e^2*ln
(exp(f*x+e))-6/f*b*a^2*c^2*ln(exp(f*x+e))+1/f*b^3*ln(exp(2*f*x+2*e)+1)*d^2*
x^2+1/f^2*b^3*polylog(2,-exp(2*f*x+2*e))*d^2*x-1/2*b^3*d^2*polylog(3,-exp(2
*f*x+2*e))/f^3-a^2*b*d^2*x^3+a*b^2*d^2*x^3+a^3*c*d*x^2-b^3*c*d*x^2+1/3*a^3*
d^2*x^3-1/3*b^3*d^2*x^3+c^2*a^3*x+b^3*c^2*x+4/3/f^3*b^3*d^2*e^3-2/f^3*b^3*d
^2*ln(exp(f*x+e))+1/f^3*b^3*d^2*ln(exp(2*f*x+2*e)+1)+1/f*b^3*c^2*ln(exp(2*f
*x+2*e)+1)-2/f*b^3*c^2*ln(exp(f*x+e))+3*a*b^2*d^2*polylog(2,-exp(2*f*x+2*e)
)/f^3-3/2*a^2*b*d^2*polylog(3,-exp(2*f*x+2*e))/f^3+2*b^2*(3*a*d^2*f*x^2*exp
(2*f*x+2*e)+b*d^2*f*x^2*exp(2*f*x+2*e)+6*a*c*d*f*x*exp(2*f*x+2*e)+2*b*c*d*f
*x*exp(2*f*x+2*e)+3*a*c^2*f*exp(2*f*x+2*e)+3*a*d^2*f*x^2+b*c^2*f*exp(2*f*x+
2*e)+b*d^2*x*exp(2*f*x+2*e)+6*a*c*d*f*x+b*c*d*exp(2*f*x+2*e)+3*a*c^2*f+b*d^
2*x+c*b*d)/f^2/(exp(2*f*x+2*e)+1)^2+3*c^2*a^2*b*x+3*c^2*a*b^2*x-12/f*b*a^2*
c*d*e*x+12/f^2*b*a^2*c*d*e*ln(exp(f*x+e))+6/f*b*ln(exp(2*f*x+2*e)+1)*a^2*c*
d*x+6/f^2*b*a^2*d^2*e^2*x-6/f^2*b*a^2*c*d*e^2-4/f*b^3*c*d*e*x-12/f^2*b^2*a*
d^2*e*x+3/f^2*b*a^2*c*d*polylog(2,-exp(2*f*x+2*e))+3/f^2*b*polylog(2,-exp(2
*f*x+2*e))*a^2*d^2*x-12/f^2*b^2*a*c*d*ln(exp(f*x+e))-6/f^3*b*a^2*d^2*e^2*ln
(exp(f*x+e))+3/f*b*ln(exp(2*f*x+2*e)+1)*a^2*d^2*x^2+6/f^2*b^2*ln(exp(2*f*x+
2*e)+1)*a*d^2*x+2/f*b^3*ln(exp(2*f*x+2*e)+1)*c*d*x+6/f^2*b^2*a*c*d*ln(exp(2
*f*x+2*e)+1)+4/f^2*b^3*c*d*e*ln(exp(f*x+e))+12/f^3*b^2*a*d^2*e*ln(exp(f*x+e
))
```

maxima [B] time = 0.97, size = 869, normalized size = 2.15

$$\frac{1}{3}a^3d^2x^3+a^3cdx^2+b^3c^2\left(x+\frac{e}{f}+\frac{\log\left(e^{\left(-2fx-2e\right)}+1\right)}{f}+\frac{2\,e^{\left(-2fx-2e\right)}}{f\left(2\,e^{\left(-2fx-2e\right)}+e^{\left(-4fx-4e\right)}+1\right)}\right)+a^3c^2x+\frac{3\,a^2bc^2\log\left(\mathrm{co}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*tanh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/3*a^3*d^2*x^3 + a^3*c*d*x^2 + b^3*c^2*(x + e/f + log(e^(-2*f*x - 2*e) + 1
)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) +
 a^3*c^2*x + 3*a^2*b*c^2*log(cosh(f*x + e))/f + 1/3*(18*a*b^2*c^2*f + 6*b^3
*c*d + (3*a^2*b*d^2*f^2 + 3*a*b^2*d^2*f^2 + b^3*d^2*f^2)*x^3 + 3*(3*a^2*b*c
*d*f^2 + b^3*c*d*f^2 + 3*(c*d*f^2 + 2*d^2*f)*a*b^2)*x^2 + 3*(2*b^3*d^2 + 3*
(c^2*f^2 + 4*c*d*f)*a*b^2)*x + (9*a*b^2*c^2*f^2*x*e^(4*e) + (3*a^2*b*d^2*f^
2*e^(4*e) + 3*a*b^2*d^2*f^2*e^(4*e) + b^3*d^2*f^2*e^(4*e))*x^3 + 3*(3*a^2*b
*c*d*f^2*e^(4*e) + 3*a*b^2*c*d*f^2*e^(4*e) + b^3*c*d*f^2*e^(4*e))*x^2)*e^(4
*f*x) + 2*(9*a*b^2*c^2*f*e^(2*e) + 3*b^3*c*d*e^(2*e) + (3*a^2*b*d^2*f^2*e^(
2*e) + 3*a*b^2*d^2*f^2*e^(2*e) + b^3*d^2*f^2*e^(2*e))*x^3 + 3*(3*a^2*b*c*d*
f^2*e^(2*e) + 3*(c*d*f^2*e^(2*e) + d^2*f*e^(2*e))*a*b^2 + (c*d*f^2*e^(2*e)
+ d^2*f*e^(2*e))*b^3)*x^2 + 3*(3*(c^2*f^2*e^(2*e) + 2*c*d*f*e^(2*e))*a*b^2
+ (2*c*d*f*e^(2*e) + d^2*e^(2*e))*b^3)*x)*e^(2*f*x))/(f^2*e^(4*f*x + 4*e) +
 2*f^2*e^(2*f*x + 2*e) + f^2) - 2*(6*a*b^2*c*d*f + b^3*d^2)*x/f^2 + 1/2*(3*
a^2*b*d^2 + b^3*d^2)*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(
2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))/f^3 + (3*a^2*b*c*d*f + b^3*c*
d*f + 3*a*b^2*d^2)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)
```

```
))/f^3 + (6*a*b^2*c*d*f + b^3*d^2)*log(e^(2*f*x + 2*e) + 1)/f^3 - 2/3*((3*a
^2*b*d^2 + b^3*d^2)*f^3*x^3 + 3*(3*a^2*b*c*d*f + b^3*c*d*f + 3*a*b^2*d^2)*f
^2*x^2)/f^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \tanh\left(e + f\,x\right)\right)^3 (c + d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^3*(c + d*x)^2,x)
```

```
[Out] int((a + b*tanh(e + f*x))^3*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh\left(e + f\,x\right)\right)^3 (c + dx)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*tanh(f*x+e))**3,x)
```

```
[Out] Integral((a + b*tanh(e + f*x))**3*(c + d*x)**2, x)
```

3.65 $\int (c+dx)(a+b\tanh(e+fx))^3\,dx$

Optimal. Leaf size=261

$$\frac{a^3(c+dx)^2}{2d}+\frac{3a^2b(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{3a^2b(c+dx)^2}{2d}+\frac{3a^2bd\mathrm{Li}_2\left(-e^{2(e+fx)}\right)}{2f^2}-\frac{3ab^2(c+dx)\tanh(e+fx)}{f}+3a$$

```
[Out] 3*a*b^2*c*x+1/2*b^3*d*x/f+3/2*a*b^2*d*x^2+1/2*a^3*(d*x+c)^2/d-3/2*a^2*b*(d*
x+c)^2/d-1/2*b^3*(d*x+c)^2/d+3*a^2*b*(d*x+c)*ln(1+exp(2*f*x+2*e))/f+b^3*(d*
x+c)*ln(1+exp(2*f*x+2*e))/f+3*a*b^2*d*ln(cosh(f*x+e))/f^2+3/2*a^2*b*d*polyl
og(2,-exp(2*f*x+2*e))/f^2+1/2*b^3*d*polylog(2,-exp(2*f*x+2*e))/f^2-1/2*b^3*
d*tanh(f*x+e)/f^2-3*a*b^2*(d*x+c)*tanh(f*x+e)/f-1/2*b^3*(d*x+c)*tanh(f*x+e)
^2/f
```

Rubi [A] time = 0.35, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3722, 3718, 2190, 2279, 2391, 3720, 3475, 3473, 8}

$$\frac{3a^2bd\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{b^3d\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}+\frac{3a^2b(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{3a^2b(c+dx)^2}{2d}+\frac{a^3(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Tanh[e + f*x])^3,x]
```

```
[Out] 3*a*b^2*c*x + (b^3*d*x)/(2*f) + (3*a*b^2*d*x^2)/2 + (a^3*(c + d*x)^2)/(2*d)
 - (3*a^2*b*(c + d*x)^2)/(2*d) - (b^3*(c + d*x)^2)/(2*d) + (3*a^2*b*(c + d*
x)*Log[1 + E^(2*(e + f*x))])/f + (b^3*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f
 + (3*a*b^2*d*Log[Cosh[e + f*x]])/f^2 + (3*a^2*b*d*PolyLog[2, -E^(2*(e + f*
x))])/(2*f^2) + (b^3*d*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) - (b^3*d*Tanh[
e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)*Tanh[e + f*x])/f - (b^3*(c + d*x)*Ta
nh[e + f*x]^2)/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3722

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)(a+b\tanh(e+fx))^3\,dx &= \int \left(a^3(c+dx)+3a^2b(c+dx)\tanh(e+fx)+3ab^2(c+dx)\tanh^2(e+fx)\right.\\
&= \frac{a^3(c+dx)^2}{2d}+\left(3a^2b\right)\int (c+dx)\tanh(e+fx)\,dx+\left(3ab^2\right)\int (c+dx)\tan\\
&= \frac{a^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)^2}{2d}-\frac{3ab^2(c+dx)\tanh(e+fx)}{f}-\frac{b^3(c+dx)\ta\\
&= 3ab^2cx+\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)^2}{2d}-\frac{b^3(c+dx)^2}{2d}+\frac{3a^2b(}{}\\
&= 3ab^2cx+\frac{b^3dx}{2f}+\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)^2}{2d}-\frac{b^3(c+dx)^2}{2d}\\
&= 3ab^2cx+\frac{b^3dx}{2f}+\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)^2}{2d}-\frac{b^3(c+dx)^2}{2d}\\
&= 3ab^2cx+\frac{b^3dx}{2f}+\frac{3}{2}ab^2dx^2+\frac{a^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)^2}{2d}-\frac{b^3(c+dx)^2}{2d}
\end{aligned}$$

Mathematica [A] time = 3.68, size = 265, normalized size = 1.02

$$\frac{\cosh(e+fx)(a+b\tanh(e+fx))^3\left(-bd\left(3a^2+b^2\right)\mathrm{Li}_2\left(-e^{-2(e+fx)}\right)\cosh^2(e+fx)+\cosh^2(e+fx)\left(-2b\log(co\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Tanh[e + f*x])^3,x]
```

```
[Out] (Cosh[e + f*x]*(b^3*f*(c + d*x) + Cosh[e + f*x]^2*(-((e + f*x)*(-3*a^2*b*d*
(e + f*x) - b^3*d*(e + f*x) + a^3*(-2*c*f + d*(e - f*x)) + 3*a*b^2*(-2*c*f
+ d*(e - f*x)))) + 2*b*(3*a^2 + b^2)*d*(e + f*x)*Log[1 + E^(-2*(e + f*x))]
- 2*b*(-3*a*b*d + 3*a^2*(d*e - c*f) + b^2*(d*e - c*f))*Log[Cosh[e + f*x]])
- b*(3*a^2 + b^2)*d*Cosh[e + f*x]^2*PolyLog[2, -E^(-2*(e + f*x))] - (b^2*(b
*d + 6*a*f*(c + d*x))*Sinh[2*(e + f*x)])/2)*(a + b*Tanh[e + f*x])^3)/(2*f^2
*(a*Cosh[e + f*x] + b*Sinh[e + f*x])^3)
```

fricas [C] time = 1.03, size = 3262, normalized size = 12.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] 1/2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*c*f^2*x + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2
 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*
a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*cosh(f*x + e)^4 + 4*(
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3
)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*c*f^2)*x)*cosh(f*x + e)*sinh(f*x + e)^3 + ((a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b + b^3)*d*e^2 - 4*(3*a^2*b +
 b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2)*x)*si
nh(f*x + e)^4 + 2*b^3*d + 2*(3*a^2*b + b^3)*d*e^2 + 2*((a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + b^3*d + 2*(3*a^2*b + b^3)*d*e^2 - 2*
(2*(3*a^2*b + b^3)*c*e - (3*a*b^2 + b^3)*c)*f + 2*((a^3 - 3*a^2*b + 3*a*b^2
 - b^3)*c*f^2 - (3*a*b^2 - b^3)*d*f)*x)*cosh(f*x + e)^2 + 2*((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + b^3*d + 2*(3*a^2*b + b^3)*d*e^
2 + 3*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*
b + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*c*f^2)*x)*cosh(f*x + e)^2 - 2*(2*(3*a^2*b + b^3)*c*e - (3*
a*b^2 + b^3)*c)*f + 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*f^2 - (3*a*b^2 - b
^3)*d*f)*x)*sinh(f*x + e)^2 + 4*(3*a*b^2*c - (3*a^2*b + b^3)*c*e)*f + 2*((3
*a^2*b + b^3)*d*cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d*cosh(f*x + e)*sinh(f*
x + e)^3 + (3*a^2*b + b^3)*d*sinh(f*x + e)^4 + 2*(3*a^2*b + b^3)*d*cosh(f*x
 + e)^2 + 2*(3*(3*a^2*b + b^3)*d*cosh(f*x + e)^2 + (3*a^2*b + b^3)*d)*sinh(
f*x + e)^2 + (3*a^2*b + b^3)*d + 4*((3*a^2*b + b^3)*d*cosh(f*x + e)^3 + (3*
a^2*b + b^3)*d*cosh(f*x + e))*sinh(f*x + e))*dilog(I*cosh(f*x + e) + I*sinh
(f*x + e)) + 2*((3*a^2*b + b^3)*d*cosh(f*x + e)^4 + 4*(3*a^2*b + b^3)*d*cos
h(f*x + e)*sinh(f*x + e)^3 + (3*a^2*b + b^3)*d*sinh(f*x + e)^4 + 2*(3*a^2*b
 + b^3)*d*cosh(f*x + e)^2 + 2*(3*(3*a^2*b + b^3)*d*cosh(f*x + e)^2 + (3*a^2
*b + b^3)*d)*sinh(f*x + e)^2 + (3*a^2*b + b^3)*d + 4*((3*a^2*b + b^3)*d*cos
h(f*x + e)^3 + (3*a^2*b + b^3)*d*cosh(f*x + e))*sinh(f*x + e))*dilog(-I*cos
h(f*x + e) - I*sinh(f*x + e)) + 2*((3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^
2*b + b^3)*c*f)*cosh(f*x + e)^4 + 4*(3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a
^2*b + b^3)*c*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (3*a*b^2*d - (3*a^2*b + b^
3)*d*e + (3*a^2*b + b^3)*c*f)*sinh(f*x + e)^4 + 3*a*b^2*d - (3*a^2*b + b^3)
*d*e + (3*a^2*b + b^3)*c*f + 2*(3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b
+ b^3)*c*f)*cosh(f*x + e)^2 + 2*(3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b
 + b^3)*c*f + 3*(3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b^3)*c*f)*cos
h(f*x + e)^2)*sinh(f*x + e)^2 + 4*((3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^
2*b + b^3)*c*f)*cosh(f*x + e)^3 + (3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2
*b + b^3)*c*f)*cosh(f*x + e))*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x +
 e) + I) + 2*((3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b^3)*c*f)*cosh(
f*x + e)^4 + 4*(3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b^3)*c*f)*cosh
(f*x + e)*sinh(f*x + e)^3 + (3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b
^3)*c*f)*sinh(f*x + e)^4 + 3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b^3
```

```
)*c*f + 2*(3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b^3)*c*f)*cosh(f*x
+ e)^2 + 2*(3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b^3)*c*f + 3*(3*a*
b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b^3)*c*f)*cosh(f*x + e)^2)*sinh(f*
x + e)^2 + 4*((3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b^3)*c*f)*cosh(
f*x + e)^3 + (3*a*b^2*d - (3*a^2*b + b^3)*d*e + (3*a^2*b + b^3)*c*f)*cosh(f
*x + e))*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) - I) + 2*(((3*a^2
*b + b^3)*d*f*x + (3*a^2*b + b^3)*d*e)*cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)
*d*f*x + (3*a^2*b + b^3)*d*e)*cosh(f*x + e)*sinh(f*x + e)^3 + ((3*a^2*b + b
^3)*d*f*x + (3*a^2*b + b^3)*d*e)*sinh(f*x + e)^4 + (3*a^2*b + b^3)*d*f*x +
(3*a^2*b + b^3)*d*e + 2*((3*a^2*b + b^3)*d*f*x + (3*a^2*b + b^3)*d*e)*cosh(
f*x + e)^2 + 2*((3*a^2*b + b^3)*d*f*x + (3*a^2*b + b^3)*d*e + 3*((3*a^2*b +
 b^3)*d*f*x + (3*a^2*b + b^3)*d*e)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(((
3*a^2*b + b^3)*d*f*x + (3*a^2*b + b^3)*d*e)*cosh(f*x + e)^3 + ((3*a^2*b + b
^3)*d*f*x + (3*a^2*b + b^3)*d*e)*cosh(f*x + e))*sinh(f*x + e))*log(I*cosh(f
*x + e) + I*sinh(f*x + e) + 1) + 2*(((3*a^2*b + b^3)*d*f*x + (3*a^2*b + b^3
)*d*e)*cosh(f*x + e)^4 + 4*((3*a^2*b + b^3)*d*f*x + (3*a^2*b + b^3)*d*e)*co
sh(f*x + e)*sinh(f*x + e)^3 + ((3*a^2*b + b^3)*d*f*x + (3*a^2*b + b^3)*d*e)
*sinh(f*x + e)^4 + (3*a^2*b + b^3)*d*f*x + (3*a^2*b + b^3)*d*e + 2*((3*a^2*
b + b^3)*d*f*x + (3*a^2*b + b^3)*d*e)*cosh(f*x + e)^2 + 2*((3*a^2*b + b^3)*
d*f*x + (3*a^2*b + b^3)*d*e + 3*((3*a^2*b + b^3)*d*f*x + (3*a^2*b + b^3)*d*
e)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(((3*a^2*b + b^3)*d*f*x + (3*a^2*b
+ b^3)*d*e)*cosh(f*x + e)^3 + ((3*a^2*b + b^3)*d*f*x + (3*a^2*b + b^3)*d*e)
*cosh(f*x + e))*sinh(f*x + e))*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1)
+ 4*(((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*f^2*x^2 - 12*a*b^2*d*e + 2*(3*a^2*b
 + b^3)*d*e^2 - 4*(3*a^2*b + b^3)*c*e*f - 2*(6*a*b^2*d*f - (a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*c*f^2)*x)*cosh(f*x + e)^3 + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d*f^2*x^2 - 12*a*b^2*d*e + b^3*d + 2*(3*a^2*b + b^3)*d*e^2 - 2*(2*(3*a^2*
b + b^3)*c*e - (3*a*b^2 + b^3)*c)*f + 2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*c*
f^2 - (3*a*b^2 - b^3)*d*f)*x)*cosh(f*x + e))*sinh(f*x + e))/(f^2*cosh(f*x +
 e)^4 + 4*f^2*cosh(f*x + e)*sinh(f*x + e)^3 + f^2*sinh(f*x + e)^4 + 2*f^2*c
osh(f*x + e)^2 + 2*(3*f^2*cosh(f*x + e)^2 + f^2)*sinh(f*x + e)^2 + f^2 + 4*
(f^2*cosh(f*x + e)^3 + f^2*cosh(f*x + e))*sinh(f*x + e))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\left(b \tanh\left(fx + e\right) + a\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(b*tanh(f*x + e) + a)^3, x)
```

maple [A] time = 0.50, size = 459, normalized size = 1.76

$$-\frac{3a^2bd\,x^2}{2}+3c\,a^2bx+\frac{a^3d\,x^2}{2}-\frac{b^3d\,x^2}{2}+c\,a^3x+b^3cx+3a\,b^2cx+\frac{3a\,b^2d\,x^2}{2}-\frac{6b\,a^2dex}{f}+\frac{6bd\,a^2e\ln\left(\mathrm{e}^{fx+e}\right)}{f^2}+\frac{3b\ln\left(\mathrm{e}^{2f}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*tanh(f*x+e))^3,x)
```

```
[Out] -3/2*a^2*b*d*x^2+3*c*a^2*b*x+1/2*a^3*d*x^2-1/2*b^3*d*x^2+c*a^3*x+b^3*c*x+3*
a*b^2*c*x+3/2*a*b^2*d*x^2+1/2*b^3*d*polylog(2,-exp(2*f*x+2*e))/f^2-6/f*b*a^
2*d*e*x+6/f^2*b*d*a^2*e*ln(exp(f*x+e))+3/f*b*ln(exp(2*f*x+2*e)+1)*a^2*d*x-2
/f*b^3*d*e*x-3/f^2*b*a^2*d*e^2+1/f*b^3*ln(exp(2*f*x+2*e)+1)*d*x-6/f*b*a^2*c
*ln(exp(f*x+e))+3/f*b*a^2*c*ln(exp(2*f*x+2*e)+1)-6/f^2*b^2*d*a*ln(exp(f*x+e
))+3/f^2*b^2*d*a*ln(exp(2*f*x+2*e)+1)+2/f^2*b^3*d*e*ln(exp(f*x+e))+3/2*a^2*
b*d*polylog(2,-exp(2*f*x+2*e))/f^2-1/f^2*b^3*d*e^2-2/f*b^3*c*ln(exp(f*x+e))
+1/f*b^3*c*ln(exp(2*f*x+2*e)+1)+b^2*(6*a*d*f*x*exp(2*f*x+2*e)+2*b*d*f*x*exp
```

```
(2*f*x+2*e)+6*a*c*f*exp(2*f*x+2*e)+2*b*c*f*exp(2*f*x+2*e)+6*a*d*f*x+b*d*exp
(2*f*x+2*e)+6*a*c*f+b*d)/f^2/(exp(2*f*x+2*e)+1)^2
```

maxima [A] time = 0.62, size = 475, normalized size = 1.82

$$\frac{1}{2}a^3dx^2+b^3c\left(x+\frac{e}{f}+\frac{\log\left(e^{\left(-2fx-2e\right)}+1\right)}{f}+\frac{2e^{\left(-2fx-2e\right)}}{f\left(2e^{\left(-2fx-2e\right)}+e^{\left(-4fx-4e\right)}+1\right)}\right)+a^3cx-\frac{6ab^2dx}{f}+\frac{3a^2bc\log\left(\cosh\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a^3*d*x^2 + b^3*c*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x
- 2*e)/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) + a^3*c*x - 6*a*b^2
*d*x/f + 3*a^2*b*c*log(cosh(f*x + e))/f - (3*a^2*b*d + b^3*d)*x^2 + 3*a*b^2
*d*log(e^(2*f*x + 2*e) + 1)/f^2 + 1/2*(12*a*b^2*c*f + 6*(c*f^2 + 2*d*f)*a*b
^2*x + 2*b^3*d + (3*a^2*b*d*f^2 + 3*a*b^2*d*f^2 + b^3*d*f^2)*x^2 + (6*a*b^2
*c*f^2*x*e^(4*e) + (3*a^2*b*d*f^2*e^(4*e) + 3*a*b^2*d*f^2*e^(4*e) + b^3*d*f
^2*e^(4*e))*x^2)*e^(4*f*x) + 2*(6*a*b^2*c*f*e^(2*e) + b^3*d*e^(2*e) + (3*a^
2*b*d*f^2*e^(2*e) + 3*a*b^2*d*f^2*e^(2*e) + b^3*d*f^2*e^(2*e))*x^2 + 2*(b^3
*d*f*e^(2*e) + 3*(c*f^2*e^(2*e) + d*f*e^(2*e))*a*b^2)*x)*e^(2*f*x))/(f^2*e^
(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) + 1/2*(3*a^2*b*d + b^3*d)*(2*f
*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\tanh\left(e+fx\right)\right)^3\,(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^3*(c + d*x),x)
```

```
[Out] int((a + b*tanh(e + f*x))^3*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\tanh\left(e+fx\right)\right)^3\,(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*tanh(f*x+e))**3,x)
```

```
[Out] Integral((a + b*tanh(e + f*x))**3*(c + d*x), x)
```

3.66 $\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\tanh(e+fx))^3}{c+dx},x\right)$$

```
[Out] Unintegrable((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Tanh[e + f*x])^3/(c + d*x),x]
```

```
[Out] Defer[Int][(a + b*Tanh[e + f*x])^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$$

Mathematica [A] time = 63.49, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x),x]
```

```
[Out] Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\tanh\left(fx+e\right)^3+3\,ab^2\tanh\left(fx+e\right)^2+3\,a^2b\tanh\left(fx+e\right)+a^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b^3*tanh(f*x + e)^3 + 3*a*b^2*tanh(f*x + e)^2 + 3*a^2*b*tanh(f*x
+ e) + a^3)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\tanh\left(fx+e\right)+a\right)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*tanh(f*x + e) + a)^3/(d*x + c), x)
```

maple [A] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \tanh\left(fx + e\right)\right)^3}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

```
[Out] int((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 \log\left(dx + c\right)}{d} + \frac{\left(3\,a^2 b + 3\,ab^2 + b^3\right) \log\left(dx + c\right)}{d} + \frac{6\,ab^2 dfx + 6\,ab^2 cf - b^3 d + \left(6\,ab^2 cfe^{(2e)} + \left(2\,cfe^{(2e}\right.\right.}{d^2 f^2 x^2 + 2\,cdf^2 x + c^2 f^2 + \left(d^2 f^2 x^2 e^{(4e)} + 2\,cdf^2 xe^{(4e)} + c^2 f^2 e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] a^3*log(d*x + c)/d + (3*a^2*b + 3*a*b^2 + b^3)*log(d*x + c)/d + (6*a*b^2*d*
f*x + 6*a*b^2*c*f - b^3*d + (6*a*b^2*c*f*e^(2*e) + (2*c*f*e^(2*e) - d*e^(2*
e))*b^3 + 2*(3*a*b^2*d*f*e^(2*e) + b^3*d*f*e^(2*e))*x)*e^(2*f*x))/(d^2*f^2*
x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2*e^(4*e) + 2*c*d*f^2*x*e^(4*e) +
c^2*f^2*e^(4*e))*e^(4*f*x) + 2*(d^2*f^2*x^2*e^(2*e) + 2*c*d*f^2*x*e^(2*e) +
 c^2*f^2*e^(2*e))*e^(2*f*x)) - integrate(2*(3*a^2*b*c^2*f^2 - 3*a*b^2*c*d*f
 + (c^2*f^2 + d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 + (6*a^2*b*c*d
*f^2 + 2*b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3
*c^2*d*f^2*x + c^3*f^2 + (d^3*f^2*x^3*e^(2*e) + 3*c*d^2*f^2*x^2*e^(2*e) + 3
*c^2*d*f^2*x*e^(2*e) + c^3*f^2*e^(2*e))*e^(2*f*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b \tanh\left(e + f\,x\right)\right)^3}{c + d\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^3/(c + d*x),x)
```

```
[Out] int((a + b*tanh(e + f*x))^3/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \tanh\left(e + fx\right)\right)^3}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))**3/(d*x+c),x)
```

```
[Out] Integral((a + b*tanh(e + f*x))**3/(c + d*x), x)
```

3.67 $\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(a+b\tanh(e+fx))^3}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Tanh[e + f*x])^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(a + b*Tanh[e + f*x])^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$$

Mathematica [A] time = 55.42, size = 0, normalized size = 0.00

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x)^2,x]
```

```
[Out] Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x)^2, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^3\tanh\left(fx+e\right)^3+3\,ab^2\tanh\left(fx+e\right)^2+3\,a^2b\tanh\left(fx+e\right)+a^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*tanh(f*x + e)^3 + 3*a*b^2*tanh(f*x + e)^2 + 3*a^2*b*tanh(f*x
+ e) + a^3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\tanh\left(fx+e\right)+a\right)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*tanh(f*x + e) + a)^3/(d*x + c)^2, x)
```

maple [A] time = 1.21, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\tanh\left(fx + e\right)\right)^3}{\left(dx + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

```
[Out] int((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^3}{d^2x + cd} - \frac{3\,a^2bc^2f^2 + 3\left(c^2f^2 - 2\,cdf\right)ab^2 + \left(c^2f^2 + 2\,d^2\right)b^3 + \left(3\,a^2bd^2f^2 + 3\,ab^2d^2f^2 + b^3d^2f^2\right)x^2 + 2\left(3\,a^2bcd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -a^3/(d^2*x + c*d) - (3*a^2*b*c^2*f^2 + 3*(c^2*f^2 - 2*c*d*f)*a*b^2 + (c^2*
f^2 + 2*d^2)*b^3 + (3*a^2*b*d^2*f^2 + 3*a*b^2*d^2*f^2 + b^3*d^2*f^2)*x^2 +
2*(3*a^2*b*c*d*f^2 + b^3*c*d*f^2 + 3*(c*d*f^2 - d^2*f)*a*b^2)*x + (3*a^2*b*
c^2*f^2*e^(4*e) + 3*a*b^2*c^2*f^2*e^(4*e) + b^3*c^2*f^2*e^(4*e) + (3*a^2*b*
d^2*f^2*e^(4*e) + 3*a*b^2*d^2*f^2*e^(4*e) + b^3*d^2*f^2*e^(4*e))*x^2 + 2*(3
*a^2*b*c*d*f^2*e^(4*e) + 3*a*b^2*c*d*f^2*e^(4*e) + b^3*c*d*f^2*e^(4*e))*x)*
e^(4*f*x) + 2*(3*a^2*b*c^2*f^2*e^(2*e) + 3*(c^2*f^2*e^(2*e) - c*d*f*e^(2*e)
)*a*b^2 + (c^2*f^2*e^(2*e) - c*d*f*e^(2*e) + d^2*e^(2*e))*b^3 + (3*a^2*b*d^
2*f^2*e^(2*e) + 3*a*b^2*d^2*f^2*e^(2*e) + b^3*d^2*f^2*e^(2*e))*x^2 + (6*a^2
*b*c*d*f^2*e^(2*e) + 3*(2*c*d*f^2*e^(2*e) - d^2*f*e^(2*e))*a*b^2 + (2*c*d*f
^2*e^(2*e) - d^2*f*e^(2*e))*b^3)*x)*e^(2*f*x))/(d^4*f^2*x^3 + 3*c*d^3*f^2*x
^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3*e^(4*e) + 3*c*d^3*f^2*x^2*e
^(4*e) + 3*c^2*d^2*f^2*x*e^(4*e) + c^3*d*f^2*e^(4*e))*e^(4*f*x) + 2*(d^4*f^
2*x^3*e^(2*e) + 3*c*d^3*f^2*x^2*e^(2*e) + 3*c^2*d^2*f^2*x*e^(2*e) + c^3*d*f
^2*e^(2*e))*e^(2*f*x)) - integrate(2*(3*a^2*b*c^2*f^2 - 6*a*b^2*c*d*f + (c^
2*f^2 + 3*d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 + 2*(3*a^2*b*c*d*f
^2 + b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^4*f^2*x^4 + 4*c*d^3*f^2*x^3 + 6*c^2
*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4*e^(2*e) + 4*c*d^3*f^2
*x^3*e^(2*e) + 6*c^2*d^2*f^2*x^2*e^(2*e) + 4*c^3*d*f^2*x*e^(2*e) + c^4*f^2*
e^(2*e))*e^(2*f*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a + b\tanh\left(e + f\,x\right)\right)^3}{\left(c + d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(e + f*x))^3/(c + d*x)^2,x)
```

```
[Out] int((a + b*tanh(e + f*x))^3/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\tanh\left(e + fx\right)\right)^3}{\left(c + dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(f*x+e))**3/(d*x+c)**2,x)
[Out] Integral((a + b*tanh(e + f*x))**3/(c + d*x)**2, x)
```

3.68 $\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$

Optimal. Leaf size=212

$$\frac{3bd^2(c+dx)\mathrm{Li}_3\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^3\left(a^2-b^2\right)}+\frac{3bd(c+dx)^2\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^2\left(a^2-b^2\right)}-\frac{b(c+dx)^3\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{f\left(a^2-b^2\right)}+\frac{3bd^3\mathrm{Li}_4\left(-\frac{(a}{}\right.}$$

```
[Out] 1/4*(d*x+c)^4/(a+b)/d-b*(d*x+c)^3*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2
)/f+3/2*b*d*(d*x+c)^2*polylog(2,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2+
3/2*b*d^2*(d*x+c)*polylog(3,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^3+3/4*
b*d^3*polylog(4,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^4
```

Rubi [A] time = 0.34, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3732, 2190, 2531, 6609, 2282, 6589}

$$\frac{3bd^2(c+dx)\mathrm{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^3\left(a^2-b^2\right)}+\frac{3bd(c+dx)^2\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^2\left(a^2-b^2\right)}+\frac{3bd^3\mathrm{PolyLog}\left(4,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right.}{4f^4\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Tanh[e + f*x]),x]
```

```
[Out] (c + d*x)^4/(4*(a + b)*d) - (b*(c + d*x)^3*Log[1 + (a - b)/((a + b)*E^(2*(e
 + f*x)))])/((a^2 - b^2)*f) + (3*b*d*(c + d*x)^2*PolyLog[2, -((a - b)/((a +
 b)*E^(2*(e + f*x))))])/(2*(a^2 - b^2)*f^2) + (3*b*d^2*(c + d*x)*PolyLog[3,
 -((a - b)/((a + b)*E^(2*(e + f*x))))])/(2*(a^2 - b^2)*f^3) + (3*b*d^3*Poly
Log[4, -((a - b)/((a + b)*E^(2*(e + f*x))))])/(4*(a^2 - b^2)*f^4)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3732

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int
[((c + d*x)^m*E^Simp[2*I*(e + f*x), x])/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
```

```
*I*(e + f*x), x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \frac{(c+dx)^4}{4(a+b)d} + (2b)\int \frac{e^{-2(e+fx)}(c+dx)^3}{(a+b)^2+\left(a^2-b^2\right)e^{-2(e+fx)}}\,dx$$

$$= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{(3bd)\int(c+dx)^2\log\left(1+\frac{\left(a^2-b^2\right)e^{-2}}{(a+b)^2}\right.}{\left(a^2-b^2\right)f}$$

$$= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\left(a^2-b^2\right)f^2} -$$

$$= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\left(a^2-b^2\right)f^2} +$$

$$= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\left(a^2-b^2\right)f^2} +$$

$$= \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\left(a^2-b^2\right)f^2} +$$

Mathematica [A] time = 3.49, size = 239, normalized size = 1.13

$$\frac{x\cosh(e)\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)}{4(a\cosh(e)+b\sinh(e))} + \frac{b\left(\frac{3d\left(2f^2(c+dx)^2\mathrm{Li}_2\left(\frac{(b-a)e^{-2(e+fx)}}{a+b}\right)+d\left(2f(c+dx)\mathrm{Li}_3\left(\frac{(b-a)e^{-2(e+fx)}}{a+b}\right)+d\mathrm{Li}_4\left(\frac{(b-a)e^{-2(e+f}\right.\right.\right.}{f^4(a-b)}\right.}{4(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Tanh[e + f*x]),x]
```

```
[Out] (b*((-2*(c + d*x)^4)/(d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))) - (4*(c + d*x
)^3*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*f) + (3*d*(2*f^2*(
c + d*x)^2*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))] + d*(2*f*(c + d*x
)*PolyLog[3, (-a + b)/((a + b)*E^(2*(e + f*x)))] + d*PolyLog[4, (-a + b)/((
```

```
a + b)*E^(2*(e + f*x)))]))))/((a - b)*f^4)))/(4*(a + b)) + (x*(4*c^3 + 6*c^2
*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[e])/(4*(a*Cosh[e] + b*Sinh[e]))
```

fricas [C] time = 0.76, size = 740, normalized size = 3.49

$$\frac{(a+b)d^3f^4x^4 + 4(a+b)cd^2f^4x^3 + 6(a+b)c^2df^4x^2 + 4(a+b)c^3f^4x - 24bd^3\text{polylog}\left(4, \sqrt{-\frac{a+b}{a-b}}\left(\cosh\left(fx+e\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*((a + b)*d^3*f^4*x^4 + 4*(a + b)*c*d^2*f^4*x^3 + 6*(a + b)*c^2*d*f^4*x^
2 + 4*(a + b)*c^3*f^4*x - 24*b*d^3*polylog(4, sqrt(-(a + b)/(a - b))*(cosh(
f*x + e) + sinh(f*x + e))) - 24*b*d^3*polylog(4, -sqrt(-(a + b)/(a - b))*(c
osh(f*x + e) + sinh(f*x + e))) - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^
2*d*f^2)*dilog(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) - 12
*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-sqrt(-(a + b)/(a -
b))*(cosh(f*x + e) + sinh(f*x + e))) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b
*c^2*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x
+ e) + 2*(a - b)*sqrt(-(a + b)/(a - b))) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f +
 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(
f*x + e) - 2*(a - b)*sqrt(-(a + b)/(a - b))) - 4*(b*d^3*f^3*x^3 + 3*b*c*d^2
*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)
*log(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 4*(b*d^3
*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*
f + 3*b*c^2*d*e*f^2)*log(-sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x
+ e)) + 1) + 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, sqrt(-(a + b)/(a - b))*(
cosh(f*x + e) + sinh(f*x + e))) + 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, -sq
rt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))))/((a^2 - b^2)*f^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{b\tanh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*tanh(f*x + e) + a), x)
```

maple [B] time = 0.68, size = 1143, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*tanh(f*x+e)),x)
```

```
[Out] 1/f^4*b/(a+b)*d^3*e^3/(a-b)*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)-2/f^4
*b/(a+b)*d^3*e^3/(a-b)*ln(exp(f*x+e))-3/2/f^3*b/(a+b)*c*d^2/(-a+b)*polylog(
3,(a+b)*exp(2*f*x+2*e)/(-a+b))+3/2/f^2*b/(a+b)*c^2*d/(-a+b)*polylog(2,(a+b)
*exp(2*f*x+2*e)/(-a+b))+1/f*b/(a+b)*d^3/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-
a+b))*x^3+3/2/f^2*b/(a+b)*d^3/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))
*x^2-3/2/f^3*b/(a+b)*d^3/(-a+b)*polylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b))*x+1/
(a+b)*c*d^2*x^3+3/2/(a+b)*c^2*d*x^2+6/f^3*b/(a+b)*c*d^2*e^2/(a-b)*ln(exp(f*
x+e))+3/f^2*b/(a+b)*c^2*d*e/(a-b)*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)
+3/f^2*b/(a+b)*c^2*d/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e+3/f*b/(a+b)
*c*d^2/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x^2+3/f^2*b/(a+b)*c*d^2/(-a
```

```
+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))*x-3/f^3*b/(a+b)*c*d^2*e^2/(a-b)*
ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)-3/f^3*b/(a+b)*c*d^2*e^2/(-a+b)*ln
(1-(a+b)*exp(2*f*x+2*e)/(-a+b))+3/f*b/(a+b)*c^2*d/(-a+b)*ln(1-(a+b)*exp(2*f
*x+2*e)/(-a+b))*x-6/f^2*b/(a+b)*c^2*d*e/(a-b)*ln(exp(f*x+e))+6/f^2*b/(a+b)/
(-a+b)*c*d^2*e^2*x-6/f*b/(a+b)/(-a+b)*c^2*d*e*x+1/4/(a+b)*d^3*x^4+1/(a+b)*c
^3*x-2/f^3*b/(a+b)/(-a+b)*e^3*d^3*x-2*b/(a+b)/(-a+b)*c*d^2*x^3+4/f^3*b/(a+b
)/(-a+b)*c*d^2*e^3-3*b/(a+b)/(-a+b)*c^2*d*x^2-3/f^2*b/(a+b)/(-a+b)*c^2*d*e^
2+1/f^4*b/(a+b)*d^3*e^3/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))-3/2/f^4*b/
(a+b)/(-a+b)*e^4*d^3-1/2*b/(a+b)/(-a+b)*d^3*x^4+2/f*b/(a+b)*c^3/(a-b)*ln(ex
p(f*x+e))-1/f*b/(a+b)*c^3/(a-b)*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+3
/4/f^4*b/(a+b)*d^3/(-a+b)*polylog(4,(a+b)*exp(2*f*x+2*e)/(-a+b))
```

maxima [B] time = 0.70, size = 528, normalized size = 2.49

$$-\frac{3\left(2fx\log\left(\frac{\left(ae^{(2e)}+be^{(2e)}\right)e^{(2fx)}}{a-b}+1\right)+\mathrm{Li}_2\left(-\frac{\left(ae^{(2e)}+be^{(2e)}\right)e^{(2fx)}}{a-b}\right)\right)bc^2d}{2\left(a^2f^2-b^2f^2\right)}-\frac{3\left(2f^2x^2\log\left(\frac{\left(ae^{(2e)}+be^{(2e)}\right)e^{(2fx)}}{a-b}+1\right)+2f\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -3/2*(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(-(a*
e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c^2*d/(a^2*f^2 - b^2*f^2) - 3/2*
(2*f^2*x^2*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilog
(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*e) + b*e
^(2*e))*e^(2*f*x)/(a - b)))*b*c*d^2/(a^2*f^3 - b^2*f^3) - 1/3*(4*f^3*x^3*lo
g((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 6*f^2*x^2*dilog(-(a*e^(2
*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - 6*f*x*polylog(3, -(a*e^(2*e) + b*e^(2
*e))*e^(2*f*x)/(a - b)) + 3*polylog(4, -(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(
a - b)))*b*d^3/(a^2*f^4 - b^2*f^4) - c^3*(b*log(-(a - b)*e^(-2*f*x - 2*e) -
 a - b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f)) + 1/2*(b*d^3*f^4*x^4 + 4*b
*c*d^2*f^4*x^3 + 6*b*c^2*d*f^4*x^2)/(a^2*f^4 - b^2*f^4) + 1/4*(d^3*x^4 + 4*
c*d^2*x^3 + 6*c^2*d*x^2)/(a + b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{a+b\tanh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*tanh(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a + b*tanh(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{a+b\tanh\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*tanh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**3/(a + b*tanh(e + f*x)), x)
```

3.69 $\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$

Optimal. Leaf size=157

$$\frac{bd(c+dx)\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2\left(a^2-b^2\right)}-\frac{b(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{f\left(a^2-b^2\right)}+\frac{bd^2\mathrm{Li}_3\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^3\left(a^2-b^2\right)}+\frac{(c+dx)^3}{3d(a+b)}$$

```
[Out] 1/3*(d*x+c)^3/(a+b)/d-b*(d*x+c)^2*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2
)/f+b*d*(d*x+c)*polylog(2,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2+1/2*b*
d^2*polylog(3,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^3
```

Rubi [A] time = 0.29, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3732, 2190, 2531, 2282, 6589}

$$\frac{bd(c+dx)\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2\left(a^2-b^2\right)}+\frac{bd^2\mathrm{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^3\left(a^2-b^2\right)}-\frac{b(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{f\left(a^2-b^2\right)}+\frac{(c+dx)}{3d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Tanh[e + f*x]),x]
```

```
[Out] (c + d*x)^3/(3*(a + b)*d) - (b*(c + d*x)^2*Log[1 + (a - b)/((a + b)*E^(2*(e
 + f*x)))])/((a^2 - b^2)*f) + (b*d*(c + d*x)*PolyLog[2, -((a - b)/((a + b)*
E^(2*(e + f*x))))])/((a^2 - b^2)*f^2) + (b*d^2*PolyLog[3, -((a - b)/((a + b
)*E^(2*(e + f*x))))])/(2*(a^2 - b^2)*f^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3732

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int
[((c + d*x)^m*E^Simp[2*I*(e + f*x), x])/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2,
```

```
0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx = \frac{(c+dx)^3}{3(a+b)d} + (2b)\int \frac{e^{-2(e+fx)}(c+dx)^2}{(a+b)^2+\left(a^2-b^2\right)e^{-2(e+fx)}}\,dx$$

$$= \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{(2bd)\int(c+dx)\log\left(1+\frac{\left(a^2-b^2\right)e^{-2(e}}{(a+b)^2}\right.}{\left(a^2-b^2\right)f}$$

$$= \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{bd(c+dx)\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f^2} - \frac{(b}{}$$

$$= \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{bd(c+dx)\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f^2} + \frac{(b}{}$$

$$= \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{bd(c+dx)\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f^2} + \frac{bd}{}$$

Mathematica [A] time = 3.87, size = 191, normalized size = 1.22

$$\frac{x\cosh(e)\left(3c^2+3cdx+d^2x^2\right)}{3(a\cosh(e)+b\sinh(e))} + \frac{b\left(\frac{3d\left(2f(c+dx)\mathrm{Li}_2\left(\frac{(b-a)e^{-2(e+fx)}}{a+b}\right)+d\mathrm{Li}_3\left(\frac{(b-a)e^{-2(e+fx)}}{a+b}\right)\right)}{f^3(a-b)} - \frac{6(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{f(a-b)} - \frac{}{d(a(e}\right.}{6(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Tanh[e + f*x]),x]
```

```
[Out] (b*((-4*(c + d*x)^3)/(d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))) - (6*(c + d*x
)^2*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*f) + (3*d*(2*f*(c
+ d*x)*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))] + d*PolyLog[3, (-a +
b)/((a + b)*E^(2*(e + f*x)))]))/((a - b)*f^3)))/(6*(a + b)) + (x*(3*c^2 + 3
*c*d*x + d^2*x^2)*Cosh[e])/(3*(a*Cosh[e] + b*Sinh[e]))
```

fricas [C] time = 0.60, size = 500, normalized size = 3.18

$$\frac{(a+b)d^2f^3x^3 + 3(a+b)cdf^3x^2 + 3(a+b)c^2f^3x + 6bd^2\mathrm{polylog}\left(3,\sqrt{-\frac{a+b}{a-b}}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*((a + b)*d^2*f^3*x^3 + 3*(a + b)*c*d*f^3*x^2 + 3*(a + b)*c^2*f^3*x + 6*
b*d^2*polylog(3, sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) +
```

```
6*b*d^2*polylog(3, -sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)))
 - 6*(b*d^2*f*x + b*c*d*f)*dilog(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + si
nh(f*x + e))) - 6*(b*d^2*f*x + b*c*d*f)*dilog(-sqrt(-(a + b)/(a - b))*(cosh
(f*x + e) + sinh(f*x + e))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2
*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt(-(a + b)/
(a - b))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2*(a + b)*cosh(f*x
+ e) + 2*(a + b)*sinh(f*x + e) - 2*(a - b)*sqrt(-(a + b)/(a - b))) - 3*(b*d
^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(sqrt(-(a + b)/(a
- b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2
*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-sqrt(-(a + b)/(a - b))*(cosh(f*x + e) +
sinh(f*x + e)) + 1))/((a^2 - b^2)*f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{b\tanh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*tanh(f*x + e) + a), x)
```

maple [B] time = 0.66, size = 720, normalized size = 4.59

$$\frac{d^2x^3}{3a+3b}+\frac{cd\,x^2}{a+b}+\frac{c^2x}{a+b}+\frac{2b\,c^2\ln\left(\mathrm{e}^{fx+e}\right)}{f\,(a+b)\,(a-b)}-\frac{b\,c^2\ln\left(a\,\mathrm{e}^{2fx+2e}+b\,\mathrm{e}^{2fx+2e}+a-b\right)}{f\,(a+b)\,(a-b)}+\frac{2b\,d^2e^2\ln\left(\mathrm{e}^{fx+e}\right)}{f^3\,(a+b)\,(a-b)}-\frac{b\,d^2e^2\ln\left(a\,\mathrm{e}^{}\right.}{f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

```
[Out] 1/3/(a+b)*d^2*x^3+1/(a+b)*c*d*x^2+1/(a+b)*c^2*x+2/f*b/(a+b)*c^2/(a-b)*ln(ex
p(f*x+e))-1/f*b/(a+b)*c^2/(a-b)*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+2
/f^3*b/(a+b)*d^2*e^2/(a-b)*ln(exp(f*x+e))-1/f^3*b/(a+b)*d^2*e^2/(a-b)*ln(a*
exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)-2/3*b/(a+b)/(-a+b)*d^2*x^3+2/f^2*b/(a+
b)/(-a+b)*d^2*e^2*x+4/3/f^3*b/(a+b)/(-a+b)*d^2*e^3+1/f*b/(a+b)*d^2/(-a+b)*l
n(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x^2-1/f^3*b/(a+b)*d^2/(-a+b)*ln(1-(a+b)*ex
p(2*f*x+2*e)/(-a+b))*e^2+1/f^2*b/(a+b)*d^2/(-a+b)*polylog(2,(a+b)*exp(2*f*x
+2*e)/(-a+b))*x-1/2/f^3*b/(a+b)*d^2/(-a+b)*polylog(3,(a+b)*exp(2*f*x+2*e)/(
-a+b))-4/f^2*b/(a+b)*c*d*e/(a-b)*ln(exp(f*x+e))+2/f^2*b/(a+b)*c*d*e/(a-b)*l
n(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+2/f*b/(a+b)*c*d/(-a+b)*ln(1-(a+b)*
exp(2*f*x+2*e)/(-a+b))*x+2/f^2*b/(a+b)*c*d/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)
/(-a+b))*e-2*b/(a+b)/(-a+b)*d*c*x^2-4/f*b/(a+b)/(-a+b)*d*c*e*x-2/f^2*b/(a+b
)/(-a+b)*d*c*e^2+1/f^2*b/(a+b)*c*d/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-
a+b))
```

maxima [B] time = 0.87, size = 337, normalized size = 2.15

$$-\frac{\left(2\,fx\log\left(\frac{\left(ae^{(2e)}+be^{(2e)}\right)e^{(2fx)}}{a-b}+1\right)+\mathrm{Li}_2\left(-\frac{\left(ae^{(2e)}+be^{(2e)}\right)e^{(2fx)}}{a-b}\right)\right)bcd}{a^2f^2-b^2f^2}-\frac{\left(2\,f^2x^2\log\left(\frac{\left(ae^{(2e)}+be^{(2e)}\right)e^{(2fx)}}{a-b}+1\right)+2\,fx\mathrm{Li}_2\left(-\right.\right.}{2\left(a^2f^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(-(a*e^(2
*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c*d/(a^2*f^2 - b^2*f^2) - 1/2*(2*f^2
```

```
*x^2*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilog(-(a*e
^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*e) + b*e^(2*e)
)*e^(2*f*x)/(a - b)))*b*d^2/(a^2*f^3 - b^2*f^3) - c^2*(b*log(-(a - b)*e^(-2
*f*x - 2*e) - a - b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f)) + 2/3*(b*d^2*
f^3*x^3 + 3*b*c*d*f^3*x^2)/(a^2*f^3 - b^2*f^3) + 1/3*(d^2*x^3 + 3*c*d*x^2)/
(a + b)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{a+b\tanh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*tanh(e + f*x)),x)
```

```
[Out] int((c + d*x)^2/(a + b*tanh(e + f*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{a+b\tanh\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*tanh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**2/(a + b*tanh(e + f*x)), x)
```

3.70 $\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx$

Optimal. Leaf size=108

$$-\frac{b(c+dx)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{f\left(a^2-b^2\right)}+\frac{bd\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^2\left(a^2-b^2\right)}+\frac{(c+dx)^2}{2d(a+b)}$$

```
[Out] 1/2*(d*x+c)^2/(a+b)/d-b*(d*x+c)*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/
f+1/2*b*d*polylog(2,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2
```

Rubi [A] time = 0.17, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3732, 2190, 2279, 2391}

$$\frac{bd\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f^2\left(a^2-b^2\right)}-\frac{b(c+dx)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{f\left(a^2-b^2\right)}+\frac{(c+dx)^2}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Tanh[e + f*x]),x]
```

```
[Out] (c + d*x)^2/(2*(a + b)*d) - (b*(c + d*x)*Log[1 + (a - b)/((a + b)*E^(2*(e +
 f*x)))])/((a^2 - b^2)*f) + (b*d*PolyLog[2, -((a - b)/((a + b)*E^(2*(e + f*
x))))])/(2*(a^2 - b^2)*f^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3732

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int
[((c + d*x)^m*E^Simp[2*I*(e + f*x), x])/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx = \frac{(c+dx)^2}{2(a+b)d} + (2b)\int \frac{e^{-2(e+fx)}(c+dx)}{(a+b)^2+\left(a^2-b^2\right)e^{-2(e+fx)}}\,dx$$

$$= \frac{(c+dx)^2}{2(a+b)d} - \frac{b(c+dx)\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{(bd)\int \log\left(1+\frac{\left(a^2-b^2\right)e^{-2(e+fx)}}{(a+b)^2}\right)dx}{\left(a^2-b^2\right)f}$$

$$= \frac{(c+dx)^2}{2(a+b)d} - \frac{b(c+dx)\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} - \frac{(bd)\,\text{Subst}\left(\int \frac{\log\left(1+\frac{\left(a^2-b^2\right)x}{(a+b)^2}\right)}{x}\,dx, x, e^{-}\right.}{2\left(a^2-b^2\right)f^2}$$

$$= \frac{(c+dx)^2}{2(a+b)d} - \frac{b(c+dx)\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)f} + \frac{bd\,\text{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2\left(a^2-b^2\right)f^2}$$

Mathematica [A] time = 3.33, size = 144, normalized size = 1.33

$$\frac{b\left(-\frac{2(c+dx)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{f(a-b)} - \frac{2(c+dx)^2}{d\left(a\left(e^{2e}+1\right)+b\left(e^{2e}-1\right)\right)} + \frac{d\,\text{Li}_2\left(\frac{(b-a)e^{-2(e+fx)}}{a+b}\right)}{f^2(a-b)}\right)}{2(a+b)} + \frac{x\cosh(e)(2c+dx)}{2(a\cosh(e)+b\sinh(e))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Tanh[e + f*x]),x]
```

```
[Out] (b*((-2*(c + d*x)^2)/(d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))) - (2*(c + d*x
)*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*f) + (d*PolyLog[2, (
-a + b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*f^2)))/(2*(a + b)) + (x*(2*c +
 d*x)*Cosh[e])/(2*(a*Cosh[e] + b*Sinh[e]))
```

fricas [B] time = 0.60, size = 306, normalized size = 2.83

$$\frac{(a+b)df^2x^2 + 2(a+b)cf^2x - 2\,bd\,\text{Li}_2\left(\sqrt{-\frac{a+b}{a-b}}\left(\cosh\left(fx+e\right)+\sinh\left(fx+e\right)\right)\right) - 2\,bd\,\text{Li}_2\left(-\sqrt{-\frac{a+b}{a-b}}\left(\cosh\left(f\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((a + b)*d*f^2*x^2 + 2*(a + b)*c*f^2*x - 2*b*d*dilog(sqrt(-(a + b)/(a -
 b))*(cosh(f*x + e) + sinh(f*x + e))) - 2*b*d*dilog(-sqrt(-(a + b)/(a - b))
*(cosh(f*x + e) + sinh(f*x + e))) + 2*(b*d*e - b*c*f)*log(2*(a + b)*cosh(f*
x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt(-(a + b)/(a - b))) + 2*(b
*d*e - b*c*f)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) - 2*(a
- b)*sqrt(-(a + b)/(a - b))) - 2*(b*d*f*x + b*d*e)*log(sqrt(-(a + b)/(a - b
))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 2*(b*d*f*x + b*d*e)*log(-sqrt(-(a
 + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1))/((a^2 - b^2)*f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{b\tanh\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*tanh(f*x + e) + a), x)
```

maple [B] time = 0.66, size = 357, normalized size = 3.31

$$\frac{d\,x^2}{2b+2a}+\frac{cx}{a+b}+\frac{2bc\ln\left(\mathrm{e}^{fx+e}\right)}{f\,(a+b)\,(a-b)}-\frac{bc\ln\left(a\,\mathrm{e}^{2fx+2e}+b\,\mathrm{e}^{2fx+2e}+a-b\right)}{f\,(a+b)\,(a-b)}+\frac{bd\ln\left(1-\frac{(a+b)\mathrm{e}^{2fx+2e}}{-a+b}\right)x}{f\,(a+b)\,(-a+b)}+\frac{bd\ln\left(1-\frac{(a+b)\mathrm{e}}{-a}\right.}{f^2\,(a+b)\,(-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*tanh(f*x+e)),x)
```

```
[Out] 1/2/(a+b)*d*x^2+1/(a+b)*c*x+2/f*b/(a+b)*c/(a-b)*ln(exp(f*x+e))-1/f*b/(a+b)*
c/(a-b)*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+1/f*b/(a+b)*d/(-a+b)*ln(1
-(a+b)*exp(2*f*x+2*e)/(-a+b))*x+1/f^2*b/(a+b)*d/(-a+b)*ln(1-(a+b)*exp(2*f*x
+2*e)/(-a+b))*e-b/(a+b)/(-a+b)*d*x^2-2/f*b/(a+b)/(-a+b)*d*e*x-1/f^2*b/(a+b)
/(-a+b)*d*e^2+1/2/f^2*b/(a+b)*d/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b
))-2/f^2*b/(a+b)*d*e/(a-b)*ln(exp(f*x+e))+1/f^2*b/(a+b)*d*e/(a-b)*ln(a*exp(
2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(4\,b\int\frac{x}{a^2-b^2+\left(a^2e^{(2\,e)}+2\,abe^{(2\,e)}+b^2e^{(2\,e)}\right)e^{\left(2\,fx\right)}}\,dx+\frac{x^2}{a+b}\right)d-c\left(\frac{b\log\left(-(a-b)e^{\left(-2\,fx-2\,e\right)}-a-b\right)}{\left(a^2-b^2\right)f}-\frac{fx+}{(a+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*b*integrate(x/(a^2 - b^2 + (a^2*e^(2*e) + 2*a*b*e^(2*e) + b^2*e^(2*e
))*e^(2*f*x)), x) + x^2/(a + b))*d - c*(b*log(-(a - b)*e^(-2*f*x - 2*e) - a
 - b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{c+d\,x}{a+b\tanh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*tanh(e + f*x)),x)
```

```
[Out] int((c + d*x)/(a + b*tanh(e + f*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{c+dx}{a+b\tanh\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tanh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)/(a + b*tanh(e + f*x)), x)
```

3.71 $\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\tanh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Tanh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Tanh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$$

Mathematica [A] time = 15.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])), x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{adx+ac+(bdx+bc)\tanh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*tanh(f*x + e)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(b\tanh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*tanh(f*x + e) + a)), x)
```

maple [A] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+b\tanh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$2\,b\int \frac{1}{a^2c-b^2c+\left(a^2d-b^2d\right)x+\left(a^2ce^{(2\,e)}+2\,abce^{(2\,e)}+b^2ce^{(2\,e)}+\left(a^2de^{(2\,e)}+2\,abde^{(2\,e)}+b^2de^{(2\,e)}\right)x\right)e^{\left(2\,fx\right)}}\,dx+\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/(a^2*c - b^2*c + (a^2*d - b^2*d)*x + (a^2*c*e^(2*e) + 2*a*b
*c*e^(2*e) + b^2*c*e^(2*e) + (a^2*d*e^(2*e) + 2*a*b*d*e^(2*e) + b^2*d*e^(2*
e))*x)*e^(2*f*x)), x) + log(d*x + c)/(a*d + b*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\tanh\left(e+f\,x\right)\right)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tanh(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a + b*tanh(e + f*x))*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tanh\left(e+fx\right)\right)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*tanh(e + f*x))*(c + d*x)), x)
```

3.72 $\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\tanh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Tanh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Tanh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$$

Mathematica [A] time = 24.33, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])), x]
```

fricas [A] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{ad^2x^2+2\,acdx+ac^2+\left(bd^2x^2+2\,bcdx+bc^2\right)\tanh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*tanh(f*x + e)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(b\tanh\left(fx+e\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*tanh(f*x + e) + a)), x)
```

maple [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(a+b\tanh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$2\,b\int \frac{1}{a^2c^2-b^2c^2+\left(a^2d^2-b^2d^2\right)x^2+2\left(a^2cd-b^2cd\right)x+\left(a^2c^2e^{(2\,e)}+2\,abc^2e^{(2\,e)}+b^2c^2e^{(2\,e)}+\left(a^2d^2e^{(2\,e)}+2\,abd^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*b*integrate(1/(a^2*c^2 - b^2*c^2 + (a^2*d^2 - b^2*d^2)*x^2 + 2*(a^2*c*d -
 b^2*c*d)*x + (a^2*c^2*e^(2*e) + 2*a*b*c^2*e^(2*e) + b^2*c^2*e^(2*e) + (a^2
*d^2*e^(2*e) + 2*a*b*d^2*e^(2*e) + b^2*d^2*e^(2*e))*x^2 + 2*(a^2*c*d*e^(2*e
) + 2*a*b*c*d*e^(2*e) + b^2*c*d*e^(2*e))*x)*e^(2*f*x)), x) - 1/(a*c*d + b*c
*d + (a*d^2 + b*d^2)*x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\tanh\left(e+f\,x\right)\right)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tanh(e + f*x))*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*tanh(e + f*x))*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tanh\left(e+fx\right)\right)\,\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*tanh(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*tanh(e + f*x))*(c + d*x)**2), x)
```

3.73 $\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=642

$$\frac{3b^2d^2(c+dx)\mathrm{Li}_2\left(-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}-\frac{3b^2d^2(c+dx)\mathrm{Li}_3\left(-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}+\frac{3b^2d(c+dx)^2\mathrm{Li}_2\left(-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{3b^2d(c+d}$$

```
[Out] -2*b^2*(d*x+c)^3/(a^2-b^2)^2/f+2*b^2*(d*x+c)^3/(a-b)/(a+b)^2/(a-b+(a+b)*exp
(2*f*x+2*e))/f+1/4*(d*x+c)^4/(a-b)^2/d+3*b^2*d*(d*x+c)^2*ln(1+(a+b)*exp(2*f
*x+2*e)/(a-b))/(a^2-b^2)^2/f^2-2*b*(d*x+c)^3*ln(1+(a+b)*exp(2*f*x+2*e)/(a-b
))/(a-b)^2/(a+b)/f+2*b^2*(d*x+c)^3*ln(1+(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^
2)^2/f+3*b^2*d^2*(d*x+c)*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2
/f^3-3*b*d*(d*x+c)^2*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f
^2+3*b^2*d*(d*x+c)^2*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^2
-3/2*b^2*d^3*polylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^4+3*b*d^2
*(d*x+c)*polylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^3-3*b^2*d^2
*(d*x+c)*polylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3-3/2*b*d^3*p
olylog(4,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^4+3/2*b^2*d^3*polylog
(4,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^4
```

Rubi [A] time = 2.21, antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3734, 2190, 2531, 6609, 2282, 6589, 2254, 2185, 2184, 2191}

$$\frac{3b^2d^2(c+dx)\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}-\frac{3b^2d^2(c+dx)\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}+\frac{3b^2d(c+dx)^2\mathrm{PolyLog}\left(2,-\right.}{f^2\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] (-2*b^2*(c + d*x)^3)/((a^2 - b^2)^2*f) + (2*b^2*(c + d*x)^3)/((a - b)*(a +
b)^2*(a - b + (a + b)*E^(2e + 2*f*x))*f) + (c + d*x)^4/(4*(a - b)^2*d) + (
3*b^2*d*(c + d*x)^2*Log[1 + ((a + b)*E^(2e + 2*f*x))/(a - b)])/((a^2 - b^2
)^2*f^2) - (2*b*(c + d*x)^3*Log[1 + ((a + b)*E^(2e + 2*f*x))/(a - b)])/((a
 - b)^2*(a + b)*f) + (2*b^2*(c + d*x)^3*Log[1 + ((a + b)*E^(2e + 2*f*x))/(
a - b)])/((a^2 - b^2)^2*f) + (3*b^2*d^2*(c + d*x)*PolyLog[2, -(((a + b)*E^(
2e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f^3) - (3*b*d*(c + d*x)^2*PolyLog[2
, -(((a + b)*E^(2e + 2*f*x))/(a - b))])/((a - b)^2*(a + b)*f^2) + (3*b^2*d
*(c + d*x)^2*PolyLog[2, -(((a + b)*E^(2e + 2*f*x))/(a - b))])/((a^2 - b^2)
^2*f^2) - (3*b^2*d^3*PolyLog[3, -(((a + b)*E^(2e + 2*f*x))/(a - b))])/(2*(
a^2 - b^2)^2*f^4) + (3*b*d^2*(c + d*x)*PolyLog[3, -(((a + b)*E^(2e + 2*f*x
))/(a - b))])/((a - b)^2*(a + b)*f^3) - (3*b^2*d^2*(c + d*x)*PolyLog[3, -((
(a + b)*E^(2e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f^3) - (3*b*d^3*PolyLog[
4, -(((a + b)*E^(2e + 2*f*x))/(a - b))])/(2*(a - b)^2*(a + b)*f^4) + (3*b^
2*d^3*PolyLog[4, -(((a + b)*E^(2e + 2*f*x))/(a - b))])/(2*(a^2 - b^2)^2*f^
4)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2185

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2254

```
Int[((a_.) + (b_.)*(F_)^(u_))^(p_.)*((c_.) + (d_.)*(F_)^(v_))^(q_.)*((e_.)
+ (f_.)*(x_))^(m_.), x_Symbol] :> With[{w = ExpandIntegrand[(e + f*x)^m, (a
 + b*F^u)^p*(c + d*F^v)^q, x]}, Int[w, x] /; SumQ[w]] /; FreeQ[{F, a, b, c,
 d, e, f, m}, x] && IntegersQ[p, q] && LinearQ[{u, v}, x] && RationalQ[Simp
lify[u/v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3734

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - (2*I*b)/(a^2
+ b^2 + (a - I*b)^2*E^(2*I*(e + f*x))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^3}{(a-b)^2} + \frac{4be^{2e+2fx}(c+dx)^3}{(a-b)^2\left(-a\left(1-\frac{b}{a}\right)-a\left(1+\frac{b}{a}\right)e^{2e+2fx}\right)} + \frac{4b^2e^{4e+4fx}(c+dx)^3}{(a-b)^2\left(a\left(1-\frac{b}{a}\right)+a\left(1+\frac{b}{a}\right)e^{2e+2fx}\right)^2} \right) dx$$

$$= \frac{(c+dx)^4}{4(a-b)^2d} + \frac{(4b)\int \frac{e^{2e+2fx}(c+dx)^3}{-a\left(1-\frac{b}{a}\right)-a\left(1+\frac{b}{a}\right)e^{2e+2fx}}\,dx}{(a-b)^2} + \frac{\left(4b^2\right)\int \frac{e^{4e+4fx}(c+dx)^3}{\left(a\left(1-\frac{b}{a}\right)+a\left(1+\frac{b}{a}\right)e^{2e+2fx}\right)^2}\,dx}{(a-b)^2}$$

$$= \frac{(c+dx)^4}{4(a-b)^2d} - \frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} + \frac{\left(4b^2\right)\int \left(\frac{(c+dx)^3}{(a+b)^2} + \frac{(a-b)^2(c+dx)^3}{(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)} \right) dx}{(a-$$

$$= \frac{(c+dx)^4}{4(a-b)^2d} + \frac{b^2(c+dx)^4}{\left(a^2-b^2\right)^2d} - \frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} - \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-$$

$$= \frac{(c+dx)^4}{4(a-b)^2d} - \frac{b^2(c+dx)^4}{\left(a^2-b^2\right)^2d} - \frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} - \frac{3bd(c+dx)^2\mathrm{Li}_2\left(-$$

$$= \frac{2b^2(c+dx)^3}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^4}{4(a-b)^2d} - \frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f}$$

$$= -\frac{2b^2(c+dx)^3}{\left(a^2-b^2\right)^2f} + \frac{2b^2(c+dx)^3}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^4}{4(a-b)^2d} - \frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f}$$

$$= -\frac{2b^2(c+dx)^3}{\left(a^2-b^2\right)^2f} + \frac{2b^2(c+dx)^3}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^4}{4(a-b)^2d} + \frac{3b^2d(c+dx)^2\mathrm{Li}_2\left(-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^2} - \frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f}$$

$$= -\frac{2b^2(c+dx)^3}{\left(a^2-b^2\right)^2f} + \frac{2b^2(c+dx)^3}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^4}{4(a-b)^2d} + \frac{3b^2d(c+dx)^2\mathrm{Li}_2\left(-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^2} - \frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f}$$

$$= -\frac{2b^2(c+dx)^3}{\left(a^2-b^2\right)^2f} + \frac{2b^2(c+dx)^3}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^4}{4(a-b)^2d} + \frac{3b^2d(c+dx)^2\mathrm{Li}_2\left(-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^2} - \frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f}$$

Mathematica [A] time = 13.08, size = 1019, normalized size = 1.59

$$\frac{b\left(-4a\left(b\left(-1+e^{2e}\right)+a\left(1+e^{2e}\right)\right)f^3x^3\log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)d^4+3a\left(b\left(-1+e^{2e}\right)+a\left(1+e^{2e}\right)\right)\left(2f^2\mathrm{Li}_2\left(\frac{(b-a)e^{-2(e}\right.\right.}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] (b*(4*(a - b)*b*d*f^3*(c + d*x)^3 - 2*a*(a - b)*f^4*(c + d*x)^4 - 12*c*d^2*
(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))*f^2*(-(b*d) + a*c*f)*x*Log[1 + (a - b)
/((a + b)*E^(2*(e + f*x)))] + 6*d^3*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))*f^
2*(b*d - 2*a*c*f)*x^2*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))] - 4*a*d^4*
(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))*f^3*x^3*Log[1 + (a - b)/((a + b)*E^(2*
(e + f*x)))] + 2*c^2*d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))*f^2*(-3*b*d + 2
*a*c*f)*(2*f*x - Log[a - b + (a + b)*E^(2*(e + f*x))]) + 6*c*d^2*(b*(-1 + E
^(2*e)) + a*(1 + E^(2*e)))*f*(-(b*d) + a*c*f)*PolyLog[2, (-a + b)/((a + b)*
E^(2*(e + f*x)))] - 3*d^3*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))*(b*d - 2*a*c
*f)*(2*f*x*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[3, (-a
+ b)/((a + b)*E^(2*(e + f*x)))]) + 3*a*d^4*(b*(-1 + E^(2*e)) + a*(1 + E^(2*
e)))*(2*f^2*x^2*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))] + 2*f*x*Poly
Log[3, (-a + b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[4, (-a + b)/((a + b)*E
^(2*(e + f*x)))]))))/(2*(a - b)^2*(a + b)^2*d*(b*(-1 + E^(2*e)) + a*(1 + E^(
2*e)))*f^4) + (4*a^2*c^3*f*x*Cosh[f*x] + 4*b^2*c^3*f*x*Cosh[f*x] + 6*a^2*c^
2*d*f*x^2*Cosh[f*x] + 6*b^2*c^2*d*f*x^2*Cosh[f*x] + 4*a^2*c*d^2*f*x^3*Cosh[
f*x] + 4*b^2*c*d^2*f*x^3*Cosh[f*x] + a^2*d^3*f*x^4*Cosh[f*x] + b^2*d^3*f*x^
4*Cosh[f*x] + 4*a^2*c^3*f*x*Cosh[2*e + f*x] - 4*b^2*c^3*f*x*Cosh[2*e + f*x]
 + 6*a^2*c^2*d*f*x^2*Cosh[2*e + f*x] - 6*b^2*c^2*d*f*x^2*Cosh[2*e + f*x] +
4*a^2*c*d^2*f*x^3*Cosh[2*e + f*x] - 4*b^2*c*d^2*f*x^3*Cosh[2*e + f*x] + a^2
*d^3*f*x^4*Cosh[2*e + f*x] - b^2*d^3*f*x^4*Cosh[2*e + f*x] - 8*b^2*c^3*Sinh
[f*x] - 24*b^2*c^2*d*x*Sinh[f*x] + 8*a*b*c^3*f*x*Sinh[f*x] - 24*b^2*c*d^2*x
^2*Sinh[f*x] + 12*a*b*c^2*d*f*x^2*Sinh[f*x] - 8*b^2*d^3*x^3*Sinh[f*x] + 8*a
*b*c*d^2*f*x^3*Sinh[f*x] + 2*a*b*d^3*f*x^4*Sinh[f*x])/(8*(a - b)*(a + b)*f*
(a*Cosh[e] + b*Sinh[e])*(a*Cosh[e + f*x] + b*Sinh[e + f*x]))
```

fricas [C] time = 0.68, size = 6160, normalized size = 9.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^3 + a^2*b - a*b^2 - b^3)*d^3*f^4*x^4 + 4*(a^3 + a^2*b - a*b^2 - b^3
)*c*d^2*f^4*x^3 + 6*(a^3 + a^2*b - a*b^2 - b^3)*c^2*d*f^4*x^2 + 4*(a^3 + a^
2*b - a*b^2 - b^3)*c^3*f^4*x - 4*(a^2*b - a*b^2)*d^3*e^4 - 8*(a*b^2 - b^3)*
d^3*e^3 + 8*(2*(a^2*b - a*b^2)*c^3*e + (a*b^2 - b^3)*c^3)*f^3 - 24*((a^2*b
 - a*b^2)*c^2*d*e^2 + (a*b^2 - b^3)*c^2*d*e)*f^2 + ((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d^3*f^4*x^4 - 4*(a^2*b + a*b^2)*d^3*e^4 + 16*(a^2*b + a*b^2)*c^3*e*
f^3 - 8*(a*b^2 + b^3)*d^3*e^3 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d^2*f^
4 - 2*(a*b^2 + b^3)*d^3*f^3)*x^3 - 24*((a^2*b + a*b^2)*c^2*d*e^2 + (a*b^2 +
 b^3)*c^2*d*e)*f^2 + 6*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*d*f^4 - 4*(a*b^
2 + b^3)*c*d^2*f^3)*x^2 + 8*(2*(a^2*b + a*b^2)*c*d^2*e^3 + 3*(a*b^2 + b^3)*
c*d^2*e^2)*f + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^3*f^4 - 6*(a*b^2 + b^3)
*c^2*d*f^3)*x)*cosh(f*x + e)^2 + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^3*f^4
*x^4 - 4*(a^2*b + a*b^2)*d^3*e^4 + 16*(a^2*b + a*b^2)*c^3*e*f^3 - 8*(a*b^2
+ b^3)*d^3*e^3 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d^2*f^4 - 2*(a*b^2 +
b^3)*d^3*f^3)*x^3 - 24*((a^2*b + a*b^2)*c^2*d*e^2 + (a*b^2 + b^3)*c^2*d*e)*
f^2 + 6*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*d*f^4 - 4*(a*b^2 + b^3)*c*d^2*
f^3)*x^2 + 8*(2*(a^2*b + a*b^2)*c*d^2*e^3 + 3*(a*b^2 + b^3)*c*d^2*e^2)*f +
4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^3*f^4 - 6*(a*b^2 + b^3)*c^2*d*f^3)*x)*
cosh(f*x + e)*sinh(f*x + e) + ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^3*f^4*x^4
- 4*(a^2*b + a*b^2)*d^3*e^4 + 16*(a^2*b + a*b^2)*c^3*e*f^3 - 8*(a*b^2 + b^3
)*d^3*e^3 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d^2*f^4 - 2*(a*b^2 + b^3)*
d^3*f^3)*x^3 - 24*((a^2*b + a*b^2)*c^2*d*e^2 + (a*b^2 + b^3)*c^2*d*e)*f^2 +
 6*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*d*f^4 - 4*(a*b^2 + b^3)*c*d^2*f^3)*
x^2 + 8*(2*(a^2*b + a*b^2)*c*d^2*e^3 + 3*(a*b^2 + b^3)*c*d^2*e^2)*f + 4*((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^3*f^4 - 6*(a*b^2 + b^3)*c^2*d*f^3)*x)*sinh(
f*x + e)^2 + 8*(2*(a^2*b - a*b^2)*c*d^2*e^3 + 3*(a*b^2 - b^3)*c*d^2*e^2)*f
- 24*((a^2*b - a*b^2)*d^3*f^2*x^2 + (a^2*b - a*b^2)*c^2*d*f^2 - (a*b^2 - b^
```

```
3)*c*d^2*f + ((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*b^2)*c^2*d*f^2 - (a*
b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*b^2 + b^3)*d^3*f)*x)
*cosh(f*x + e)^2 + 2*((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*b^2)*c^2*d*f
^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*b^2 + b^3)*d
^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) + ((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*
b + a*b^2)*c^2*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2
 - (a*b^2 + b^3)*d^3*f)*x)*sinh(f*x + e)^2 + (2*(a^2*b - a*b^2)*c*d^2*f^2 -
 (a*b^2 - b^3)*d^3*f)*x)*dilog(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh
(f*x + e))) - 24*((a^2*b - a*b^2)*d^3*f^2*x^2 + (a^2*b - a*b^2)*c^2*d*f^2 -
 (a*b^2 - b^3)*c*d^2*f + ((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*b^2)*c^2
*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*b^2 + b^
3)*d^3*f)*x)*cosh(f*x + e)^2 + 2*((a^2*b + a*b^2)*d^3*f^2*x^2 + (a^2*b + a*
b^2)*c^2*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^2)*c*d^2*f^2 - (a*
b^2 + b^3)*d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) + ((a^2*b + a*b^2)*d^3*f^2
*x^2 + (a^2*b + a*b^2)*c^2*d*f^2 - (a*b^2 + b^3)*c*d^2*f + (2*(a^2*b + a*b^
2)*c*d^2*f^2 - (a*b^2 + b^3)*d^3*f)*x)*sinh(f*x + e)^2 + (2*(a^2*b - a*b^2)
*c*d^2*f^2 - (a*b^2 - b^3)*d^3*f)*x)*dilog(-sqrt(-(a + b)/(a - b))*(cosh(f*
x + e) + sinh(f*x + e))) + 4*(2*(a^2*b - a*b^2)*d^3*e^3 - 2*(a^2*b - a*b^2)
*c^3*f^3 + 3*(a*b^2 - b^3)*d^3*e^2 + 3*(2*(a^2*b - a*b^2)*c^2*d*e + (a*b^2
 - b^3)*c^2*d)*f^2 + (2*(a^2*b + a*b^2)*d^3*e^3 - 2*(a^2*b + a*b^2)*c^3*f^3
 + 3*(a*b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b + a*b^2)*c^2*d*e + (a*b^2 + b^3)*c^
2*d)*f^2 - 6*((a^2*b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f)*cosh(f*
x + e)^2 + 2*(2*(a^2*b + a*b^2)*d^3*e^3 - 2*(a^2*b + a*b^2)*c^3*f^3 + 3*(a*
b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b + a*b^2)*c^2*d*e + (a*b^2 + b^3)*c^2*d)*f^
2 - 6*((a^2*b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f)*cosh(f*x + e)*
sinh(f*x + e) + (2*(a^2*b + a*b^2)*d^3*e^3 - 2*(a^2*b + a*b^2)*c^3*f^3 + 3*
(a*b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b + a*b^2)*c^2*d*e + (a*b^2 + b^3)*c^2*d)
*f^2 - 6*((a^2*b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f)*sinh(f*x +
e)^2 - 6*((a^2*b - a*b^2)*c*d^2*e^2 + (a*b^2 - b^3)*c*d^2*e)*f)*log(2*(a +
b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt(-(a + b)/(a - b
))) + 4*(2*(a^2*b - a*b^2)*d^3*e^3 - 2*(a^2*b - a*b^2)*c^3*f^3 + 3*(a*b^2 -
 b^3)*d^3*e^2 + 3*(2*(a^2*b - a*b^2)*c^2*d*e + (a*b^2 - b^3)*c^2*d)*f^2 + (
2*(a^2*b + a*b^2)*d^3*e^3 - 2*(a^2*b + a*b^2)*c^3*f^3 + 3*(a*b^2 + b^3)*d^3
*e^2 + 3*(2*(a^2*b + a*b^2)*c^2*d*e + (a*b^2 + b^3)*c^2*d)*f^2 - 6*((a^2*b
+ a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f)*cosh(f*x + e)^2 + 2*(2*(a^2*
b + a*b^2)*d^3*e^3 - 2*(a^2*b + a*b^2)*c^3*f^3 + 3*(a*b^2 + b^3)*d^3*e^2 +
3*(2*(a^2*b + a*b^2)*c^2*d*e + (a*b^2 + b^3)*c^2*d)*f^2 - 6*((a^2*b + a*b^2
)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f)*cosh(f*x + e)*sinh(f*x + e) + (2*(a
^2*b + a*b^2)*d^3*e^3 - 2*(a^2*b + a*b^2)*c^3*f^3 + 3*(a*b^2 + b^3)*d^3*e^2
 + 3*(2*(a^2*b + a*b^2)*c^2*d*e + (a*b^2 + b^3)*c^2*d)*f^2 - 6*((a^2*b + a*
b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f)*sinh(f*x + e)^2 - 6*((a^2*b - a*
b^2)*c*d^2*e^2 + (a*b^2 - b^3)*c*d^2*e)*f)*log(2*(a + b)*cosh(f*x + e) + 2*
(a + b)*sinh(f*x + e) - 2*(a - b)*sqrt(-(a + b)/(a - b))) - 4*(2*(a^2*b - a
*b^2)*d^3*f^3*x^3 + 2*(a^2*b - a*b^2)*d^3*e^3 + 6*(a^2*b - a*b^2)*c^2*d*e*f
^2 + 3*(a*b^2 - b^3)*d^3*e^2 + 3*(2*(a^2*b - a*b^2)*c*d^2*f^3 - (a*b^2 - b^
3)*d^3*f^2)*x^2 + (2*(a^2*b + a*b^2)*d^3*f^3*x^3 + 2*(a^2*b + a*b^2)*d^3*e^
3 + 6*(a^2*b + a*b^2)*c^2*d*e*f^2 + 3*(a*b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b +
 a*b^2)*c*d^2*f^3 - (a*b^2 + b^3)*d^3*f^2)*x^2 - 6*((a^2*b + a*b^2)*c*d^2*e
^2 + (a*b^2 + b^3)*c*d^2*e)*f + 6*((a^2*b + a*b^2)*c^2*d*f^3 - (a*b^2 + b^3
)*c*d^2*f^2)*x)*cosh(f*x + e)^2 + 2*(2*(a^2*b + a*b^2)*d^3*f^3*x^3 + 2*(a^2
*b + a*b^2)*d^3*e^3 + 6*(a^2*b + a*b^2)*c^2*d*e*f^2 + 3*(a*b^2 + b^3)*d^3*e
^2 + 3*(2*(a^2*b + a*b^2)*c*d^2*f^3 - (a*b^2 + b^3)*d^3*f^2)*x^2 - 6*((a^2*
b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f + 6*((a^2*b + a*b^2)*c^2*d*
f^3 - (a*b^2 + b^3)*c*d^2*f^2)*x)*cosh(f*x + e)*sinh(f*x + e) + (2*(a^2*b +
 a*b^2)*d^3*f^3*x^3 + 2*(a^2*b + a*b^2)*d^3*e^3 + 6*(a^2*b + a*b^2)*c^2*d*e
*f^2 + 3*(a*b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b + a*b^2)*c*d^2*f^3 - (a*b^2 +
b^3)*d^3*f^2)*x^2 - 6*((a^2*b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f
 + 6*((a^2*b + a*b^2)*c^2*d*f^3 - (a*b^2 + b^3)*c*d^2*f^2)*x)*sinh(f*x + e)
^2 - 6*((a^2*b - a*b^2)*c*d^2*e^2 + (a*b^2 - b^3)*c*d^2*e)*f + 6*((a^2*b -
```

```
a*b^2)*c^2*d*f^3 - (a*b^2 - b^3)*c*d^2*f^2)*x)*log(sqrt(-(a + b)/(a - b))*(
cosh(f*x + e) + sinh(f*x + e)) + 1) - 4*(2*(a^2*b - a*b^2)*d^3*f^3*x^3 + 2*
(a^2*b - a*b^2)*d^3*e^3 + 6*(a^2*b - a*b^2)*c^2*d*e*f^2 + 3*(a*b^2 - b^3)*d
^3*e^2 + 3*(2*(a^2*b - a*b^2)*c*d^2*f^3 - (a*b^2 - b^3)*d^3*f^2)*x^2 + (2*(
a^2*b + a*b^2)*d^3*f^3*x^3 + 2*(a^2*b + a*b^2)*d^3*e^3 + 6*(a^2*b + a*b^2)*
c^2*d*e*f^2 + 3*(a*b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b + a*b^2)*c*d^2*f^3 - (a
*b^2 + b^3)*d^3*f^2)*x^2 - 6*((a^2*b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d
^2*e)*f + 6*((a^2*b + a*b^2)*c^2*d*f^3 - (a*b^2 + b^3)*c*d^2*f^2)*x)*cosh(f
*x + e)^2 + 2*(2*(a^2*b + a*b^2)*d^3*f^3*x^3 + 2*(a^2*b + a*b^2)*d^3*e^3 +
6*(a^2*b + a*b^2)*c^2*d*e*f^2 + 3*(a*b^2 + b^3)*d^3*e^2 + 3*(2*(a^2*b + a*b
^2)*c*d^2*f^3 - (a*b^2 + b^3)*d^3*f^2)*x^2 - 6*((a^2*b + a*b^2)*c*d^2*e^2 +
 (a*b^2 + b^3)*c*d^2*e)*f + 6*((a^2*b + a*b^2)*c^2*d*f^3 - (a*b^2 + b^3)*c*
d^2*f^2)*x)*cosh(f*x + e)*sinh(f*x + e) + (2*(a^2*b + a*b^2)*d^3*f^3*x^3 +
2*(a^2*b + a*b^2)*d^3*e^3 + 6*(a^2*b + a*b^2)*c^2*d*e*f^2 + 3*(a*b^2 + b^3)
*d^3*e^2 + 3*(2*(a^2*b + a*b^2)*c*d^2*f^3 - (a*b^2 + b^3)*d^3*f^2)*x^2 - 6*
((a^2*b + a*b^2)*c*d^2*e^2 + (a*b^2 + b^3)*c*d^2*e)*f + 6*((a^2*b + a*b^2)*
c^2*d*f^3 - (a*b^2 + b^3)*c*d^2*f^2)*x)*sinh(f*x + e)^2 - 6*((a^2*b - a*b^2
)*c*d^2*e^2 + (a*b^2 - b^3)*c*d^2*e)*f + 6*((a^2*b - a*b^2)*c^2*d*f^3 - (a*
b^2 - b^3)*c*d^2*f^2)*x)*log(-sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(
f*x + e)) + 1) - 48*((a^2*b + a*b^2)*d^3*cosh(f*x + e)^2 + 2*(a^2*b + a*b^2
)*d^3*cosh(f*x + e)*sinh(f*x + e) + (a^2*b + a*b^2)*d^3*sinh(f*x + e)^2 + (
a^2*b - a*b^2)*d^3)*polylog(4, sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh
(f*x + e))) - 48*((a^2*b + a*b^2)*d^3*cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d
^3*cosh(f*x + e)*sinh(f*x + e) + (a^2*b + a*b^2)*d^3*sinh(f*x + e)^2 + (a^2
*b - a*b^2)*d^3)*polylog(4, -sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f
*x + e))) + 24*(2*(a^2*b - a*b^2)*d^3*f*x + 2*(a^2*b - a*b^2)*c*d^2*f - (a*
b^2 - b^3)*d^3 + (2*(a^2*b + a*b^2)*d^3*f*x + 2*(a^2*b + a*b^2)*c*d^2*f - (
a*b^2 + b^3)*d^3)*cosh(f*x + e)^2 + 2*(2*(a^2*b + a*b^2)*d^3*f*x + 2*(a^2*b
 + a*b^2)*c*d^2*f - (a*b^2 + b^3)*d^3)*cosh(f*x + e)*sinh(f*x + e) + (2*(a^
2*b + a*b^2)*d^3*f*x + 2*(a^2*b + a*b^2)*c*d^2*f - (a*b^2 + b^3)*d^3)*sinh(
f*x + e)^2)*polylog(3, sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e
))) + 24*(2*(a^2*b - a*b^2)*d^3*f*x + 2*(a^2*b - a*b^2)*c*d^2*f - (a*b^2 -
b^3)*d^3 + (2*(a^2*b + a*b^2)*d^3*f*x + 2*(a^2*b + a*b^2)*c*d^2*f - (a*b^2
+ b^3)*d^3)*cosh(f*x + e)^2 + 2*(2*(a^2*b + a*b^2)*d^3*f*x + 2*(a^2*b + a*b
^2)*c*d^2*f - (a*b^2 + b^3)*d^3)*cosh(f*x + e)*sinh(f*x + e) + (2*(a^2*b +
a*b^2)*d^3*f*x + 2*(a^2*b + a*b^2)*c*d^2*f - (a*b^2 + b^3)*d^3)*sinh(f*x +
e)^2)*polylog(3, -sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))))/
((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^4*cosh(f*x + e)^2 +
2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^4*cosh(f*x + e)*sin
h(f*x + e) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^4*sinh(f
*x + e)^2 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*f^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{\left(b\tanh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*tanh(f*x + e) + a)^2, x)
```

maple [B] time = 0.94, size = 2278, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*tanh(f*x+e))^2,x)
```

```
[Out] -6/(a-b)/(a+b)^2*b*a*c^2*d/(-a+b)*x^2+12/f^2/(a-b)/(a+b)^2*b/(-a+b)*e^2*a*c
*d^2*x+6/f^2/(a-b)/(a+b)^2*b*a*c*d^2/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/
(-a+b))*x+6/f/(a-b)/(a+b)^2*b*a*c^2*d/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+
b))*x+6/f/(a-b)/(a+b)^2*b*a*c*d^2/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*
x^2+6/f^2/(a-b)/(a+b)^2*b*a*c^2*d/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*
e-6/f^3/(a-b)/(a+b)^2*b*a*c*d^2*e^2/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b)
)+12/f^3/(a-b)^2/(a+b)^2*b^2*c*d^2*e*ln(exp(f*x+e))+3/2/f^4/(a-b)/(a+b)^2*b
*a*d^3/(-a+b)*polylog(4,(a+b)*exp(2*f*x+2*e)/(-a+b))-6/f^3/(a-b)^2/(a+b)^2*
b^2*c*d^2*e*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+2/f^4/(a-b)^2/(a+b)^2
*b*a*d^3*e^3*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)-3/f^3/(a-b)/(a+b)^2*
b^2*c*d^2/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))-4/f^4/(a-b)^2/(a+b)
^2*b*a*d^3*e^3*ln(exp(f*x+e))-3/f^2/(a-b)/(a+b)^2*b^2*d^3/(-a+b)*ln(1-(a+b)
*exp(2*f*x+2*e)/(-a+b))*x^2+3/f^4/(a-b)/(a+b)^2*b^2*d^3/(-a+b)*ln(1-(a+b)*e
xp(2*f*x+2*e)/(-a+b))*e^2-3/f^3/(a-b)/(a+b)^2*b^2*d^3/(-a+b)*polylog(2,(a+b
)*exp(2*f*x+2*e)/(-a+b))*x-1/(a-b)/(a+b)^2*b/(-a+b)*a*d^3*x^4-3/f^4/(a-b)/(
a+b)^2*b/(-a+b)*e^4*a*d^3-6/f^3/(a-b)/(a+b)^2*b^2/(-a+b)*e^2*d^3*x+6/f/(a-b
)/(a+b)^2*b^2/(-a+b)*d^2*c*x^2+6/f^3/(a-b)/(a+b)^2*b^2/(-a+b)*d^2*c*e^2+1/(
a^2+2*a*b+b^2)*c*d^2*x^3-4/f^4/(a-b)/(a+b)^2*b^2/(-a+b)*e^3*d^3+2/f/(a-b)/(
a+b)^2*b^2/(-a+b)*d^3*x^3+3/f^4/(a-b)^2/(a+b)^2*b^2*d^3*e^2*ln(a*exp(2*f*x+
2*e)+b*exp(2*f*x+2*e)+a-b)-6/f^4/(a-b)^2/(a+b)^2*b^2*d^3*e^2*ln(exp(f*x+e))
-2/f/(a-b)^2/(a+b)^2*b*a*c^3*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+4/f/
(a-b)^2/(a+b)^2*b*a*c^3*ln(exp(f*x+e))-6/f^2/(a-b)^2/(a+b)^2*b^2*c^2*d*ln(e
xp(f*x+e))+3/2/f^4/(a-b)/(a+b)^2*b^2*d^3/(-a+b)*polylog(3,(a+b)*exp(2*f*x+2
*e)/(-a+b))+3/f^2/(a-b)^2/(a+b)^2*b^2*c^2*d*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x
+2*e)+a-b)+2/(a-b)/f/(a^2+2*a*b+b^2)*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*b^
2/(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+1/4/(a^2+2*a*b+b^2)*d^3*x^4+1/(a^
2+2*a*b+b^2)*c^3*x+12/f^2/(a-b)/(a+b)^2*b^2/(-a+b)*d^2*c*e*x-6/f^2/(a-b)/(a
+b)^2*b*a*c^2*d/(-a+b)*e^2+8/f^3/(a-b)/(a+b)^2*b/(-a+b)*e^3*a*c*d^2-4/(a-b)
/(a+b)^2*b/(-a+b)*a*c*d^2*x^3-12/f/(a-b)/(a+b)^2*b*a*c^2*d/(-a+b)*e*x-4/f^3
/(a-b)/(a+b)^2*b/(-a+b)*e^3*a*d^3*x-6/f^2/(a-b)/(a+b)^2*b^2*c*d^2/(-a+b)*ln
(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x-6/f^3/(a-b)/(a+b)^2*b^2*c*d^2/(-a+b)*ln(1
-(a+b)*exp(2*f*x+2*e)/(-a+b))*e+6/f^2/(a-b)^2/(a+b)^2*b*a*c^2*d*e*ln(a*exp(
2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)-3/f^3/(a-b)/(a+b)^2*b*a*c*d^2/(-a+b)*polyl
og(3,(a+b)*exp(2*f*x+2*e)/(-a+b))+2/f/(a-b)/(a+b)^2*b*a*d^3/(-a+b)*ln(1-(a+
b)*exp(2*f*x+2*e)/(-a+b))*x^3+2/f^4/(a-b)/(a+b)^2*b*a*d^3/(-a+b)*ln(1-(a+b)
*exp(2*f*x+2*e)/(-a+b))*e^3+3/f^2/(a-b)/(a+b)^2*b*a*d^3/(-a+b)*polylog(2,(a
+b)*exp(2*f*x+2*e)/(-a+b))*x^2-6/f^3/(a-b)^2/(a+b)^2*b*a*c*d^2*e^2*ln(a*exp
(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+12/f^3/(a-b)^2/(a+b)^2*b*a*c*d^2*e^2*ln(e
xp(f*x+e))-12/f^2/(a-b)^2/(a+b)^2*b*a*c^2*d*e*ln(exp(f*x+e))+3/f^2/(a-b)/(a
+b)^2*b*a*c^2*d/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))-3/f^3/(a-b)/(
a+b)^2*b*a*d^3/(-a+b)*polylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b))*x+3/2/(a^2+2*a
*b+b^2)*c^2*d*x^2
```

maxima [A] time = 1.86, size = 1060, normalized size = 1.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -6*b^2*c^2*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - 2/3*(4*f^3*x^3*log((
a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 6*f^2*x^2*dilog(-(a*e^(2*e)
 + b*e^(2*e))*e^(2*f*x)/(a - b)) - 6*f*x*polylog(3, -(a*e^(2*e) + b*e^(2*e)
)*e^(2*f*x)/(a - b)) + 3*polylog(4, -(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a -
 b)))*a*b*d^3/(a^4*f^4 - 2*a^2*b^2*f^4 + b^4*f^4) + 3*b^2*c^2*d*log((a*e^(2
*e) + b*e^(2*e))*e^(2*f*x) + a - b)/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - c
^3*(2*a*b*log(-(a - b)*e^(-2*f*x - 2*e) - a - b)/((a^4 - 2*a^2*b^2 + b^4)*f
) + 2*b^2/((a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*f
*x - 2*e))*f) - (f*x + e)/((a^2 + 2*a*b + b^2)*f)) - 3/2*(2*a*b*c*d^2*f - b
^2*d^3)*(2*f^2*x^2*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f
```

```
*x*dilog(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*
e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(a^4*f^4 - 2*a^2*b^2*f^4 + b^4*f^4) - 3
*(a*b*c^2*d*f - b^2*c*d^2)*(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a
- b) + 1) + dilog(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(a^4*f^3 - 2
*a^2*b^2*f^3 + b^4*f^3) + (a*b*d^3*f^4*x^4 + 2*(2*a*b*c*d^2*f - b^2*d^3)*f^
3*x^3 + 6*(a*b*c^2*d*f^2 - b^2*c*d^2*f)*f^2*x^2)/(a^4*f^4 - 2*a^2*b^2*f^4 +
 b^4*f^4) + 1/4*(24*b^2*c^2*d*x + (a^2*d^3*f - 2*a*b*d^3*f + b^2*d^3*f)*x^4
 + 4*(a^2*c*d^2*f - 2*a*b*c*d^2*f + (c*d^2*f + 2*d^3)*b^2)*x^3 + 6*(a^2*c^2
*d*f - 2*a*b*c^2*d*f + (c^2*d*f + 4*c*d^2)*b^2)*x^2 + ((a^2*d^3*f*e^(2*e) -
 b^2*d^3*f*e^(2*e))*x^4 + 4*(a^2*c*d^2*f*e^(2*e) - b^2*c*d^2*f*e^(2*e))*x^3
 + 6*(a^2*c^2*d*f*e^(2*e) - b^2*c^2*d*f*e^(2*e))*x^2)*e^(2*f*x))/(a^4*f - 2
*a^2*b^2*f + b^4*f + (a^4*f*e^(2*e) + 2*a^3*b*f*e^(2*e) - 2*a*b^3*f*e^(2*e)
 - b^4*f*e^(2*e))*e^(2*f*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{\left(a+b\tanh\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*tanh(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^3/(a + b*tanh(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{\left(a+b\tanh\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*tanh(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**3/(a + b*tanh(e + f*x))**2, x)
```

3.74 $\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=476

$$\frac{2b^2d(c+dx)\mathrm{Li}_2\left(-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{2b^2d(c+dx)\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f^2\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^2\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f\left(a^2-b^2\right)^2}-\frac{2b^2(c+dx}{f\left(a^2-b^2\right)}$$

```
[Out] -2*b^2*(d*x+c)^2/(a^2-b^2)^2/f+2*b^2*(d*x+c)^2/(a-b)/(a+b)^2/(a-b+(a+b)*exp
(2*f*x+2*e))/f+1/3*(d*x+c)^3/(a-b)^2/d+2*b^2*d*(d*x+c)*ln(1+(a+b)*exp(2*f*x
+2*e)/(a-b))/(a^2-b^2)^2/f^2-2*b*(d*x+c)^2*ln(1+(a+b)*exp(2*f*x+2*e)/(a-b))
/(a-b)^2/(a+b)/f+2*b^2*(d*x+c)^2*ln(1+(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)
^2/f+b^2*d^2*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3-2*b*d*(
d*x+c)*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^2+2*b^2*d*(d*
x+c)*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^2+b*d^2*polylog(3
,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^3-b^2*d^2*polylog(3,-(a+b)*ex
p(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3
```

Rubi [A] time = 1.65, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3734, 2190, 2531, 2282, 6589, 2254, 2185, 2184, 2191, 2279, 2391}

$$\frac{2b^2d(c+dx)\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{b^2d^2\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}-\frac{b^2d^2\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}-\frac{2bd(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] (-2*b^2*(c + d*x)^2)/((a^2 - b^2)^2*f) + (2*b^2*(c + d*x)^2)/((a - b)*(a +
b)^2*(a - b + (a + b)*E^(2*e + 2*f*x))*f) + (c + d*x)^3/(3*(a - b)^2*d) + (
2*b^2*d*(c + d*x)*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 - b^2)^
2*f^2) - (2*b*(c + d*x)^2*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a -
 b)^2*(a + b)*f) + (2*b^2*(c + d*x)^2*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a
 - b)])/((a^2 - b^2)^2*f) + (b^2*d^2*PolyLog[2, -(((a + b)*E^(2*e + 2*f*x))/
(a - b))])/((a^2 - b^2)^2*f^3) - (2*b*d*(c + d*x)*PolyLog[2, -(((a + b)*E^(
2*e + 2*f*x))/(a - b))])/((a - b)^2*(a + b)*f^2) + (2*b^2*d*(c + d*x)*PolyL
og[2, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f^2) + (b*d^2*P
olyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a - b)^2*(a + b)*f^3) -
(b^2*d^2*PolyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f
^3)
```

Rule 2184

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Dist[
b/a, Int[((c + d*x)^m*(F^(g*(e + f*x)))^n)/(a + b*(F^(g*(e + f*x)))^n), x],
 x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2185

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Dist[1/a, Int[(c + d*x)^m*(a + b*(F^(g*(e +
 f*x)))^n)^(p + 1), x], x] - Dist[b/a, Int[(c + d*x)^m*(F^(g*(e + f*x)))^n*
(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n},
 x] && ILtQ[p, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2254

```
Int[((a_.) + (b_.)*(F_)^(u_))^(p_.)*((c_.) + (d_.)*(F_)^(v_))^(q_.)*((e_.)
+ (f_.)*(x_))^(m_.), x_Symbol] :> With[{w = ExpandIntegrand[(e + f*x)^m, (a
 + b*F^u)^p*(c + d*F^v)^q, x]}, Int[w, x] /; SumQ[w]] /; FreeQ[{F, a, b, c,
 d, e, f, m}, x] && IntegersQ[p, q] && LinearQ[{u, v}, x] && RationalQ[Simp
lify[u/v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3734

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - (2*I*b)/(a^2
+ b^2 + (a - I*b)^2*E^(2*I*(e + f*x))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \int \left(\frac{(c+dx)^2}{(a-b)^2} + \frac{4be^{2e+2fx}(c+dx)^2}{(a-b)^2\left(-a\left(1-\frac{b}{a}\right)-a\left(1+\frac{b}{a}\right)e^{2e+2fx}\right)} + \frac{4b^2e^{4e+4fx}(c+$$

$$= \frac{(c+dx)^3}{3(a-b)^2d} + \frac{(4b)\int \frac{e^{2e+2fx}(c+dx)^2}{-a\left(1-\frac{b}{a}\right)-a\left(1+\frac{b}{a}\right)e^{2e+2fx}}\,dx}{(a-b)^2} + \frac{\left(4b^2\right)\int \frac{e^{4e+4fx}(c+dx)^2}{\left(a\left(1-\frac{b}{a}\right)+a\left(1+\frac{b}{a}\right)e^{2e+2fx}\right)^2}\,dx}{(a-b)^2}$$

$$= \frac{(c+dx)^3}{3(a-b)^2d} - \frac{2b(c+dx)^2\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} + \frac{\left(4b^2\right)\int\left(\frac{(c+dx)^2}{(a+b)^2} + \frac{(a-b)^2(c+dx)^2}{(a+b)^2\left(a-b+(a+b)e^{2}\right.}\right.}{(a-$$

$$= \frac{(c+dx)^3}{3(a-b)^2d} + \frac{4b^2(c+dx)^3}{3\left(a^2-b^2\right)^2d} - \frac{2b(c+dx)^2\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} - \frac{2bd(c+dx)\mathrm{Li}_2\left(-\right.}{(a-b)^2(a+$$

$$= \frac{(c+dx)^3}{3(a-b)^2d} - \frac{4b^2(c+dx)^3}{3\left(a^2-b^2\right)^2d} - \frac{2b(c+dx)^2\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} - \frac{2bd(c+dx)\mathrm{Li}_2\left(-\right.}{(a-b)^2(a+$$

$$= \frac{2b^2(c+dx)^2}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^3}{3(a-b)^2d} - \frac{2b(c+dx)^2\log\left(1+\frac{(a+b)e}{a-}\right.}{(a-b)^2(a+b)f}$$

$$= -\frac{2b^2(c+dx)^2}{\left(a^2-b^2\right)^2f} + \frac{2b^2(c+dx)^2}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^3}{3(a-b)^2d} - \frac{2b(c+d}{($$

$$= -\frac{2b^2(c+dx)^2}{\left(a^2-b^2\right)^2f} + \frac{2b^2(c+dx)^2}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^3}{3(a-b)^2d} + \frac{2b^2d(c+}{}$$

$$= -\frac{2b^2(c+dx)^2}{\left(a^2-b^2\right)^2f} + \frac{2b^2(c+dx)^2}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^3}{3(a-b)^2d} + \frac{2b^2d(c+}{}$$

$$= -\frac{2b^2(c+dx)^2}{\left(a^2-b^2\right)^2f} + \frac{2b^2(c+dx)^2}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} + \frac{(c+dx)^3}{3(a-b)^2d} + \frac{2b^2d(c+}{}$$

Mathematica [A] time = 9.39, size = 506, normalized size = 1.06

$$\frac{f^2(a-b)(a+b)\left(fx\left(a^2-b^2\right)\left(3c^2+3cdx+d^2x^2\right)\cosh(2e+fx)+fx\left(a^2+b^2\right)\left(3c^2+3cdx+d^2x^2\right)\cosh(fx)+2b\sinh(fx)\left(afx\left(3c^2+3cdx+d^2x^2\right)-3b(c+dx)^2\right)\right)}{(a\cosh(e)+b\sinh(e))(a\cosh(e+fx)+b\sinh(e+fx))} - \frac{1}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] ((-24*(a - b)*b*c*f^2*(-(b*d) + a*c*f)*x)/(b*(-1 + E^(2*e)) + a*(1 + E^(2*e
))) - (12*(a - b)*b*d*f^2*(-(b*d) + 2*a*c*f)*x^2)/(b*(-1 + E^(2*e)) + a*(1
+ E^(2*e))) - (8*a*(a - b)*b*d^2*f^3*x^3)/(b*(-1 + E^(2*e)) + a*(1 + E^(2*e
))) + 12*b*d*f*(b*d - 2*a*c*f)*x*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))]
 - 12*a*b*d^2*f^2*x^2*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))] + 12*b*c*f
*(-(b*d) + a*c*f)*(2*f*x - Log[a - b + (a + b)*E^(2*(e + f*x))]) - 6*b*d*(b
*d - 2*a*c*f)*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))] + 6*a*b*d^2*(2
*f*x*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[3, (-a + b)/(
(a + b)*E^(2*(e + f*x)))]) + ((a - b)*(a + b)*f^2*((a^2 + b^2)*f*x*(3*c^2 +
 3*c*d*x + d^2*x^2)*Cosh[f*x] + (a^2 - b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)
*Cosh[2*e + f*x] + 2*b*(-3*b*(c + d*x)^2 + a*f*x*(3*c^2 + 3*c*d*x + d^2*x^2
))*Sinh[f*x]))/((a*Cosh[e] + b*Sinh[e])*(a*Cosh[e + f*x] + b*Sinh[e + f*x])
))/(6*(a - b)^2*(a + b)^2*f^3)
```

fricas **[C]** time = 0.56, size = 3693, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/3*((a^3 + a^2*b - a*b^2 - b^3)*d^2*f^3*x^3 + 3*(a^3 + a^2*b - a*b^2 - b^3
)*c*d*f^3*x^2 + 3*(a^3 + a^2*b - a*b^2 - b^3)*c^2*f^3*x + 4*(a^2*b - a*b^2)
*d^2*e^3 + 6*(a*b^2 - b^3)*d^2*e^2 + 6*(2*(a^2*b - a*b^2)*c^2*e + (a*b^2 -
b^3)*c^2)*f^2 + ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^2*f^3*x^3 + 4*(a^2*b + a
*b^2)*d^2*e^3 + 12*(a^2*b + a*b^2)*c^2*e*f^2 + 6*(a*b^2 + b^3)*d^2*e^2 + 3*
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*d*f^3 - 2*(a*b^2 + b^3)*d^2*f^2)*x^2 - 1
2*((a^2*b + a*b^2)*c*d*e^2 + (a*b^2 + b^3)*c*d*e)*f + 3*((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*c^2*f^3 - 4*(a*b^2 + b^3)*c*d*f^2)*x)*cosh(f*x + e)^2 + 2*((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d^2*f^3*x^3 + 4*(a^2*b + a*b^2)*d^2*e^3 + 12*
(a^2*b + a*b^2)*c^2*e*f^2 + 6*(a*b^2 + b^3)*d^2*e^2 + 3*((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*c*d*f^3 - 2*(a*b^2 + b^3)*d^2*f^2)*x^2 - 12*((a^2*b + a*b^2)*
c*d*e^2 + (a*b^2 + b^3)*c*d*e)*f + 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*f
^3 - 4*(a*b^2 + b^3)*c*d*f^2)*x)*cosh(f*x + e)*sinh(f*x + e) + ((a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d^2*f^3*x^3 + 4*(a^2*b + a*b^2)*d^2*e^3 + 12*(a^2*b +
a*b^2)*c^2*e*f^2 + 6*(a*b^2 + b^3)*d^2*e^2 + 3*((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*c*d*f^3 - 2*(a*b^2 + b^3)*d^2*f^2)*x^2 - 12*((a^2*b + a*b^2)*c*d*e^2 +
 (a*b^2 + b^3)*c*d*e)*f + 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c^2*f^3 - 4*(a
*b^2 + b^3)*c*d*f^2)*x)*sinh(f*x + e)^2 - 12*((a^2*b - a*b^2)*c*d*e^2 + (a*
b^2 - b^3)*c*d*e)*f - 6*(2*(a^2*b - a*b^2)*d^2*f*x + 2*(a^2*b - a*b^2)*c*d*
f - (a*b^2 - b^3)*d^2 + (2*(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*
f - (a*b^2 + b^3)*d^2)*cosh(f*x + e)^2 + 2*(2*(a^2*b + a*b^2)*d^2*f*x + 2*(
a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^2)*cosh(f*x + e)*sinh(f*x + e) + (2*
(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^2)*sinh
(f*x + e)^2)*dilog(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)))
- 6*(2*(a^2*b - a*b^2)*d^2*f*x + 2*(a^2*b - a*b^2)*c*d*f - (a*b^2 - b^3)*d^
2 + (2*(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^
2)*cosh(f*x + e)^2 + 2*(2*(a^2*b + a*b^2)*d^2*f*x + 2*(a^2*b + a*b^2)*c*d*f
 - (a*b^2 + b^3)*d^2)*cosh(f*x + e)*sinh(f*x + e) + (2*(a^2*b + a*b^2)*d^2*
f*x + 2*(a^2*b + a*b^2)*c*d*f - (a*b^2 + b^3)*d^2)*sinh(f*x + e)^2)*dilog(-
sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) - 6*((a^2*b - a*b^2
)*d^2*e^2 + (a^2*b - a*b^2)*c^2*f^2 + (a*b^2 - b^3)*d^2*e + ((a^2*b + a*b^2
)*d^2*e^2 + (a^2*b + a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b
^2)*c*d*e + (a*b^2 + b^3)*c*d)*f)*cosh(f*x + e)^2 + 2*((a^2*b + a*b^2)*d^2*
e^2 + (a^2*b + a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2)*c*
d*e + (a*b^2 + b^3)*c*d)*f)*cosh(f*x + e)*sinh(f*x + e) + ((a^2*b + a*b^2)*
d^2*e^2 + (a^2*b + a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2
)*c*d*e + (a*b^2 + b^3)*c*d)*f)*sinh(f*x + e)^2 - (2*(a^2*b - a*b^2)*c*d*e
```

```
+ (a*b^2 - b^3)*c*d)*f)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x +
e) + 2*(a - b)*sqrt(-(a + b)/(a - b))) - 6*((a^2*b - a*b^2)*d^2*e^2 + (a^2*
b - a*b^2)*c^2*f^2 + (a*b^2 - b^3)*d^2*e + ((a^2*b + a*b^2)*d^2*e^2 + (a^2*
b + a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2)*c*d*e + (a*b^
2 + b^3)*c*d)*f)*cosh(f*x + e)^2 + 2*((a^2*b + a*b^2)*d^2*e^2 + (a^2*b + a*
b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2)*c*d*e + (a*b^2 + b^
3)*c*d)*f)*cosh(f*x + e)*sinh(f*x + e) + ((a^2*b + a*b^2)*d^2*e^2 + (a^2*b
+ a*b^2)*c^2*f^2 + (a*b^2 + b^3)*d^2*e - (2*(a^2*b + a*b^2)*c*d*e + (a*b^2
+ b^3)*c*d)*f)*sinh(f*x + e)^2 - (2*(a^2*b - a*b^2)*c*d*e + (a*b^2 - b^3)*c
*d)*f)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) - 2*(a - b)*sq
rt(-(a + b)/(a - b))) - 6*((a^2*b - a*b^2)*d^2*f^2*x^2 - (a^2*b - a*b^2)*d^
2*e^2 + 2*(a^2*b - a*b^2)*c*d*e*f - (a*b^2 - b^3)*d^2*e + ((a^2*b + a*b^2)*
d^2*f^2*x^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^2*b + a*b^2)*c*d*e*f - (a*b^2
+ b^3)*d^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (a*b^2 + b^3)*d^2*f)*x)*cosh(f*
x + e)^2 + 2*((a^2*b + a*b^2)*d^2*f^2*x^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^
2*b + a*b^2)*c*d*e*f - (a*b^2 + b^3)*d^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (
a*b^2 + b^3)*d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e) + ((a^2*b + a*b^2)*d^2*f
^2*x^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^2*b + a*b^2)*c*d*e*f - (a*b^2 + b^3
)*d^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (a*b^2 + b^3)*d^2*f)*x)*sinh(f*x + e
)^2 + (2*(a^2*b - a*b^2)*c*d*f^2 - (a*b^2 - b^3)*d^2*f)*x)*log(sqrt(-(a + b
)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 6*((a^2*b - a*b^2)*d^2*f^
2*x^2 - (a^2*b - a*b^2)*d^2*e^2 + 2*(a^2*b - a*b^2)*c*d*e*f - (a*b^2 - b^3)
*d^2*e + ((a^2*b + a*b^2)*d^2*f^2*x^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^2*b
+ a*b^2)*c*d*e*f - (a*b^2 + b^3)*d^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (a*b^
2 + b^3)*d^2*f)*x)*cosh(f*x + e)^2 + 2*((a^2*b + a*b^2)*d^2*f^2*x^2 - (a^2*
b + a*b^2)*d^2*e^2 + 2*(a^2*b + a*b^2)*c*d*e*f - (a*b^2 + b^3)*d^2*e + (2*(
a^2*b + a*b^2)*c*d*f^2 - (a*b^2 + b^3)*d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e
) + ((a^2*b + a*b^2)*d^2*f^2*x^2 - (a^2*b + a*b^2)*d^2*e^2 + 2*(a^2*b + a*b
^2)*c*d*e*f - (a*b^2 + b^3)*d^2*e + (2*(a^2*b + a*b^2)*c*d*f^2 - (a*b^2 + b
^3)*d^2*f)*x)*sinh(f*x + e)^2 + (2*(a^2*b - a*b^2)*c*d*f^2 - (a*b^2 - b^3)*
d^2*f)*x)*log(-sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1)
+ 12*((a^2*b + a*b^2)*d^2*cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d^2*cosh(f*x
+ e)*sinh(f*x + e) + (a^2*b + a*b^2)*d^2*sinh(f*x + e)^2 + (a^2*b - a*b^2)*
d^2)*polylog(3, sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) + 1
2*((a^2*b + a*b^2)*d^2*cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d^2*cosh(f*x + e
)*sinh(f*x + e) + (a^2*b + a*b^2)*d^2*sinh(f*x + e)^2 + (a^2*b - a*b^2)*d^2
)*polylog(3, -sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))))/((a^
5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^3*cosh(f*x + e)^2 + 2*(a
^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^3*cosh(f*x + e)*sinh(f*
x + e) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^3*sinh(f*x +
 e)^2 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5)*f^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{\left(b\tanh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*tanh(f*x + e) + a)^2, x)
```

maple [B] time = 0.85, size = 1352, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

```
[Out] 4/f/(a-b)/(a+b)^2*b*a*c*d/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x+4/f^2/
(a-b)/(a+b)^2*b*a*c*d/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e-4/3/(a-b)/
(a+b)^2*b/(-a+b)*a*d^2*x^3+8/3/f^3/(a-b)/(a+b)^2*b/(-a+b)*a*d^2*e^3+4/f^2/(
a-b)/(a+b)^2*b^2/(-a+b)*d^2*e*x-1/f^3/(a-b)/(a+b)^2*b*a*d^2/(-a+b)*polylog(
3,(a+b)*exp(2*f*x+2*e)/(-a+b))-2/f^3/(a-b)^2/(a+b)^2*b*a*d^2*e^2*ln(a*exp(2
*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+4/f^3/(a-b)^2/(a+b)^2*b*a*d^2*e^2*ln(exp(f*
x+e))-2/f^2/(a-b)/(a+b)^2*b^2*d^2/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*
x-2/f^3/(a-b)/(a+b)^2*b^2*d^2/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e+2/
f/(a-b)/(a+b)^2*b^2/(-a+b)*d^2*x^2+2/f^3/(a-b)/(a+b)^2*b^2/(-a+b)*d^2*e^2-2
/f/(a-b)^2/(a+b)^2*b*a*c^2*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)+4/f/(a
-b)^2/(a+b)^2*b*a*c^2*ln(exp(f*x+e))+2/f^2/(a-b)^2/(a+b)^2*b^2*c*d*ln(a*exp
(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)-4/f^2/(a-b)^2/(a+b)^2*b^2*c*d*ln(exp(f*x+
e))-1/f^3/(a-b)/(a+b)^2*b^2*d^2/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b
))-2/f^3/(a-b)^2/(a+b)^2*b^2*d^2*e*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b
)+4/f^3/(a-b)^2/(a+b)^2*b^2*d^2*e*ln(exp(f*x+e))+1/3/(a^2+2*a*b+b^2)*d^2*x^
3+1/(a^2+2*a*b+b^2)*c^2*x-8/f/(a-b)/(a+b)^2*b*a*c*d/(-a+b)*e*x-4/(a-b)/(a+b
)^2*b*a*c*d/(-a+b)*x^2+4/f^2/(a-b)/(a+b)^2*b/(-a+b)*a*d^2*e^2*x-4/f^2/(a-b)
/(a+b)^2*b*a*c*d/(-a+b)*e^2+4/f^2/(a-b)^2/(a+b)^2*b*a*c*d*e*ln(a*exp(2*f*x+
2*e)+b*exp(2*f*x+2*e)+a-b)+2/f^2/(a-b)/(a+b)^2*b*a*c*d/(-a+b)*polylog(2,(a+
b)*exp(2*f*x+2*e)/(-a+b))-8/f^2/(a-b)^2/(a+b)^2*b*a*c*d*e*ln(exp(f*x+e))-2/
f^3/(a-b)/(a+b)^2*b*a*d^2*e^2/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))+2/f/
(a-b)/(a+b)^2*b*a*d^2/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x^2+2/f^2/(a
-b)/(a+b)^2*b*a*d^2/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))*x+1/(a^2+
2*a*b+b^2)*c*d*x^2+2/(a-b)/f/(a^2+2*a*b+b^2)*(d^2*x^2+2*c*d*x+c^2)*b^2/(a*e
xp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)
```

maxima [A] time = 1.92, size = 753, normalized size = 1.58

$$-\frac{4\,b^2cdfx}{a^4f^2-2\,a^2b^2f^2+b^4f^2}-\frac{\left(2\,f^2x^2\log\left(\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}+1\right)+2\,fx\mathrm{Li}_2\left(-\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right)e^{(2\,fx)}}{a-b}\right)-\mathrm{Li}_3(-\frac{\left(ae^{(2\,e)}+be^{(2\,e)}\right.}{a-b}}{a^4f^3-2\,a^2b^2f^3+b^4f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -4*b^2*c*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - (2*f^2*x^2*log((a*e^(2
*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilog(-(a*e^(2*e) + b*e^(2*
e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a -
 b)))*a*b*d^2/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 2*b^2*c*d*log((a*e^(2*e
) + b*e^(2*e))*e^(2*f*x) + a - b)/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - c^2
*(2*a*b*log(-(a - b)*e^(-2*f*x - 2*e) - a - b)/((a^4 - 2*a^2*b^2 + b^4)*f)
+ 2*b^2/((a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*f*x
 - 2*e))*f) - (f*x + e)/((a^2 + 2*a*b + b^2)*f)) - (2*a*b*c*d*f - b^2*d^2)*
(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(-(a*e^(2*
e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 2
/3*(2*a*b*d^2*f^3*x^3 + 3*(2*a*b*c*d*f - b^2*d^2)*f^2*x^2)/(a^4*f^3 - 2*a^2
*b^2*f^3 + b^4*f^3) + 1/3*(12*b^2*c*d*x + (a^2*d^2*f - 2*a*b*d^2*f + b^2*d^
2*f)*x^3 + 3*(a^2*c*d*f - 2*a*b*c*d*f + (c*d*f + 2*d^2)*b^2)*x^2 + ((a^2*d^
2*f*e^(2*e) - b^2*d^2*f*e^(2*e))*x^3 + 3*(a^2*c*d*f*e^(2*e) - b^2*c*d*f*e^(
2*e))*x^2)*e^(2*f*x))/(a^4*f - 2*a^2*b^2*f + b^4*f + (a^4*f*e^(2*e) + 2*a^3
*b*f*e^(2*e) - 2*a*b^3*f*e^(2*e) - b^4*f*e^(2*e))*e^(2*f*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{\left(a+b\tanh\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*tanh(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^2/(a + b*tanh(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{\left(a+b\tanh\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*tanh(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)**2/(a + b*tanh(e + f*x))**2, x)
```

3.75 $\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=196

$$\frac{b(-2acf-2adfx+bd)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{f^2\left(a^2-b^2\right)^2}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}+\frac{abd\mathrm{Li}_2\left(-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2\left(a^2-b^2\right)^2}$$

```
[Out] -1/2*(d*x+c)^2/(a^2-b^2)/d+1/4*(-2*a*d*f*x-2*a*c*f+b*d)^2/a/(a-b)/(a+b)^2/d
/f^2+b*(-2*a*d*f*x-2*a*c*f+b*d)*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)^
2/f^2+a*b*d*polylog(2,(-a+b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)^2/f^2+b*(d*x+c
)/(a^2-b^2)/f/(a+b*tanh(f*x+e))
```

Rubi [A] time = 0.29, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3733, 3732, 2190, 2279, 2391}

$$\frac{abd\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{b(-2acf-2adfx+bd)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{f^2\left(a^2-b^2\right)^2}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] -(c + d*x)^2/(2*(a^2 - b^2)*d) + (b*d - 2*a*c*f - 2*a*d*f*x)^2/(4*a*(a - b)
*(a + b)^2*d*f^2) + (b*(b*d - 2*a*c*f - 2*a*d*f*x)*Log[1 + (a - b)/((a + b)
*E^(2*(e + f*x)))])/((a^2 - b^2)^2*f^2) + (a*b*d*PolyLog[2, -((a - b)/((a +
 b)*E^(2*(e + f*x))))])/((a^2 - b^2)^2*f^2) + (b*(c + d*x))/((a^2 - b^2)*f*
(a + b*Tanh[e + f*x]))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3732

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Dist[2*I*b, Int
[((c + d*x)^m*E^Simp[2*I*(e + f*x), x])/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2,
 0] && IGtQ[m, 0]
```

Rule 3733

```
Int[((c_.) + (d_.)*(x_))/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol
] :> -Simp[(c + d*x)^2/(2*d*(a^2 + b^2)), x] + (Dist[1/(f*(a^2 + b^2)), Int
[(b*d + 2*a*c*f + 2*a*d*f*x)/(a + b*Tan[e + f*x]), x], x] - Simp[(b*(c + d*
x))/(f*(a^2 + b^2)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = -\frac{(c+dx)^2}{2\left(a^2-b^2\right)d} + \frac{b(c+dx)}{\left(a^2-b^2\right)f(a+b\tanh(e+fx))} - \frac{i\int \frac{-ibd+2iacf+2iadfx}{a+b\tanh(e+fx)}\,dx}{\left(a^2-b^2\right)f}$$

$$= -\frac{(c+dx)^2}{2\left(a^2-b^2\right)d} + \frac{(bd-2acf-2adfx)^2}{4a(a-b)(a+b)^2df^2} + \frac{b(c+dx)}{\left(a^2-b^2\right)f(a+b\tanh(e+fx))} - \frac{(2ib)}{}$$

$$= -\frac{(c+dx)^2}{2\left(a^2-b^2\right)d} + \frac{(bd-2acf-2adfx)^2}{4a(a-b)(a+b)^2df^2} + \frac{b(bd-2acf-2adfx)\log\left(1+\frac{(a-b)e^{-2(e+}}{a+b}\right.}{\left(a^2-b^2\right)^2f^2}$$

$$= -\frac{(c+dx)^2}{2\left(a^2-b^2\right)d} + \frac{(bd-2acf-2adfx)^2}{4a(a-b)(a+b)^2df^2} + \frac{b(bd-2acf-2adfx)\log\left(1+\frac{(a-b)e^{-2(e+}}{a+b}\right.}{\left(a^2-b^2\right)^2f^2}$$

$$= -\frac{(c+dx)^2}{2\left(a^2-b^2\right)d} + \frac{(bd-2acf-2adfx)^2}{4a(a-b)(a+b)^2df^2} + \frac{b(bd-2acf-2adfx)\log\left(1+\frac{(a-b)e^{-2(e+}}{a+b}\right.}{\left(a^2-b^2\right)^2f^2}$$

Mathematica [C] time = 7.06, size = 476, normalized size = 2.43

$$\frac{\operatorname{sech}^2(e+fx)(a\cosh(e+fx)+b\sinh(e+fx))\left(2b^2f\left(b^2-a^2\right)(c+dx)\sinh(e+fx)-a\left(a^2-b^2\right)(e+fx)(d(e-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)/(a + b*Tanh[e + f*x])^2,x]
```

```
[Out] (Sech[e + f*x]^2*(a*Cosh[e + f*x] + b*Sinh[e + f*x])*(2*b^2*(-a^2 + b^2)*f*
(c + d*x)*Sinh[e + f*x] - a*(a^2 - b^2)*(e + f*x)*(-2*c*f + d*(e - f*x))*(a
*Cosh[e + f*x] + b*Sinh[e + f*x]) - 2*b^2*d*(b*(e + f*x) - a*Log[a*Cosh[e +
 f*x] + b*Sinh[e + f*x]])*(a*Cosh[e + f*x] + b*Sinh[e + f*x]) - 4*a*b*d*e*(
b*(e + f*x) - a*Log[a*Cosh[e + f*x] + b*Sinh[e + f*x]])*(a*Cosh[e + f*x] +
b*Sinh[e + f*x]) + 4*a*b*c*f*(b*(e + f*x) - a*Log[a*Cosh[e + f*x] + b*Sinh[
e + f*x]])*(a*Cosh[e + f*x] + b*Sinh[e + f*x]) + 2*a*b*d*((Sqrt[1 - a^2/b^2
]*b*(e + f*x)^2)/E^ArcTanh[a/b] - I*a*(e + f*x)*(Pi - (2*I)*ArcTanh[a/b]) +
 I*a*Pi*Log[1 + E^(2*(e + f*x))] - 2*a*(e + f*x + ArcTanh[a/b])*Log[1 - E^(
-2*(e + f*x + ArcTanh[a/b]))] - I*a*Pi*Log[Cosh[e + f*x]] + 2*a*ArcTanh[a/b
]*Log[I*Sinh[e + f*x + ArcTanh[a/b]]] + a*PolyLog[2, E^(-2*(e + f*x + ArcTa
nh[a/b]))])*(a*Cosh[e + f*x] + b*Sinh[e + f*x])))/(2*a*(a^2 - b^2)^2*f^2*(a
 + b*Tanh[e + f*x])^2)
```

fricas [B] time = 0.77, size = 1790, normalized size = 9.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^3 + a^2*b - a*b^2 - b^3)*d*f^2*x^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)*
c*f^2*x - 4*(a^2*b - a*b^2)*d*e^2 - 4*(a*b^2 - b^3)*d*e + ((a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*f^2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*c*e
*f - 4*(a*b^2 + b^3)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2*(a*
b^2 + b^3)*d*f)*x)*cosh(f*x + e)^2 + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*f
^2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*c*e*f - 4*(a*b^2 + b^3
)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2*(a*b^2 + b^3)*d*f)*x)*
cosh(f*x + e)*sinh(f*x + e) + ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*f^2*x^2 -
4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*c*e*f - 4*(a*b^2 + b^3)*d*e + 2
*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2*(a*b^2 + b^3)*d*f)*x)*sinh(f*x
+ e)^2 + 4*(2*(a^2*b - a*b^2)*c*e + (a*b^2 - b^3)*c)*f - 4*((a^2*b + a*b^2)
*d*cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d*cosh(f*x + e)*sinh(f*x + e) + (a^2
*b + a*b^2)*d*sinh(f*x + e)^2 + (a^2*b - a*b^2)*d)*dilog(sqrt(-(a + b)/(a -
 b))*(cosh(f*x + e) + sinh(f*x + e))) - 4*((a^2*b + a*b^2)*d*cosh(f*x + e)^
2 + 2*(a^2*b + a*b^2)*d*cosh(f*x + e)*sinh(f*x + e) + (a^2*b + a*b^2)*d*sin
h(f*x + e)^2 + (a^2*b - a*b^2)*d)*dilog(-sqrt(-(a + b)/(a - b))*(cosh(f*x +
 e) + sinh(f*x + e))) + 2*(2*(a^2*b - a*b^2)*d*e - 2*(a^2*b - a*b^2)*c*f +
(2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*c*f + (a*b^2 + b^3)*d)*cosh(f*x
+ e)^2 + 2*(2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*c*f + (a*b^2 + b^3)*d
)*cosh(f*x + e)*sinh(f*x + e) + (2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*
c*f + (a*b^2 + b^3)*d)*sinh(f*x + e)^2 + (a*b^2 - b^3)*d)*log(2*(a + b)*cos
h(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt(-(a + b)/(a - b))) +
2*(2*(a^2*b - a*b^2)*d*e - 2*(a^2*b - a*b^2)*c*f + (2*(a^2*b + a*b^2)*d*e -
 2*(a^2*b + a*b^2)*c*f + (a*b^2 + b^3)*d)*cosh(f*x + e)^2 + 2*(2*(a^2*b + a
*b^2)*d*e - 2*(a^2*b + a*b^2)*c*f + (a*b^2 + b^3)*d)*cosh(f*x + e)*sinh(f*x
 + e) + (2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*c*f + (a*b^2 + b^3)*d)*s
inh(f*x + e)^2 + (a*b^2 - b^3)*d)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*s
inh(f*x + e) - 2*(a - b)*sqrt(-(a + b)/(a - b))) - 4*((a^2*b - a*b^2)*d*f*x
 + (a^2*b - a*b^2)*d*e + ((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^2)*d*e)*cosh
(f*x + e)^2 + 2*((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^2)*d*e)*cosh(f*x + e)
*sinh(f*x + e) + ((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^2)*d*e)*sinh(f*x + e
)^2)*log(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 4*((
a^2*b - a*b^2)*d*f*x + (a^2*b - a*b^2)*d*e + ((a^2*b + a*b^2)*d*f*x + (a^2*
b + a*b^2)*d*e)*cosh(f*x + e)^2 + 2*((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^2
)*d*e)*cosh(f*x + e)*sinh(f*x + e) + ((a^2*b + a*b^2)*d*f*x + (a^2*b + a*b^
2)*d*e)*sinh(f*x + e)^2)*log(-sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(
f*x + e)) + 1))/((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^2*co
sh(f*x + e)^2 + 2*(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*f^2*c
osh(f*x + e)*sinh(f*x + e) + (a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 +
 b^5)*f^2*sinh(f*x + e)^2 + (a^5 - a^4*b - 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 -
b^5)*f^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx + c}{\left(b \tanh\left(fx + e\right) + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*tanh(f*x + e) + a)^2, x)
```

maple [B] time = 0.78, size = 565, normalized size = 2.88

$$\frac{d\,x^2}{2a^2 + 4ab + 2b^2} + \frac{cx}{a^2 + 2ab + b^2} + \frac{2b^2\,(dx + c)}{(a - b)\,f\left(a^2 + 2ab + b^2\right)\left(a\,\mathrm{e}^{2fx+2e} + b\,\mathrm{e}^{2fx+2e} + a - b\right)} - \frac{2b^2 d \ln\left(\mathrm{e}^{fx+e}\right)}{f^2\,(a - b)^2\,(a + b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

```
[Out] 1/2/(a^2+2*a*b+b^2)*d*x^2+1/(a^2+2*a*b+b^2)*c*x+2/(a-b)/f/(a^2+2*a*b+b^2)*b
^2*(d*x+c)/(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)-2/f^2/(a-b)^2/(a+b)^2*b^
2*d*ln(exp(f*x+e))+1/f^2/(a-b)^2/(a+b)^2*b^2*d*ln(a*exp(2*f*x+2*e)+b*exp(2*
f*x+2*e)+a-b)+4/f/(a-b)^2/(a+b)^2*b*a*c*ln(exp(f*x+e))-2/f/(a-b)^2/(a+b)^2*
b*a*c*ln(a*exp(2*f*x+2*e)+b*exp(2*f*x+2*e)+a-b)-4/f^2/(a-b)^2/(a+b)^2*b*d*a
*e*ln(exp(f*x+e))+2/f^2/(a-b)^2/(a+b)^2*b*d*a*e*ln(a*exp(2*f*x+2*e)+b*exp(2
*f*x+2*e)+a-b)+2/f/(a-b)/(a+b)^2*b*d*a/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a
+b))*x+2/f^2/(a-b)/(a+b)^2*b*d*a/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e
-2/(a-b)/(a+b)^2*b*d*a/(-a+b)*x^2-4/f/(a-b)/(a+b)^2*b*d*a/(-a+b)*e*x-2/f^2/
(a-b)/(a+b)^2*b*d*a/(-a+b)*e^2+1/f^2/(a-b)/(a+b)^2*b*d*a/(-a+b)*polylog(2,(
a+b)*exp(2*f*x+2*e)/(-a+b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(8\,abf\int\frac{x}{a^4fe^{\left(2fx+2e\right)}+2\,a^3bfe^{\left(2fx+2e\right)}-2\,ab^3fe^{\left(2fx+2e\right)}-b^4fe^{\left(2fx+2e\right)}+a^4f-2\,a^2b^2f+b^4f}\,dx-2\,b^2\left(\frac{}{\left(a^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a*b*f*integrate(x/(a^4*f*e^(2*f*x + 2*e) + 2*a^3*b*f*e^(2*f*x + 2*e)
 - 2*a*b^3*f*e^(2*f*x + 2*e) - b^4*f*e^(2*f*x + 2*e) + a^4*f - 2*a^2*b^2*f
+ b^4*f), x) - 2*b^2*(2*(f*x + e)/((a^4 - 2*a^2*b^2 + b^4)*f^2) - log((a +
b)*e^(2*f*x + 2*e) + a - b)/((a^4 - 2*a^2*b^2 + b^4)*f^2)) + ((a^2*f*e^(2*e
) - b^2*f*e^(2*e))*x^2*e^(2*f*x) + 4*b^2*x + (a^2*f - 2*a*b*f + b^2*f)*x^2)
/(a^4*f - 2*a^2*b^2*f + b^4*f + (a^4*f*e^(2*e) + 2*a^3*b*f*e^(2*e) - 2*a*b^
3*f*e^(2*e) - b^4*f*e^(2*e))*e^(2*f*x)))*d - c*(2*a*b*log(-(a - b)*e^(-2*f*
x - 2*e) - a - b)/((a^4 - 2*a^2*b^2 + b^4)*f) + 2*b^2/((a^4 - 2*a^2*b^2 + b
^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*f*x - 2*e))*f) - (f*x + e)/((a^2
 + 2*a*b + b^2)*f))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{c+d\,x}{\left(a+b\tanh\left(e+f\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*tanh(e + f*x))^2,x)
```

```
[Out] int((c + d*x)/(a + b*tanh(e + f*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{c+dx}{\left(a+b\tanh\left(e+fx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*tanh(f*x+e))**2,x)
```

```
[Out] Integral((c + d*x)/(a + b*tanh(e + f*x))**2, x)
```

3.76 $\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\tanh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Tanh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)*(a + b*Tanh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$$

Mathematica [A] time = 156.93, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])^2), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2dx + a^2c + \left(b^2dx + b^2c\right)\tanh\left(fx+e\right)^2 + 2\left(abdx + abc\right)\tanh\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*tanh(f*x + e)^2 + 2*(a*b*d*
x + a*b*c)*tanh(f*x + e)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(b\tanh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*tanh(f*x + e) + a)^2), x)
```

maple [A] time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+b\tanh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2\,b^2}{a^4cf - 2\,a^2b^2cf + b^4cf + \left(a^4df - 2\,a^2b^2df + b^4df\right)x + \left(a^4cfe^{(2\,e)} + 2\,a^3bcfe^{(2\,e)} - 2\,ab^3cfe^{(2\,e)} - b^4cfe^{(2\,e)} + \left(a^4d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*b^2/(a^4*c*f - 2*a^2*b^2*c*f + b^4*c*f + (a^4*d*f - 2*a^2*b^2*d*f + b^4*d
*f)*x + (a^4*c*f*e^(2*e) + 2*a^3*b*c*f*e^(2*e) - 2*a*b^3*c*f*e^(2*e) - b^4*
c*f*e^(2*e) + (a^4*d*f*e^(2*e) + 2*a^3*b*d*f*e^(2*e) - 2*a*b^3*d*f*e^(2*e)
- b^4*d*f*e^(2*e))*x)*e^(2*f*x)) + log(d*x + c)/(a^2*d + 2*a*b*d + b^2*d) +
 integrate(2*(2*a*b*d*f*x + 2*a*b*c*f + b^2*d)/(a^4*c^2*f - 2*a^2*b^2*c^2*f
 + b^4*c^2*f + (a^4*d^2*f - 2*a^2*b^2*d^2*f + b^4*d^2*f)*x^2 + 2*(a^4*c*d*f
 - 2*a^2*b^2*c*d*f + b^4*c*d*f)*x + (a^4*c^2*f*e^(2*e) + 2*a^3*b*c^2*f*e^(2
*e) - 2*a*b^3*c^2*f*e^(2*e) - b^4*c^2*f*e^(2*e) + (a^4*d^2*f*e^(2*e) + 2*a^
3*b*d^2*f*e^(2*e) - 2*a*b^3*d^2*f*e^(2*e) - b^4*d^2*f*e^(2*e))*x^2 + 2*(a^4
*c*d*f*e^(2*e) + 2*a^3*b*c*d*f*e^(2*e) - 2*a*b^3*c*d*f*e^(2*e) - b^4*c*d*f*
e^(2*e))*x)*e^(2*f*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\tanh\left(e+f\,x\right)\right)^2\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tanh(e + f*x))^2*(c + d*x)),x)
```

```
[Out] int(1/((a + b*tanh(e + f*x))^2*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tanh\left(e+fx\right)\right)^2\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*tanh(f*x+e))**2,x)
```

```
[Out] Integral(1/((a + b*tanh(e + f*x))**2*(c + d*x)), x)
```

3.77 $\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$$

Mathematica [A] time = 128.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2d^2x^2 + 2\,a^2cdx + a^2c^2 + \left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\tanh\left(fx+e\right)^2 + 2\left(abd^2x^2 + 2\,abcdx + abc^2\right)\text{t}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*tanh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*tanh
(f*x + e)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(b\tanh\left(fx+e\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*tanh(f*x + e) + a)^2), x)
```

maple [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(a+b\tanh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2cf-2\,abcf+\left(cf}{a^4c^2df-2\,a^2b^2c^2df+b^4c^2df+\left(a^4d^3f-2\,a^2b^2d^3f+b^4d^3f\right)x^2+2\left(a^4cd^2f-2\,a^2b^2cd^2f+b^4cd^2f\right)x+\left(a^4c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -(a^2*c*f - 2*a*b*c*f + (c*f - 2*d)*b^2 + (a^2*d*f - 2*a*b*d*f + b^2*d*f)*x
 + (a^2*c*f*e^(2*e) - b^2*c*f*e^(2*e) + (a^2*d*f*e^(2*e) - b^2*d*f*e^(2*e))
*x)*e^(2*f*x))/(a^4*c^2*d*f - 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f + (a^4*d^3*f
- 2*a^2*b^2*d^3*f + b^4*d^3*f)*x^2 + 2*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f + b
^4*c*d^2*f)*x + (a^4*c^2*d*f*e^(2*e) + 2*a^3*b*c^2*d*f*e^(2*e) - 2*a*b^3*c^
2*d*f*e^(2*e) - b^4*c^2*d*f*e^(2*e) + (a^4*d^3*f*e^(2*e) + 2*a^3*b*d^3*f*e^
(2*e) - 2*a*b^3*d^3*f*e^(2*e) - b^4*d^3*f*e^(2*e))*x^2 + 2*(a^4*c*d^2*f*e^(
2*e) + 2*a^3*b*c*d^2*f*e^(2*e) - 2*a*b^3*c*d^2*f*e^(2*e) - b^4*c*d^2*f*e^(2
*e))*x)*e^(2*f*x)) + integrate(4*(a*b*d*f*x + a*b*c*f + b^2*d)/(a^4*c^3*f -
 2*a^2*b^2*c^3*f + b^4*c^3*f + (a^4*d^3*f - 2*a^2*b^2*d^3*f + b^4*d^3*f)*x^
3 + 3*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f + b^4*c*d^2*f)*x^2 + 3*(a^4*c^2*d*f
- 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f)*x + (a^4*c^3*f*e^(2*e) + 2*a^3*b*c^3*f*e
^(2*e) - 2*a*b^3*c^3*f*e^(2*e) - b^4*c^3*f*e^(2*e) + (a^4*d^3*f*e^(2*e) + 2
*a^3*b*d^3*f*e^(2*e) - 2*a*b^3*d^3*f*e^(2*e) - b^4*d^3*f*e^(2*e))*x^3 + 3*(
a^4*c*d^2*f*e^(2*e) + 2*a^3*b*c*d^2*f*e^(2*e) - 2*a*b^3*c*d^2*f*e^(2*e) - b
^4*c*d^2*f*e^(2*e))*x^2 + 3*(a^4*c^2*d*f*e^(2*e) + 2*a^3*b*c^2*d*f*e^(2*e)
- 2*a*b^3*c^2*d*f*e^(2*e) - b^4*c^2*d*f*e^(2*e))*x)*e^(2*f*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\tanh\left(e+f\,x\right)\right)^2(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*tanh(e + f*x))^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*tanh(e + f*x))^2*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\tanh\left(e+fx\right)\right)^2(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*tanh(f*x+e))**2,x)
```

```
[Out] Integral(1/((a + b*tanh(e + f*x))**2*(c + d*x)**2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#               Port of original Maple grading function by
#               Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#               added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```